

ROTATION OF A RIGID BODY

Conceptual Questions

12.1. As suggested by the figure, we will assume that the larger sphere is more massive. Then the center of gravity would be at point a because if we suspend the dumbbell from point a then the counterclockwise torque due to the large sphere (large weight times small lever arm) will be equal to the clockwise torque due to the small sphere (small weight times large lever arm).

Look at the figure and mentally balance the dumbbell on your finger; your finger would have to be at point a. The sun-earth system is similar to this except that the sun's mass is so much greater than the earth's that the center of mass (called the barycenter for astronomical objects orbiting each other) is only 450 km from the center of the sun.

12.2. To double the rotational energy without changing ω requires doubling the moment of inertia. The moment of inertia is proportional to R^2 so R must increase by $\sqrt{2}$.

12.3. The rotational kinetic energy is $K_{\text{rot}} = \frac{1}{2}I\omega^2$. For a disk, $I = \frac{1}{2}MR^2$. Since the mass is the same for all three disks, the quantity $R^2\omega^2$ determines the ranking. Thus $K_a = K_b > K_c$.

12.4. No. The moment of inertia does not have any dependence on a quantity that indicates an object is rotating, such as ω or α , so an object does not have to be rotating to have a moment of inertia.

12.5. Mass that is farther away from the axis of rotation contributes more to the moment of inertia $I = \int r^2 dm$.

Here, r is the distance from the axis of rotation to the mass element dm . Note r^2 is always positive. For a rod, there is more mass farther away from an axis through the rod's end than one through its middle.

12.6. Because sphere 2 has twice the radius, its mass is greater by a factor of $2^3 = 8$, since $m = \frac{4}{3}\pi r^3 \rho_{\text{steel}}$. The added mass is also distributed farther from the center, so, $I \propto mr^2$ leads to $I_2 \propto (8m_1)(2r_1)^2 = 32I_1$.

12.7. It will be easier to rotate the solid sphere because the hollow sphere's mass is generally distributed farther from its center. If you roll both simultaneously down an incline, the solid sphere will win.

12.8. $\tau_e > \tau_a = \tau_b > \tau_c = \tau_d > \tau_f$ The torque $\tau = rF|\sin \theta|$. We must calculate each torque:

$$\tau_a = \left(\frac{L}{2}\right)F \quad \tau_d = \frac{L}{2}F \sin 45^\circ = \frac{\sqrt{2}}{4}LF$$

$$\tau_b = \left(\frac{L}{4}\right)F \quad \tau_e = L(2F)$$

$$\tau_c = \left(\frac{L}{2}\right)F \sin 45^\circ = \frac{\sqrt{2}}{4}LF \quad \tau_f = LF \sin 0^\circ = 0$$

12.9. (a) Negative, since it causes the ball to rotate clockwise.

(b) The angular velocity holds steady (iii) after the push has ended, since the torque has ended, and there is no more angular acceleration.

(c) The torque is zero after the push has ended because there is no longer an applied force.

12.10. Since $\tau = I\alpha$, $\alpha = \frac{\tau}{I}$. Also, $\tau = Fr$ and $I = mr^2 \Rightarrow \alpha = \frac{F}{mr}$. Calculate α for each case:

$$\alpha_a = \frac{F_0}{m_0 r_0} \quad \alpha_c = \frac{F_0}{m_0(2r_0)} = \frac{1}{2}\alpha_a$$

$$\alpha_b = \frac{2F_0}{2m_0 r_0} = \alpha_a \quad \alpha_d = \frac{2F_0}{(2m_0)(2r_0)} = \frac{1}{2}\alpha_a$$

So $\alpha_a = \alpha_b > \alpha_c = \alpha_d$.

12.11. The block attached to the solid cylinder hits first. The solid cylinder has a smaller moment of inertia since more of its mass is closer to the rotation axis, so it has less resistance to a change in its rotational motion. The torque applied by the string attached to the block makes the solid cylinder change its rotation and unwind the string faster.

12.12. The moment of inertia for the tuck position is smaller than that of the pike position. Since the angular momentum of the diver is conserved, any initial angular velocity is increased more when the diver moves to the tuck position relative to the pike position.

12.13. The angular momentum L of disk b is larger than the angular momentum of disk a. Calculate L for each:

$$L_a = I_a \omega_a = \frac{1}{2}mr_a^2 \omega_a$$

$$L_b = I_b \omega_b = \frac{1}{2}m(2r_a^2)^2 \left(\frac{1}{2}\omega_a\right) = 2\left(\frac{1}{2}mr_a^2 \omega_a\right) = 2L_a$$

Exercises and Problems

Exercises

Section 12.1 Rotational Motion

12.1. Model: Assume constant angular acceleration.

Solve: (a) The final angular velocity is $\omega_f = (2000 \text{ rpm})\left(\frac{2\pi \text{ rad}}{\text{rev}}\right)\left(\frac{\text{min}}{60 \text{ s}}\right) = 209.4 \text{ rad/s}$. The definition of angular acceleration gives us

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{\omega_f - \omega_i}{\Delta t} = \frac{209.4 \text{ rad/s} - 0 \text{ rad/s}}{0.50 \text{ s}} = 419 \text{ rad/s}^2.$$

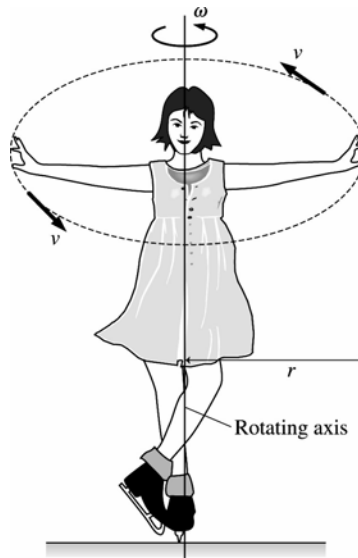
The angular acceleration of the drill is $4.2 \times 10^2 \text{ rad/s}^2$.

$$(b) \theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2 = 0 \text{ rad} + 0 \text{ rad} + \frac{1}{2} (419 \text{ rad/s}^2) (0.50 \text{ s})^2 = 52.4 \text{ rad}$$

The drill makes $(52.4 \text{ rad}) \left(\frac{\text{rev}}{2\pi \text{ rad}} \right) = 8.3 \text{ revolutions}$.

12.2. Model: A spinning skater, whose arms are outstretched, is a rigid rotating body.

Visualize:



Solve: The speed $v = r\omega$, where $r = 140 \text{ cm}/2 = 0.70 \text{ m}$. Also, $180 \text{ rpm} = (180)2\pi/60 \text{ rad/s} = 6\pi \text{ rad/s}$. Thus, $v = (0.70 \text{ m})(6\pi \text{ rad/s}) = 13.2 \text{ m/s}$.

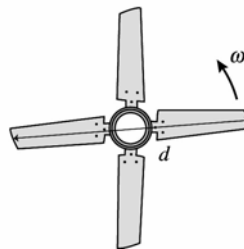
Assess: A speed of $13.2 \text{ m/s} \approx 26 \text{ mph}$ for the hands is a little high, but reasonable.

12.3. Model: Assume constant angular acceleration.

Visualize:

Pictorial representation

Known
 $d = 80 \text{ cm}$
 $w_i = 60 \text{ rpm}$
 $w_f = 0 \text{ rpm}$
 $\Delta t = 25 \text{ s}$



Solve: The initial angular velocity is $\omega_i = (60 \text{ rpm}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) = 2\pi \text{ rad/s}$.

The angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 \text{ rad/s} - 2\pi \text{ rad/s}}{25 \text{ s}} = -0.251 \text{ rad/s}^2$$

The angular velocity of the fan blade after 10 s is

$$\omega_f = \omega_i + \alpha(t - t_0) = 2\pi \text{ rad/s} + (-0.251 \text{ rad/s}^2)(10 \text{ s} - 0 \text{ s}) = 3.77 \text{ rad/s}$$

The tangential speed of the tip of the fan blade is

$$v_t = r\omega = (0.40 \text{ m})(3.77 \text{ rad/s}) = 1.5 \text{ m/s}$$

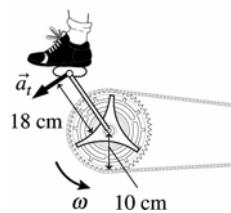
$$(b) \theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2 = 0 \text{ rad} + (2\pi \text{ rad/s})(25 \text{ s}) + \frac{1}{2} (-0.251 \text{ rad/s}^2)(25 \text{ s})^2 = 78.64 \text{ rad}$$

The fan turns $78.64 \text{ rad} = 12.5 \text{ rev} \approx 13 \text{ rev}$ while coming to a stop.

12.4. Model: Assume constant angular acceleration.

Visualize:

Pictorial representation



Known

$$w_i = 60 \text{ rpm}$$

$$w_f = 90 \text{ rpm}$$

$$\Delta t = 10 \text{ s}$$

Find

$$a_t, \text{ length of chain}$$

Solve: (a) Since $a_t = r\alpha$, find α first. With $90 \text{ rpm} = 9.43 \text{ rad/s}$ and $60 \text{ rpm} = 6.28 \text{ rad/s}$,

$$\alpha = \frac{\Delta \omega}{\Delta t} = \frac{9.43 \text{ rad} - 6.28 \text{ rad/s}}{10 \text{ s}} = 0.314 \text{ rad/s}^2$$

The angular acceleration of the sprocket and pedal are the same. So

$$a_t = r\alpha = (0.18 \text{ m})(0.314 \text{ rad/s}^2) = 0.057 \text{ m/s}^2$$

(b) The length of chain that passes over the sprocket during this time is $L = r\Delta\theta$. Find $\Delta\theta$:

$$\theta_f = \theta_i + \omega_i \Delta t + \frac{1}{2} \alpha (\Delta t)^2$$

$$\theta_f - \theta_i = \Delta\theta = (6.28 \text{ rad/s})(10 \text{ s}) + \frac{1}{2} (0.314 \text{ rad/s}^2)(10 \text{ s})^2 = 78.5 \text{ rad}$$

The length of chain which has passed over the top of the sprocket is

$$L = (0.10 \text{ m})(78.5 \text{ rad}) = 7.9 \text{ m}$$

Section 12.2 Rotation About the Center of Mass

12.5. Model: The earth and moon are particles.

Visualize:

Known

$$x_E = 0 \text{ m}$$

$$x_M = 13.84 \times 10^8 \text{ m}$$

$$m_E = 5.98 \times 10^{24} \text{ kg}$$

$$m_M = 7.36 \times 10^{22} \text{ kg}$$



Choosing $x_E = 0 \text{ m}$ sets the coordinate origin at the center of the earth so that the center of mass location is the distance from the center of the earth.

Solve:

$$x_{\text{cm}} = \frac{m_E x_E + m_M x_M}{m_E + m_M} = \frac{(5.98 \times 10^{24} \text{ kg})(0 \text{ m}) + (7.36 \times 10^{22} \text{ kg})(3.84 \times 10^8 \text{ m})}{5.98 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}}$$

$$= 4.67 \times 10^6 \text{ m} \approx 4.7 \times 10^6 \text{ m}$$

Assess: The center of mass of the earth-moon system is called the barycenter and is located beneath the surface of the earth. Even though $x_E = 0 \text{ m}$ the earth influences the center of mass location because m_E is in the denominator of the expression for x_{cm} .

12.6. Visualize: The coordinates of the three masses m_A , m_B , and m_C are (0 cm, 0 cm), (0 cm, 10 cm), and (10 cm, 0 cm), respectively.

Solve: The coordinates of the center of mass are

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(300 \text{ g})(0 \text{ cm}) + (200 \text{ g})(0 \text{ cm}) + (100 \text{ g})(10 \text{ cm})}{(300 \text{ g} + 200 \text{ g} + 100 \text{ g})} = 1.7 \text{ cm}$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(300 \text{ g})(0 \text{ cm}) + (200 \text{ g})(10 \text{ cm}) + (100 \text{ g})(0 \text{ cm})}{(300 \text{ g} + 200 \text{ g} + 100 \text{ g})} = 3.3 \text{ cm}$$

12.7. The coordinates of the three masses m_A , m_B , and m_C are (0 cm, 10 cm), (10 cm, 10 cm), and (10 cm, 0 cm), respectively.

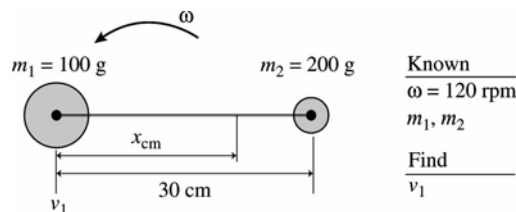
Solve: The coordinates of the center of mass are

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(200 \text{ g})(0 \text{ cm}) + (300 \text{ g})(12 \text{ cm}) + (100 \text{ g})(12 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 8.0 \text{ cm}$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(200 \text{ g})(0 \text{ cm}) + (300 \text{ g})(10 \text{ cm}) + (100 \text{ g})(0 \text{ cm})}{(200 \text{ g} + 300 \text{ g} + 100 \text{ g})} = 5.0 \text{ cm}$$

12.8. Model: The balls are particles located at the ball's respective centers.

Visualize:



Solve: The center of mass of the two balls measured from the left hand ball is

$$x_{\text{cm}} = \frac{(100 \text{ g})(0 \text{ cm}) + (200 \text{ g})(30 \text{ cm})}{100 \text{ g} + 200 \text{ g}} = 20 \text{ cm}$$

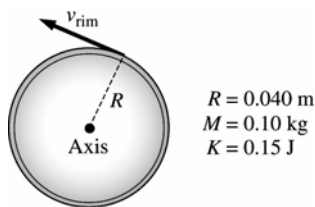
The linear speed of the 100 g ball is

$$v_1 = r\omega = x_{\text{cm}}\omega = (0.20 \text{ m})(120 \text{ rev/min}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) = 2.5 \text{ m/s}$$

Section 12.3 Rotational Energy

12.9. Model: The disk is a rigid body rotating about an axis through its center.

Visualize:



Solve: The speed of the point on the rim is given by $v_{\text{rim}} = R\omega$. The angular velocity ω of the disk can be determined from its rotational kinetic energy which is $K = \frac{1}{2}I\omega^2 = 0.15 \text{ J}$. The moment of inertia I of the disk about its center and perpendicular to the plane of the disk is given by

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(0.10 \text{ kg})(0.040 \text{ m})^2 = 8.0 \times 10^{-5} \text{ kg m}^2$$

$$\Rightarrow \omega^2 = \frac{2(0.15 \text{ J})}{I} = \frac{0.30 \text{ J}}{8.0 \times 10^{-5} \text{ kg m}^2} \Rightarrow \omega = 61.237 \text{ rad/s}$$

Now, we can go back to the first equation to find v_{rim} . We get $v_{\text{rim}} = R\omega = (0.040 \text{ m})(61.237 \text{ rad/s}) = 2.4 \text{ m/s}$.

12.10. Model: The earth is a rigid, spherical rotating body.

Solve: The rotational kinetic energy of the earth is $K_{\text{rot}} = \frac{1}{2}I\omega^2$. The moment of inertia of a sphere about its diameter (see Table 12.2) is $I = \frac{2}{5}M_{\text{earth}}R^2$ and the angular velocity of the earth is

$$\omega = \frac{2\pi \text{ rad}}{24 \times 3600 \text{ s}} = 7.27 \times 10^{-5} \text{ rad/s}$$

Thus, the rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2} \left(\frac{2}{5} M_{\text{earth}} R^2 \right) \omega^2$$

$$= \frac{1}{5} (5.98 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 (7.27 \times 10^{-5} \text{ rad/s})^2 = 2.57 \times 10^{29} \text{ J}$$

12.11. Model: The triangle is a rigid body rotating about an axis through the center.

Visualize: Please refer to Figure EX12.11. Each 200 g mass is a distance r away from the axis of rotation, where r is given by

$$\frac{0.20 \text{ m}}{r} = \cos 30^\circ \Rightarrow r = \frac{0.20 \text{ m}}{\cos 30^\circ} = 0.2309 \text{ m}$$

Solve: (a) The moment of inertia of the triangle is $I = 3 \times mr^2 = 3(0.200 \text{ kg})(0.2309 \text{ m})^2 = 0.032 \text{ kg m}^2$.

(b) The frequency of rotation is given as 5.0 revolutions per s or $10\pi \text{ rad/s}$. The rotational kinetic energy is

$$K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(0.0320 \text{ kg m}^2)(10.0\pi \text{ rad/s})^2 = 15.8 \text{ J} = 16 \text{ J}$$

12.12. Model: The baton is a thin rod rotating about a perpendicular axis through its center of mass.

Solve: The moment of inertia of a thin rod rotating about its center is $I = \frac{1}{12}ML^2$. For the baton,

$$I = \frac{1}{12}(0.400 \text{ kg})(0.96 \text{ m})^2 = 0.031 \text{ kg m}^2$$

The rotational kinetic energy of the baton is

$$K_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} (0.031 \text{ kg} \cdot \text{m}^2) \left((100 \text{ rev/min}) \left(\frac{2\pi \text{ rad}}{\text{rev}} \right) \left(\frac{\text{min}}{60 \text{ s}} \right) \right)^2 = 1.68 \text{ J} \approx 1.7 \text{ J}$$

Section 12.4 Calculating Moment of Inertia

12.13. Model: The moment of inertia of any object depends on the axis of rotation. In the present case, the rotation axis passes through mass A and is perpendicular to the page.

Solve: (a) $x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0 \text{ m}) + (300 \text{ g})(0.10 \text{ m}) + (200 \text{ g})(0.10 \text{ m})}{100 \text{ g} + 200 \text{ g} + 300 \text{ g} + 200 \text{ g}} = 0.063 \text{ m}$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0.08 \text{ m}) + (300 \text{ g})(0.08 \text{ m}) + (200 \text{ g})(0 \text{ m})}{800 \text{ g}} = 0.050 \text{ m}$$

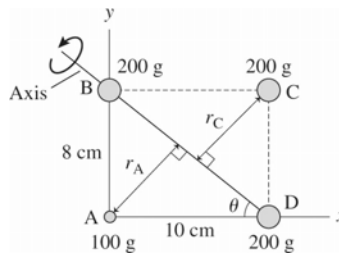
(b) The distance from the axis to mass C is 12.81 cm. The moment of inertia through A and perpendicular to the page is

$$I_A = \sum_i m_i r_i^2 = m_A r_A^2 + m_B r_B^2 + m_C r_C^2 + m_D r_D^2$$

$$= (0.100 \text{ kg})(0 \text{ m})^2 + (0.200 \text{ kg})(0.08 \text{ m})^2 + (0.300 \text{ kg})(0.1281 \text{ m})^2 + (0.200 \text{ kg})(0.10 \text{ m})^2 = 0.0082 \text{ kg} \cdot \text{m}^2$$

12.14. Model: The moment of inertia of any object depends on the axis of rotation.

Visualize:



Solve: (a) $x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m_A x_A + m_B x_B + m_C x_C + m_D x_D}{m_A + m_B + m_C + m_D}$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0 \text{ m}) + (200 \text{ g})(0.10 \text{ m}) + (200 \text{ g})(0.10 \text{ m})}{100 \text{ g} + 200 \text{ g} + 200 \text{ g} + 200 \text{ g}} = 0.057 \text{ m}$$

$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C + m_D y_D}{m_A + m_B + m_C + m_D}$$

$$= \frac{(100 \text{ g})(0 \text{ m}) + (200 \text{ g})(0.08 \text{ m}) + (200 \text{ g})(0.08 \text{ m}) + (200 \text{ g})(0 \text{ m})}{700 \text{ g}} = 0.046 \text{ m}$$

(b) The moment of inertia about a diagonal that passes through B and D is

$$I_{BD} = m_A r_A^2 + m_C r_C^2$$

where we must compute $r_A = r_C$ which are the distances from the diagonal. From triangle ABD we see that

$$\theta = \tan^{-1} \left(\frac{8}{10} \right) = 38.66^\circ. \text{ Now } r_A = (0.10 \text{ m}) \sin 38.66^\circ = 0.06247 \text{ m. Thus,}$$

$$I_{BD} = (0.100 \text{ kg})(0.06247)^2 + (0.200 \text{ kg})(0.06247)^2 = 0.0012 \text{ kg} \cdot \text{m}^2$$

Assess: Note that the masses B and D, being on the axis of rotation, do not contribute to the moment of inertia.

12.15. Model: The three masses connected by massless rigid rods are a rigid body.

Solve: (a) $x_{\text{cm}} = \frac{\sum m_i x_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg})(0.06 \text{ m}) + (0.100 \text{ kg})(0.12 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.060 \text{ m}$

$$y_{\text{cm}} = \frac{\sum m_i y_i}{\sum m_i} = \frac{(0.100 \text{ kg})(0 \text{ m}) + (0.200 \text{ kg})\left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2}\right) + (0.100 \text{ kg})(0 \text{ m})}{0.100 \text{ kg} + 0.200 \text{ kg} + 0.100 \text{ kg}} = 0.040 \text{ m}$$

(b) The moment of inertia about an axis through A and perpendicular to the page is

$$I_A = \sum m_i r_i^2 = m_B (0.10 \text{ m})^2 + m_C (0.10 \text{ m})^2 = (0.100 \text{ kg})[(0.10 \text{ m})^2 + (0.10 \text{ m})^2] = 0.0020 \text{ kg m}^2$$

(c) The moment of inertia about an axis that passes through B and C is

$$I_{BC} = m_A \left(\sqrt{(0.10 \text{ m})^2 - (0.06 \text{ m})^2} \right)^2 = 0.00128 \text{ kg m}^2 \approx 0.0013 \text{ kg m}^2$$

Assess: Note that mass m_A does not contribute to I_A , and the masses m_B and m_C do not contribute to I_{BC} .

12.16. Model: The CD is a disk of uniform density.

Solve: (a) The center of the CD is its center of mass. Using Table 12.2,

$$I_{\text{cm}} = \frac{1}{2} MR^2 = \frac{1}{2} (0.021 \text{ kg})(0.060 \text{ m})^2 = 3.8 \times 10^{-5} \text{ kg m}^2$$

(b) Using the parallel-axis theorem with $d = 0.060 \text{ m}$,

$$I = I_{\text{cm}} + Md^2 = 3.8 \times 10^{-5} \text{ kg m}^2 + (0.021 \text{ kg})(0.060 \text{ m})^2 = 1.14 \times 10^{-4} \text{ kg m}^2$$

12.17. Model: The door is a slab of uniform density.

Solve: (a) The hinges are at the edge of the door, so from Table 12.2,

$$I = \frac{1}{3} (25 \text{ kg})(0.91 \text{ m})^2 = 6.9 \text{ kg m}^2$$

(b) The distance from the axis through the center of mass along the height of the door is $d = \left(\frac{0.91 \text{ m}}{2} - 0.15 \text{ m} \right) =$

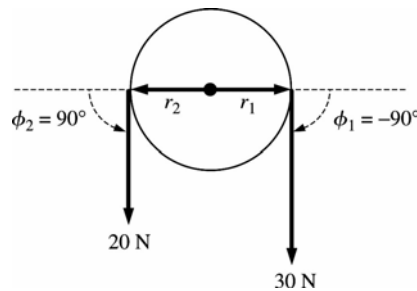
0.305 m. Using the parallel-axis theorem,

$$I = I_{\text{cm}} + Md^2 = \frac{1}{12} (25 \text{ kg})(0.91 \text{ m})^2 + (25 \text{ kg})(0.305 \text{ m})^2 = 4.1 \text{ kg m}^2$$

Assess: The moment of inertia is less for a parallel axis through a point closer to the center of mass.

Section 12.5 Torque

12.18. Visualize:



Solve: Torque by a force is defined as $\tau = Fr \sin \phi$ where ϕ is measured counterclockwise from the $\vec{r}$ vector to the $\vec{F}$ vector. The net torque on the pulley about the axle is the torque due to the 30 N force plus the torque due to the 20 N force:

$$(30 \text{ N})r_1 \sin \phi_1 + (20 \text{ N})r_2 \sin \phi_2 = (30 \text{ N})(0.02 \text{ m}) \sin(-90^\circ) + (20 \text{ N})(0.02 \text{ m}) \sin(90^\circ) \\ = (-0.60 \text{ N} \cdot \text{m}) + (0.40 \text{ N} \cdot \text{m}) = -0.20 \text{ N} \cdot \text{m}$$

Assess: A negative torque causes a clockwise acceleration of the pulley.

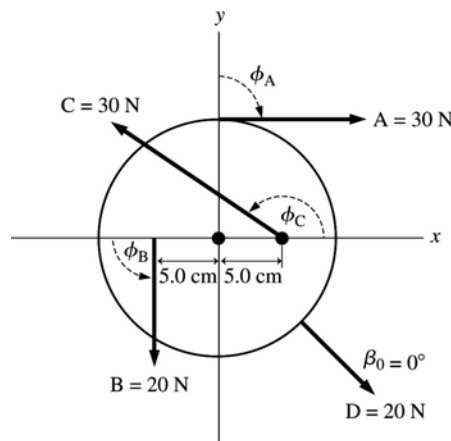
12.19. Visualize: The downward force that points at the axle provides no torque at all.

Solve: The line of action of the upward force is 10 cm away from the axis.

$$\tau = rF \Rightarrow F = \frac{\tau}{r} = \frac{5.0 \text{ N} \cdot \text{m}}{0.10 \text{ m}} = 50 \text{ N}$$

12.20. Model: The disk is a rotating rigid body.

Visualize:



The radius of the disk is 10 cm and the disk rotates on an axle through its center.

Solve: The net torque on the axle is

$$\tau = F_A r_A \sin \phi_A + F_B r_B \sin \phi_B + F_C r_C \sin \phi_C + F_D r_D \sin \phi_D \\ = (30 \text{ N})(0.10 \text{ m}) \sin(-90^\circ) + (20 \text{ N})(0.050 \text{ m}) \sin 90^\circ + (30 \text{ N})(0.050 \text{ m}) \sin 135^\circ + (20 \text{ N})(0.10 \text{ m}) \sin 0^\circ \\ = -3 \text{ N} \cdot \text{m} + 1 \text{ N} \cdot \text{m} + 1.0607 \text{ N} \cdot \text{m} = -0.94 \text{ N} \cdot \text{m}$$

Assess: A negative torque means a clockwise rotation of the disk.

12.21. Model: The pulley acts as if all of its mass were concentrated at its center of mass.

Visualize: There are three torques: two clockwise and one counterclockwise. All of the forces that produce torques act straight down.

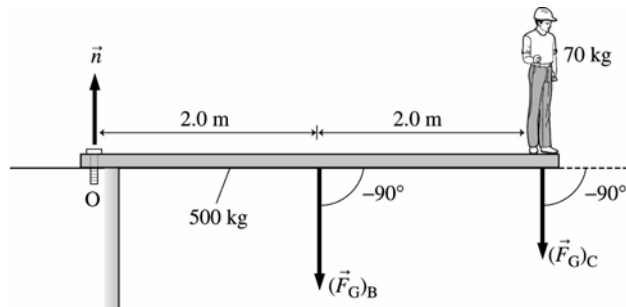
Solve:

$$\Sigma \tau = r_1 F_1 + r_2 F_2 + r_3 F_3 = (0.075 \text{ m})(15 \text{ kg})(9.8 \text{ m/s}^2) - (0.075 \text{ m})(5.0 \text{ kg})(9.8 \text{ m/s}^2) - (0.225 \text{ m})(10 \text{ kg})(9.8 \text{ m/s}^2) \\ = -14.7 \text{ N} \cdot \text{m}$$

Assess: The answer is negative because the clockwise torques are greater than the counterclockwise torques.

12.22. Model: The beam is a solid rigid body.

Visualize:



The steel beam experiences a torque due to the gravitational force on the construction worker $(\vec{F}_G)_C$ and the gravitational force on the beam $(\vec{F}_G)_B$. The normal force exerts no torque since the net torque is calculated about the point where the beam is bolted into place.

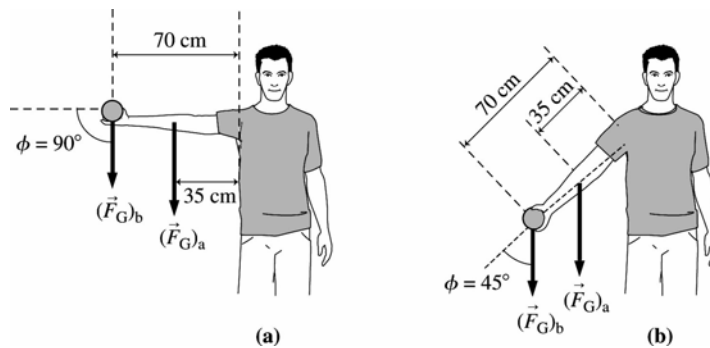
Solve: The net torque on the steel beam about point O is the sum of the torque due to $(\vec{F}_G)_C$ and the torque due to $(\vec{F}_G)_B$. The gravitational force on the beam acts at the center of mass.

$$\begin{aligned}\tau &= ((F_G)_C)(4.0 \text{ m})\sin(-90^\circ) + ((F_G)_B)(2.0 \text{ m})\sin(-90^\circ) \\ &= -(70 \text{ kg})(9.80 \text{ m/s}^2)(4.0 \text{ m}) - (500 \text{ kg})(9.80 \text{ m/s}^2)(2.0 \text{ m}) = -12.5 \text{ kN m}\end{aligned}$$

The negative torque means these forces would cause the beam to rotate clockwise. The magnitude of the torque is 12.5 kN m.

12.23. Model: Model the arm as a uniform rigid rod. Its mass acts at the center of mass.

Visualize:



Solve: (a) The torque is due both to the gravitational force on the ball and the gravitational force on the arm:

$$\begin{aligned}\tau &= \tau_{\text{ball}} + \tau_{\text{arm}} = (m_b g)r_b \sin 90^\circ + (m_a g)r_a \sin 90^\circ \\ &= (3.0 \text{ kg})(9.8 \text{ m/s}^2)(0.70 \text{ m}) + (4.0 \text{ kg})(9.8 \text{ m/s}^2)(0.35 \text{ m}) = 34 \text{ N m}\end{aligned}$$

(b) The torque is reduced because the moment arms are reduced. Both forces act at $\phi = 45^\circ$ from the radial line, so

$$\begin{aligned}\tau &= \tau_{\text{ball}} + \tau_{\text{arm}} = (m_b g)r_b \sin 45^\circ + (m_a g)r_a \sin 45^\circ \\ &= (3.0 \text{ kg})(9.8 \text{ m/s}^2)(0.70 \text{ m})(0.707) + (4.0 \text{ kg})(9.8 \text{ m/s}^2)(0.35 \text{ m})(0.707) = 24 \text{ N m}\end{aligned}$$

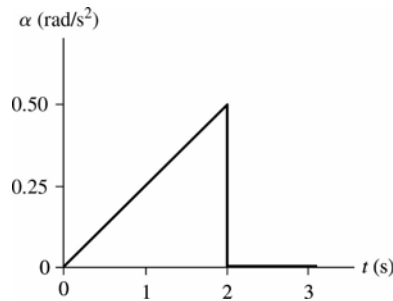
Section 12.6 Rotational Dynamics

Section 12.7 Rotation About a Fixed Axis

12.24. Solve: $\tau = I\alpha$ is the rotational analog of Newton's second law $F = ma$. We have

$$\tau = (2.0 \text{ kg m}^2)(4.0 \text{ rad/s}^2) = 8.0 \text{ kg m}^2/\text{s}^2 = 8.0 \text{ N m}.$$

12.25. Visualize: Since $\alpha = \tau/I$, a graph of the angular acceleration looks just like the torque graph with the numerical values divided by $I = 4.0 \text{ kg m}^2$.



Solve: From the discussion in Chapter 4,

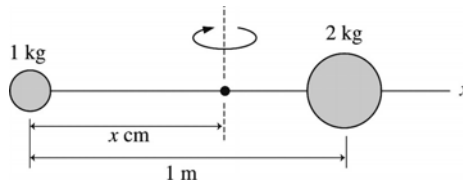
$$\omega_f = \omega_i + \text{area under the angular acceleration } \alpha \text{ curve between } t_i \text{ and } t_f$$

The area under the curve between $t = 0 \text{ s}$ and $t = 3 \text{ s}$ is 0.50 rad/s . With $\omega_i = 0 \text{ rad/s}$, we have

$$\omega_f = 0 \text{ rad/s} + 0.50 \text{ rad/s} = 0.50 \text{ rad/s}$$

12.26. Model: Two balls connected by a rigid, massless rod are a rigid body rotating about an axis through the center of mass. Assume that the size of the balls is small compared to 1 m .

Visualize:



We placed the origin of the coordinate system on the 1.0 kg ball.

Solve: The center of mass and the moment of inertia are

$$x_{\text{cm}} = \frac{(1.0 \text{ kg})(0 \text{ m}) + (2.0 \text{ kg})(1.0 \text{ m})}{(1.0 \text{ kg} + 2.0 \text{ kg})} = 0.667 \text{ m} \quad \text{and} \quad y_{\text{cm}} = 0 \text{ m}$$

$$I_{\text{about cm}} = \sum m_i r_i^2 = (1.0 \text{ kg})(0.667 \text{ m})^2 + (2.0 \text{ kg})(0.333 \text{ m})^2 = 0.667 \text{ kg m}^2$$

We have $\omega_f = 0 \text{ rad/s}$, $t_f - t_i = 5.0 \text{ s}$, and $\omega_i = -20 \text{ rpm} = -20(2\pi \text{ rad}/60 \text{ s}) = -\frac{2}{3}\pi \text{ rad/s}$, so $\omega_f = \omega_i + \alpha(t_f - t_i)$ becomes

$$0 \text{ rad/s} = \left(-\frac{2\pi}{3} \text{ rad/s}\right) + \alpha(5.0 \text{ s}) \Rightarrow \alpha = \frac{2\pi}{15} \text{ rad/s}^2$$

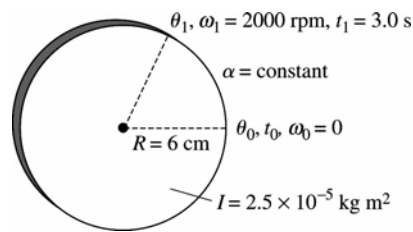
Having found I and α , we can now find the torque τ that will bring the balls to a halt in 5.0 s :

$$\tau = I_{\text{about cm}} \alpha = \left(\frac{2}{3} \text{ kg m}^2\right) \left(\frac{2\pi}{15} \text{ rad/s}^2\right) = \frac{4\pi}{45} \text{ N m} = 0.28 \text{ N m}$$

The magnitude of the torque is 0.28 N m , applied in the counterclockwise direction.

12.27. Model: The compact disk is a rigid body rotating about its center.

Visualize:



Solve: (a) The rotational kinematic equation $\omega_1 = \omega_0 + \alpha(t_1 - t_0)$ gives

$$(2000 \text{ rpm}) \left(\frac{2\pi}{60} \right) \text{ rad/s} = 0 \text{ rad} + \alpha(3.0 \text{ s} - 0 \text{ s}) \Rightarrow \alpha = \frac{200\pi}{9} \text{ rad/s}^2$$

The torque needed to obtain this operating angular velocity is

$$\tau = I\alpha = (2.5 \times 10^{-5} \text{ kg m}^2) \left(\frac{200\pi}{9} \text{ rad/s}^2 \right) = 1.75 \times 10^{-3} \text{ N m}$$

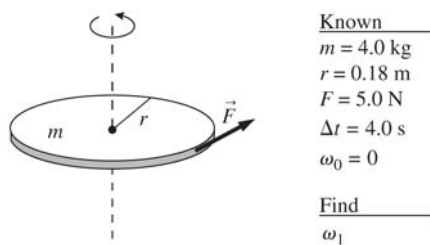
(b) From the rotational kinematic equation,

$$\begin{aligned} \theta_1 &= \theta_0 + \omega_0(t_1 - t_0) + \frac{1}{2}\alpha(t_1 - t_0)^2 = 0 \text{ rad} + 0 \text{ rad} + \frac{1}{2} \left(\frac{200\pi}{9} \text{ rad/s}^2 \right) (3.0 \text{ s} - 0 \text{ s})^2 \\ &= 100\pi \text{ rad} = \frac{100\pi}{2\pi} \text{ revolutions} = 50 \text{ rev} \end{aligned}$$

Assess: Fifty revolutions in 3 seconds is a reasonable value.

12.28. Model: Model the disk as solid. The torque is constant so the angular acceleration is constant.

Visualize: The disk starts from rest, so $\omega_0 = 0$.



Solve:

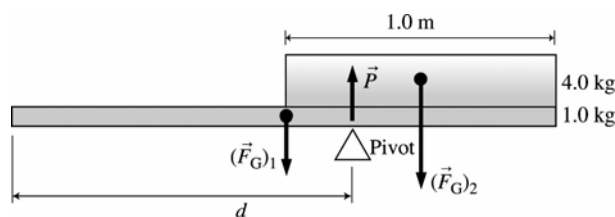
$$\tau = I\alpha = I \frac{\Delta\omega}{\Delta t} \Rightarrow \Delta\omega = \omega_1 - \omega_0 = \omega_1 - 0 = \omega_1 = \frac{\tau \Delta t}{I} = \frac{rF\Delta t}{\frac{1}{2}mr^2} = \frac{F\Delta t}{\frac{1}{2}mr} = \frac{(5.0 \text{ N})(4.0 \text{ s})}{\frac{1}{2}(4.0 \text{ kg})(0.18 \text{ m})} = 55.6 \text{ rad/s} = 530 \text{ rpm}$$

Assess: 530 rpm is pretty fast but in the reasonable range.

Section 12.8 Static Equilibrium

12.29. Model: The object balanced on the pivot is a rigid body.

Visualize:



Since the object is balanced on the pivot, it is in both translational equilibrium and rotational equilibrium.

Solve: There are three forces acting on the object: the gravitational force $(\vec{F}_G)_1$ acting through the center of mass of the long rod, the gravitational force $(\vec{F}_G)_2$ acting through the center of mass of the short rod, and the normal force $\vec{P}$ on the object applied by the pivot. The translational equilibrium equation $(F_{\text{net}})_y = 0 \text{ N}$ is

$$-(F_G)_1 - (F_G)_2 + P = 0 \text{ N} \Rightarrow P = (F_G)_1 + (F_G)_2 = (1.0 \text{ kg})(9.8 \text{ m/s}^2) + (4.0 \text{ kg})(9.8 \text{ m/s}^2) = 49 \text{ N}$$

Measuring torques about the left end, the equation for rotational equilibrium $\tau_{\text{net}} = 0 \text{ N}\cdot\text{m}$ is

$$Pd - w_1(1.0 \text{ m}) - w_2(1.5 \text{ m}) = 0 \text{ N}\cdot\text{m} \\ \Rightarrow (49 \text{ N})d - (1.0 \text{ kg})(9.8 \text{ m/s}^2)(1.0 \text{ m}) - (4.0 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) = 0 \text{ N}\cdot\text{m} \Rightarrow d = 1.40 \text{ m}$$

Thus, the pivot is 1.4 m from the left end.

12.30. Model: The object is in equilibrium, so $\Sigma F = 0$, and $\Sigma \tau = 0$.

Visualize: There are three forces:

Solve:

$$\Sigma F_y = 40 \text{ N} + F_2 - F_1 = 0 \Rightarrow F_1 = 40 \text{ N} + F_2$$

Take the torques around the left end.

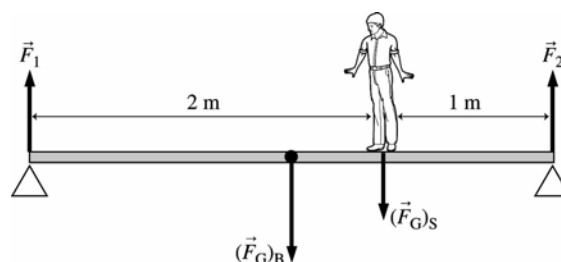
$$\Sigma \tau = (3.0 \text{ m})F_2 - (2.0 \text{ m})F_1 = 0 \\ (3.0 \text{ m})F_2 - (2.0 \text{ m})(40 \text{ N} + F_2) = 0 \\ (3.0 \text{ m} - 2.0 \text{ m})F_2 = 80 \text{ N}\cdot\text{m} \Rightarrow F_2 = 80 \text{ N}$$

Plug back into the force equation to get $F_1 = 120 \text{ N}$

Assess: This can be checked by taking torques around another convenient axis.

12.31. Model: The beam is a rigid body of length 3.0 m and the student is a particle.

Visualize:



Solve: To stay in place, the beam must be in both translational equilibrium ($\vec{F}_{\text{net}} = \vec{0}$ N) and rotational equilibrium ($\tau_{\text{net}} = 0$ N m). The first condition is

$$\begin{aligned}\Sigma F_y &= -(F_G)_{\text{beam}} - (F_G)_{\text{student}} + F_1 + F_2 = 0 \text{ N} \\ \Rightarrow F_1 + F_2 &= (F_G)_{\text{beam}} + (F_G)_{\text{student}} = (100 \text{ kg} + 80 \text{ kg})(9.80 \text{ m/s}^2) = 1764 \text{ N}\end{aligned}$$

Taking the torques about the left end of the beam, the second condition is

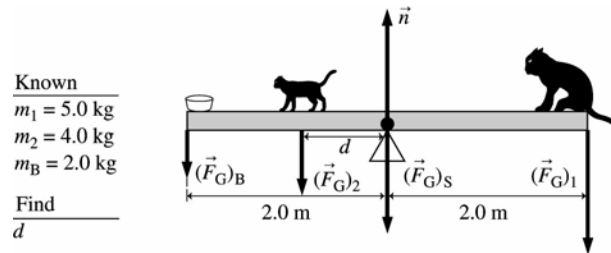
$$\begin{aligned}-(F_G)_{\text{beam}}(1.5 \text{ m}) - (F_G)_{\text{student}}(2.0 \text{ m}) + F_2(3.0 \text{ m}) &= 0 \text{ N m} \\ -(100 \text{ kg})(9.8 \text{ m/s}^2)(1.5 \text{ m}) - (80 \text{ kg})(9.8 \text{ m/s}^2)(2.0 \text{ m}) + F_2(3.0 \text{ m}) &= 0 \text{ N m} \\ \Rightarrow F_2 &= 1013 \text{ N} \approx 1000 \text{ N}\end{aligned}$$

From $F_1 + F_2 = 1764$ N, we get $F_1 = 1764 \text{ N} - 1013 \text{ N} = 0.75 \text{ kN}$.

Assess: To establish rotational equilibrium, the choice for the pivot is arbitrary. We can take torques about any point on the body of interest.

12.32. Model: The see-saw is a rigid body. The cats and bowl are particles.

Visualize:



Solve: The see-saw is in rotational equilibrium. Calculate the net torque about the pivot point.

$$\begin{aligned}\tau_{\text{net}} &= 0 = (F_G)_1(2.0 \text{ m}) - (F_G)_2(d) - (F_G)_B(2.0 \text{ m}) \\ m_2gd &= m_1g(2.0 \text{ m}) - m_Bg(2.0 \text{ m}) \\ d &= \frac{(m_1 - m_B)(2.0 \text{ m})}{m_2} = \frac{(5.0 \text{ kg} - 2.0 \text{ kg})(2.0 \text{ m})}{4.0 \text{ kg}} = 1.5 \text{ m}\end{aligned}$$

Assess: The smaller cat is close but not all the way to the end by the bowl, which makes sense since the combined mass of the smaller cat and bowl of tuna is greater than the mass of the larger cat.

Section 12.9 Rolling Motion

12.33. Solve: (a) According to Equation 12.36, the speed of the center of mass of the tire is

$$v_{\text{cm}} = R\omega = 20 \text{ m/s} \Rightarrow \omega = \frac{v_{\text{cm}}}{R} = \frac{20 \text{ m/s}}{0.30 \text{ m}} = 66.67 \text{ rad/s} = (66.7) \left(\frac{60}{2\pi} \right) \text{ rpm} = 6.4 \times 10^2 \text{ rpm}$$

(b) The speed at the top edge of the tire relative to the ground is $v_{\text{top}} = 2v_{\text{cm}} = 2(20 \text{ m/s}) = 40 \text{ m/s}$.

(c) The speed at the bottom edge of the tire relative to ground is $v_{\text{bottom}} = 0 \text{ m/s}$.

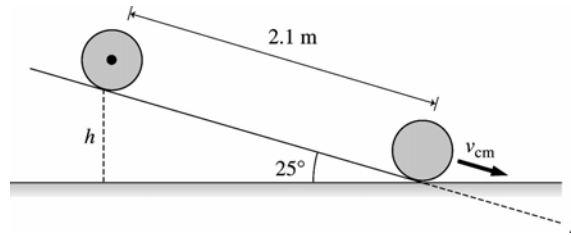
12.34. Model: The can is a rigid body rolling across the floor.

Solve: The rolling motion of the can is a translation of its center of mass plus a rotation about the center of mass. The moment of inertia of the can about the center of mass is $\frac{1}{2}MR^2$, where R is the radius of the can. Also $v_{\text{cm}} = R\omega$, where ω is the angular velocity of the can. The total kinetic energy of the can is

$$\begin{aligned} K &= K_{\text{cm}} + K_{\text{rot}} = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2 \\ &= \frac{3}{4}Mv_{\text{cm}}^2 = \frac{3}{4}(0.50 \text{ kg})(1.0 \text{ m/s})^2 = 0.38 \text{ J} \end{aligned}$$

12.35. Model: The sphere is a rigid body rolling down the incline without slipping.

Visualize:



The initial gravitational potential energy of the sphere is transformed into kinetic energy as it rolls down.

Solve: (a) If we choose the bottom of the incline as the zero of potential energy, the energy conservation equation will be $K_f = U_i$. The kinetic energy consists of both translational and rotational energy. This means

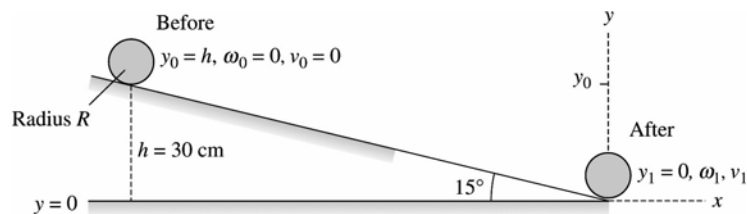
$$\begin{aligned} K_f &= \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 = Mgh \Rightarrow \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 + \frac{1}{2}M(R\omega)^2 = Mgh \\ &\Rightarrow \frac{7}{10}MR^2\omega^2 = Mg(2.1 \text{ m})\sin 25^\circ \\ \Rightarrow \omega &= \sqrt{\frac{\frac{10}{7}g(2.1 \text{ m})(\sin 25^\circ)}{R^2}} = \sqrt{\frac{\frac{10}{7}g(2.1 \text{ m})(\sin 25^\circ)}{(0.04 \text{ m})^2}} = 88 \text{ rad/s} \end{aligned}$$

(b) From part (a)

$$\begin{aligned} K_{\text{total}} &= \frac{1}{2}I_{\text{cm}}\omega^2 + \frac{1}{2}Mv_{\text{cm}}^2 = \frac{7}{10}MR^2\omega^2 \quad \text{and} \quad K_{\text{rot}} = \frac{1}{2}I_{\text{cm}}\omega^2 = \frac{1}{2}\left(\frac{2}{5}MR^2\right)\omega^2 = \frac{1}{5}MR^2\omega^2 \\ \Rightarrow \frac{K_{\text{rot}}}{K_{\text{total}}} &= \frac{\frac{1}{5}MR^2\omega^2}{\frac{7}{10}MR^2\omega^2} = \frac{1}{5} \times \frac{10}{7} = \frac{2}{7} \end{aligned}$$

12.36. Model: The mechanical energy of both the hoop (h) and the sphere (s) is conserved. The initial gravitational potential energy is transformed into kinetic energy as the objects roll down the slope. The kinetic energy is a combination of translational and rotational kinetic energy. We also assume no slipping of the hoop or of the sphere.

Visualize:



The zero of gravitational potential energy is chosen at the bottom of the slope.

Solve: The energy conservation equation for the sphere or hoop $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}I(\omega_1)^2 + \frac{1}{2}m(v_1)^2 + mgy_1 = \frac{1}{2}I(\omega_0)^2 + \frac{1}{2}m(v_0)^2 + mgy_0$$

For the sphere, this becomes

$$\begin{aligned} \frac{1}{2}\left(\frac{2}{5}mR^2\right)\frac{(v_1)_s^2}{R^2} + \frac{1}{2}m(v_1)_s^2 + 0 \text{ J} &= 0 \text{ J} + 0 \text{ J} + mgh_s \\ \Rightarrow \frac{7}{10}(v_1)_s^2 = gh \Rightarrow (v_1)_s &= \sqrt{10gh/7} = \sqrt{10(9.8 \text{ m/s}^2)(0.30 \text{ m})/7} = 2.05 \text{ m/s} \end{aligned}$$

For the hoop, this becomes

$$\begin{aligned} \frac{1}{2}(mR^2)\frac{(v_1)_h^2}{R^2} + \frac{1}{2}m(v_1)_h^2 + 0 \text{ J} &= 0 \text{ J} + 0 \text{ J} + mgh_{\text{hoop}} \\ \Rightarrow h_{\text{hoop}} &= \frac{(v_1)_h^2}{g} \end{aligned}$$

For the hoop to have the same velocity as that of the sphere,

$$h_{\text{hoop}} = \frac{(v_1)_s^2}{g} = \frac{(2.05 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 42.9 \text{ cm}$$

The hoop should be released from a height of 43 cm.

Section 12.10 The Vector Description of Rotational Motion

12.37. Visualize: To determine angle α , put the *tails* of the vectors together.

Solve: (a) The angle between the vectors when their tails coincide is 135° . The magnitude of $\vec{A} \times \vec{B}$ is $AB \sin \alpha = (6)(4) \sin 135^\circ = 17$. The direction of $\vec{A} \times \vec{B}$, using the right-hand rule, is out of the page. Thus, $\vec{A} \times \vec{B} = (17, \text{out of the page})$.

(b) The magnitude of $\vec{C} \times \vec{D}$ is $CD \sin \alpha = (6)(4) \sin 180^\circ = 0$. Thus $\vec{C} \times \vec{D} = \vec{0}$.

12.38. Solve: (a) The magnitude of $\vec{A} \times \vec{B}$ is $AB \sin \alpha = (6)(4) \sin 45^\circ = 21.21$. The direction of $\vec{A} \times \vec{B}$ is given by the right-hand rule. To curl our fingers from $\vec{A}$ to $\vec{B}$, we have to point our thumb into the page. Thus, $\vec{A} \times \vec{B} = (21, \text{into the page})$.

(b) $\vec{C} \times \vec{D} = ((6)(4) \sin 90^\circ, \text{out of the page}) = (24, \text{out of the page})$.

12.39. Solve: $\vec{A} \times \vec{B} = (3\hat{i} + \hat{j}) \times (3\hat{i} - 2\hat{j} + 2\hat{k})$

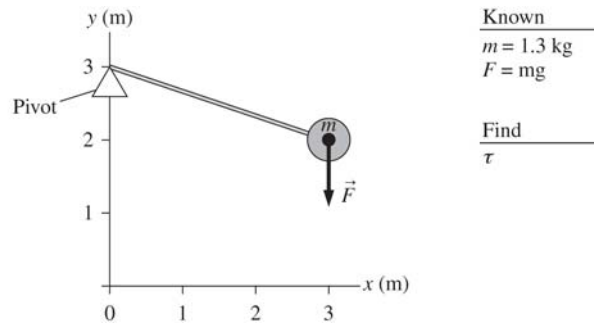
$$\begin{aligned} &= 9\hat{i} \times \hat{i} - 6\hat{i} \times \hat{j} + 6\hat{i} \times \hat{k} + 3\hat{j} \times \hat{i} - 2\hat{j} \times \hat{j} + 2\hat{j} \times \hat{k} \\ &= 0 - 6\hat{k} + 6(-\hat{j}) + 3(\hat{k}) - 0 + 2\hat{i} = 2\hat{i} - 6\hat{j} - 9\hat{k} \end{aligned}$$

12.40. Solve: $\vec{\tau} = \vec{r} \times \vec{F} = (5\hat{i} + 5\hat{j}) \times (-10\hat{j}) \text{ N m}$

$$= [-50(\hat{i} \times \hat{j}) - 50(\hat{j} \times \hat{j})] \text{ N m} = [-50(+\hat{k}) - \vec{0}] \text{ N m} = -50\hat{k} \text{ N m}$$

12.41. Model: Model the ball as a particle, and the rod as massless.

Visualize:



Solve: The vector for $\vec{r}$ is $\vec{r} = (3.0\hat{i} - 1.0\hat{j}) \text{ m}$ and $\vec{F} = -(1.3 \text{ kg})(9.8 \text{ m/s}^2)\hat{j}$

$$\vec{\tau} = \vec{r} \times \vec{F} = ((3.0\hat{i} - 1.0\hat{j}) \text{ m}) \times (-(1.3 \text{ kg})(9.8 \text{ m/s}^2)\hat{j}) = -(38 \text{ N} \cdot \text{m})\hat{k}$$

Assess: The answer is negative because the vector points into the page.

Section 12.11 Angular Momentum

12.42. Solve:

$$\begin{aligned}\vec{L} &= \vec{r} \times m\vec{v} = (1.0\hat{i} + 2.0\hat{j}) \text{ m} \times (0.200 \text{ kg})(3.0 \text{ m/s})(\cos 45^\circ\hat{i} - \sin 45^\circ\hat{j}) \\ &= (0.42\hat{i} \times \hat{i} - 0.42\hat{i} \times \hat{j} + 0.85\hat{j} \times \hat{i} - 0.85\hat{j} \times \hat{j}) \text{ kg m}^2/\text{s} = -(1.27\hat{k}) \text{ kg m}^2/\text{s} \text{ or } (1.27 \text{ kg m}^2/\text{s}, \text{ into page})\end{aligned}$$

12.43. Model: The disk is a rotating rigid body.

Solve: From Table 12.2, the moment of inertia of the disk about its center is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0 \text{ kg})(0.020 \text{ m})^2 = 4.0 \times 10^{-4} \text{ kg m}^2$$

The angular velocity ω is $600 \text{ rpm} = 600 \times 2\pi/60 \text{ rad/s} = 20\pi \text{ rad/s}$. Thus, $L = I\omega = (4.0 \times 10^{-4} \text{ kg m}^2)(20\pi \text{ rad/s}) = 0.025 \text{ kg m}^2/\text{s}$. If we wrap our right fingers in the direction of the disk's rotation, our thumb will point in the $-x$ direction. Consequently,

$$\vec{L} = -0.025 \hat{i} \text{ kg m}^2/\text{s} = (0.025 \text{ kg m}^2/\text{s}, \text{ into page})$$

12.44. Model: The bar is a rotating rigid body. Assume that the bar is thin.

Solve: The angular velocity $\omega = 120 \text{ rpm} = (120)(2\pi)/60 \text{ rad/s} = 4\pi \text{ rad/s}$. From Table 12.2, the moment of inertia of a rod about its center is $I = \frac{1}{12}ML^2$. The angular momentum is

$$L = I\omega = \left(\frac{1}{12}\right)(0.50 \text{ kg})(2.0 \text{ m})^2(4\pi \text{ rad/s}) = 2.1 \text{ kg m}^2/\text{s}$$

If we wrap our fingers in the direction of the rod's rotation, our thumb will point in the z direction or out of the page. Consequently,

$$\vec{L} = (2.1 \text{ kg m}^2/\text{s}, \text{ out of the page})$$

12.45. Model: The bowling ball is a solid sphere.

Solve: From Table 12.2, the moment of inertia about a diameter of a solid sphere is

$$I = \frac{2}{5}MR^2 = \frac{2}{5}(5.0 \text{ kg})(0.11 \text{ m})^2 = 0.0242 \text{ kg m}^2$$

Require

$$L = 0.23 \text{ kg m}^2/\text{s} = I\omega = (0.0243 \text{ kg m}^2)\omega$$

$$\Rightarrow \omega = (9.5 \text{ rad/s})$$

In rpm, this is $(9.5 \text{ rad/s}) \left(\frac{\text{rev}}{2\pi \text{ rad}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) = 91 \text{ rpm}$.

12.46. Model: Model the turntable as a rigid disk rotating on frictionless bearings. As the blocks fall from above and stick on the turntable, the turntable slows down due to increased rotational inertia of the (turntable + blocks) system. Any torques between the turntable and the blocks are internal to the system, so angular momentum of the system is conserved.

Visualize: The initial moment of inertia is I_1 and the final moment of inertia is I_2 .

Solve: The initial moment of inertia is $I_1 = I_{\text{disk}} = \frac{1}{2}mR^2 = \frac{1}{2}(2.0 \text{ kg})(0.10 \text{ m})^2 = 0.010 \text{ kg m}^2$ and the final moment of inertia is

$$I_2 = I_1 + 2mR^2 = 0.010 \text{ kg m}^2 + 2(0.500 \text{ kg}) \times (0.10 \text{ m})^2 = 0.010 \text{ kg m}^2 + 0.010 \text{ kg m}^2 = 0.020 \text{ kg m}^2$$

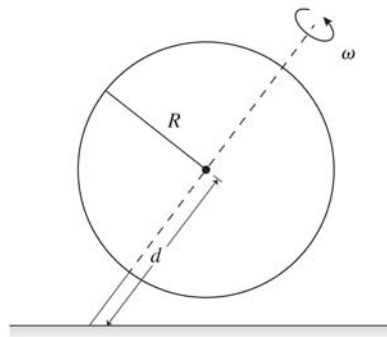
Let ω_1 and ω_2 be the initial and final angular velocities. Then

$$L_f = L_i \Rightarrow \omega_2 I_2 = \omega_1 I_1 \Rightarrow \omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{(0.010 \text{ kg m}^2)(100 \text{ rpm})}{0.020 \text{ kg m}^2} = 50 \text{ rpm}$$

Section 12.12 Precession of a Gyroscope

12.47. Model: Model the spherical top as uniformly dense with the protruding axle massless.

Visualize:



Known
 $M = 75 \text{ kg}$
 $R = 0.030 \text{ m}$
 $d = 0.040 \text{ m}$
 $\omega = 1200 \text{ rpm} = 125.7 \text{ rad/s}$
 $I_{\text{sph}} = \frac{2}{5}MR^2$

Find
 Ω

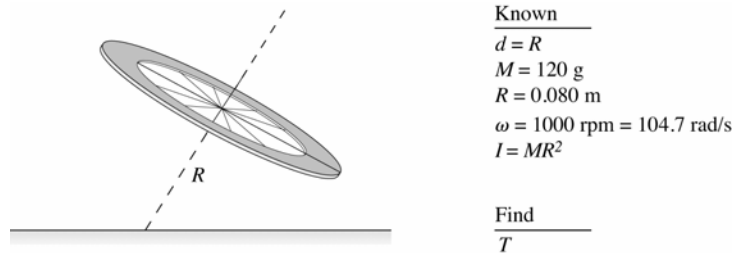
Solve:

$$\Omega = \frac{Mgd}{I\omega} = \frac{Mgd}{\frac{2}{5}MR^2\omega} = \frac{5dg}{2R^2\omega} = \frac{5(0.040 \text{ m})(9.8 \text{ m/s}^2)}{2(0.030 \text{ m})^2(125.7 \text{ rad/s})} = 8.65 \text{ rad/s} = 83 \text{ rpm}$$

Assess: The precession frequency is fairly large—not many times smaller than ω —so it is a little iffy to use our precession model.

12.48. Model: Model the gyroscope as a hoop with massless spokes.

Visualize:



Solve: (a)

$$\Omega = \frac{Mgd}{I\omega} = \frac{MgR}{MR^2\omega} = \frac{g}{R\omega}$$

This is independent of M .

(b)

$$\Omega = \frac{g}{R\omega} = \frac{(9.8 \text{ m/s}^2)}{(0.080 \text{ m})(104.7 \text{ rad/s})} = 1.17 \text{ rad/s}$$

Now find the period.

$$T = \frac{2\pi \text{ rad}}{\Omega} = \frac{2\pi \text{ rad}}{(1.17 \text{ rad/s})} = 5.4 \text{ s}$$

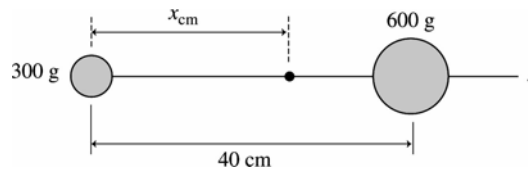
Assess: This is a fairly large period, which means the precession frequency is low; this is reasonable for the given numbers.

both expected and good because we didn't know $(y_{\text{cg}})_{\text{trunk}}$.

Problems

12.49. Model: The structure is a rigid body rotating about its center of mass.

Visualize:



We placed the origin of the coordinate system on the 300 g ball.

Solve: First, we calculate the center of mass:

$$x_{\text{cm}} = \frac{(300 \text{ g})(0 \text{ cm}) + (600 \text{ g})(40 \text{ cm})}{300 \text{ g} + 600 \text{ g}} = 26.67 \text{ cm}$$

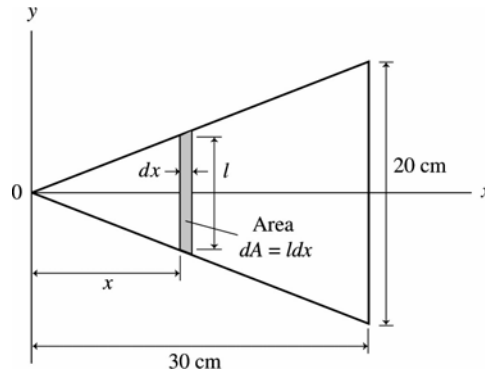
Next, we will calculate the moment of inertia about the structure's center of mass:

$$\begin{aligned} I &= (300 \text{ g})(x_{\text{cm}})^2 + (600 \text{ g})(40 \text{ cm} - x_{\text{cm}})^2 \\ &= (0.300 \text{ kg})(0.2667 \text{ m})^2 + (0.600 \text{ kg})(0.1333 \text{ m})^2 = 0.032 \text{ kg m}^2 \end{aligned}$$

Finally, we calculate the rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2} I \omega^2 = \frac{1}{2} (0.032 \text{ kg m}^2) \left(\frac{100 \times 2\pi}{60} \text{ rad/s} \right)^2 = 1.75 \text{ J} \approx 1.8 \text{ J}$$

12.50. Visualize:



Solve: We will consider a vertical strip of width dx and of mass dm at a position x from the origin. The formula for the x component of the center of mass is

$$x_{\text{cm}} = \frac{1}{M} \int x \, dm$$

The area of the steel plate is $A = \frac{1}{2} (0.2 \text{ m})(0.3 \text{ m}) = 0.030 \text{ m}^2$. Mass dm in the strip is the same fraction of M as dA is of A . Thus

$$\frac{dm}{M} = \frac{dA}{A} \Rightarrow dm = \frac{M}{A} dA = \left(\frac{0.800 \text{ kg}}{0.030 \text{ m}^2} \right) dA = (26.67 \text{ kg/m}^2) l \, dx$$

The relationship between l and x is

$$\frac{l}{0.20 \text{ m}} = \frac{x}{0.30 \text{ m}} \Rightarrow l = \frac{2}{3} x$$

Therefore,

$$x_{\text{cm}} = \frac{1}{M} \int (26.67 \text{ kg/m}^2) \left(\frac{2}{3} \right) x^2 \, dx = \frac{(17.78 \text{ kg/m}^2)}{M} \frac{x^3}{3} \bigg|_{0 \text{ m}}^{0.3 \text{ m}} = \frac{(17.78 \text{ kg/m}^2) (0.3 \text{ m})^2}{0.8 \text{ kg} \cdot 3} = 20 \text{ cm}$$

Due to symmetry $y_{\text{cm}} = 0 \text{ cm}$.

12.51. Model: The object is a rigid rotating body. Assume the masses m_1 and m_2 are small and the rod is thin.

Visualize: Please refer to Figure P12.51.

Solve: The moment of inertia of the object is the sum of the moment of inertia of the rod, mass m_1 , and mass m_2 .

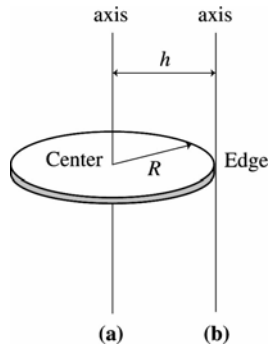
Using Table 12.2 for the moment of inertia of the rod, we get

$$\begin{aligned} I_{\text{object}} &= I_{\text{rod about center}} + I_{m_1} + I_{m_2} = \frac{1}{12} M L^2 + m_1 \left(\frac{L}{2} \right)^2 + m_2 \left(\frac{L}{4} \right)^2 \\ &= \frac{1}{12} M L^2 + \frac{1}{4} m_1 L^2 + \frac{1}{16} m_2 L^2 = \frac{L^2}{4} \left(\frac{M}{3} + m_1 + \frac{m_2}{4} \right) \end{aligned}$$

Assess: With $m_1 = m_2 = 0 \text{ kg}$, $I_{\text{rod}} = \frac{1}{12} M L^2$, as expected.

12.52. Model: The disk is a rigid rotating body. The axis is perpendicular to the plane of the disk.

Visualize:



Solve: (a) From Table 12.2, the moment of inertia of a disk about its center is

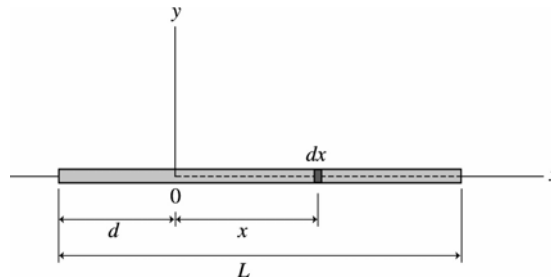
$$I = \frac{1}{2}MR^2 = \frac{1}{2}(2.0 \text{ kg})(0.10 \text{ m})^2 = 0.010 \text{ kg m}^2$$

(b) To find the moment of inertia of the disk through the edge, we can make use of the parallel-axis theorem:

$$I = I_{\text{center}} + Mh^2 = (0.010 \text{ kg m}^2) + (2.0 \text{ kg})(0.10 \text{ m})^2 = 0.030 \text{ kg m}^2$$

Assess: The larger moment of inertia about the edge means there is more inertia to rotational motion about the edge than about the center.

12.53. Visualize:



We chose the origin of the coordinate system to be on the axis of rotation, that is, at a distance d from one end of the rod.

Solve: The moment of inertia can be calculated as follows:

$$I = \int_{x_1}^{x_2} x^2 dm \quad \text{and} \quad \frac{dm}{M} = \frac{dx}{L} \Rightarrow dm = \frac{M}{L} dx$$

$$\Rightarrow I = \frac{M}{L} \int_{-d}^{L-d} x^2 dx = \left(\frac{M}{L} \right) \frac{x^3}{3} \Big|_{-d}^{L-d} = \frac{1}{3} \left(\frac{M}{L} \right) [(L-d)^3 - (-d)^3] = \frac{M}{3L} [(L-d)^3 + d^3]$$

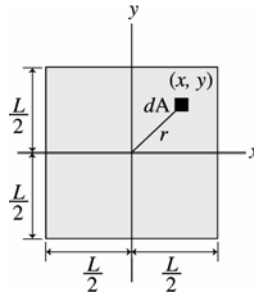
For $d = 0 \text{ m}$, $I = \frac{1}{3}ML^2$, and for $d = \frac{1}{2}L$,

$$I = \frac{M}{3L} \left[\left(\frac{L}{2} \right)^3 + \left(\frac{L}{2} \right)^3 \right] = \frac{1}{12}ML^2$$

Assess: The special cases $d = 0 \text{ m}$ and $d = L/2$ of the general formula give the same results that are found in Table 12.2.

12.54. Model: The plate has uniform density.

Visualize:



Solve: The moment of inertia is

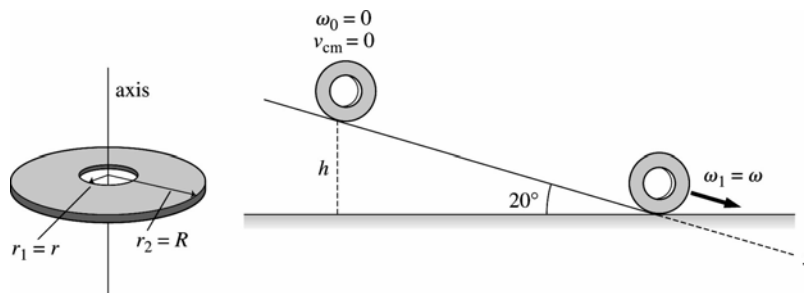
$$I = \int r^2 dm.$$

Let the mass of the plate be M . Its area is L^2 . A region of area dA located at (x, y) has mass $dm = \frac{M}{A} dA = \frac{M}{L^2} dx dy$.

The distance from the axis of rotation to the point (x, y) is $r = \sqrt{x^2 + y^2}$. With $-\frac{L}{2} \leq x \leq \frac{L}{2}$ and $-\frac{L}{2} \leq y \leq \frac{L}{2}$,

$$\begin{aligned} I &= \int_{-\frac{L}{2}}^{\frac{L}{2}} \int_{-\frac{L}{2}}^{\frac{L}{2}} (x^2 + y^2) \left(\frac{M}{L^2} \right) dx dy = \frac{M}{L^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{x^3}{3} + y^2 x \right) \bigg|_{-\frac{L}{2}}^{\frac{L}{2}} dy \\ &= \frac{M}{L^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{L^3}{24} + \frac{y^2 L}{2} - \left(-\frac{L^3}{24} - \frac{y^2 L}{2} \right) \right) dy = \frac{M}{L^2} \int_{-\frac{L}{2}}^{\frac{L}{2}} \left(\frac{L^3}{12} + Ly^2 \right) dy = \frac{M}{L^2} \left(\frac{L^4}{12} + L \frac{y^3}{3} \bigg|_{-\frac{L}{2}}^{\frac{L}{2}} \right) \\ &= \frac{M}{L^2} \left(\frac{L^4}{12} + L \left(\frac{L^3}{24} + \frac{L^3}{24} \right) \right) = \frac{1}{6} ML^2 \end{aligned}$$

12.55. Visualize:



Solve: We solve this problem by dividing the disk between radii r_1 and r_2 into narrow rings of mass dm . Let $dA = 2\pi r dr$ be the area of a ring of radius r . The mass dm in this ring is the same fraction of the total mass M as dA is of the total area A .

(a) The moment of inertia can be calculated as follows:

$$I_{\text{disk}} = \int r^2 dm \quad \text{and} \quad dm = \frac{M}{A} dA = \frac{M}{\pi(r_2^2 - r_1^2)} (2\pi r) dr$$

$$\Rightarrow I_{\text{disk}} = \frac{M}{\pi(r_2^2 - r_1^2)} \int_{r_1}^{r_2} r^2 (2\pi r) dr = \frac{2M}{(r_2^2 - r_1^2)} \int_{r_1}^{r_2} r^3 dr = \frac{2M}{(r_2^2 - r_1^2)} \frac{r^4}{4} \Big|_{r_1}^{r_2}$$

$$= \frac{2M}{4(r_2^2 - r_1^2)} (r_2^4 - r_1^4) = \frac{M}{2} (r_2^2 - r_1^2)$$

Replacing r_1 with r and r_2 with R , the moment of inertia of the disk through its center is $I_{\text{disk}} = \frac{1}{2} M(R^2 + r^2)$.

(b) For $r = 0$ m, $I_{\text{disk}} = \frac{1}{2} MR^2$. This is the moment of inertia for a solid disk or cylinder about the center. Additionally, for $r \cong R$, we have $I = MR^2$. This is the expression for the moment of inertia of a cylindrical hoop or ring about the center.

(c) The initial gravitational potential energy of the disk is transformed into kinetic energy as it rolls down. If we choose the bottom of the incline as the zero of potential energy, and use $v_{\text{cm}} = \omega R$, the energy conservation equation $K_f = U_i$ is

$$\frac{1}{2} I \omega^2 + \frac{1}{2} M v_{\text{cm}}^2 = Mgh \Rightarrow \frac{1}{2} \left(\frac{M}{2} \right) (R^2 + r^2) \frac{v_{\text{cm}}^2}{R^2} + \frac{1}{2} M v_{\text{cm}}^2 = Mgy_i = Mg(0.50 \text{ m}) \sin 20^\circ$$

$$\Rightarrow v_{\text{cm}}^2 \left(\frac{R^2 + r^2}{4R^2} \right) + \frac{1}{2} v_{\text{cm}}^2 = v_{\text{cm}}^2 \left(\frac{1}{2} + \frac{1}{4} + \frac{r^2}{4R^2} \right) = 1.6759 \text{ m}^2/\text{s}^2$$

$$v_{\text{cm}}^2 \left(\frac{3}{4} + \frac{(0.015 \text{ m})^2}{4(0.020 \text{ m})^2} \right) = 1.6759 \text{ m}^2/\text{s}^2 \Rightarrow v_{\text{cm}} = 1.37 \text{ m/s} \approx 1.4 \text{ m/s}$$

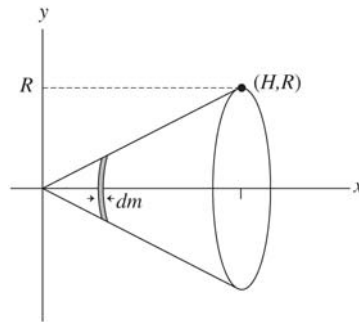
For a sliding particle on a frictionless surface $K_f = U_i$, so

$$\frac{1}{2} m v_f^2 = mgy_i \Rightarrow v_f = \sqrt{2gy_i} = \sqrt{2g(0.50 \text{ m}) \sin 20^\circ} = 1.83 \text{ m/s} \Rightarrow \frac{v_{\text{cm}}}{v_f} = 0.75$$

That is, v_{cm} is 75% of the speed of a particle sliding down a frictionless ramp.

12.56. Model: Assume the density is a constant ρ .

Visualize: Consider the cone as a stack of thin disks. Note that $y = (R/H)x$ by similar triangles.



The mass of one thin disk is

$$dm = \rho dV = \frac{M}{V} A dx = \frac{M}{\frac{1}{3} \pi H R^2} (\pi y^2) dx = \frac{3M}{H R^2} \left(\frac{R}{H} x \right)^2 dx = \frac{3M}{H^3} x^2 dx$$

Solve: (a)

$$x_{\text{cm}} = \frac{\int x dm}{\int dm} = \frac{\frac{3M}{H^3} \int x(x)^2 dx}{\frac{3M}{H^3} \int (x)^2 dx} = \frac{\left[\frac{1}{4} x^4 \right]_0^H}{\left[\frac{1}{3} x^3 \right]_0^H} = \frac{3}{4} H$$

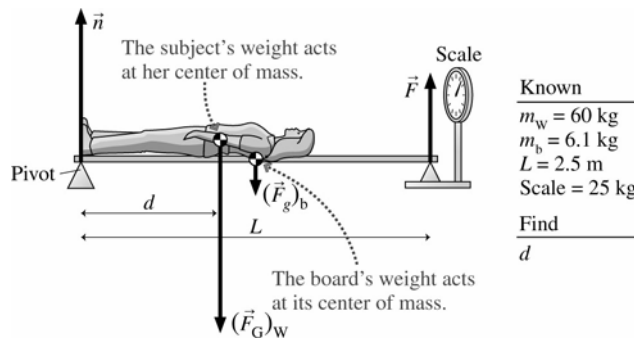
(b)

$$\begin{aligned} I &= \int dI = \int \frac{1}{2} r^2 dm = \frac{1}{2} \int y^2 dm = \frac{1}{2} \int \left(\frac{R}{H} x \right)^2 \left(\frac{3M}{H^3} x^2 dx \right) \\ &= \frac{1}{2} \left(\frac{R}{H} \right)^2 \left(\frac{3M}{H^3} \right) \int_0^H x^4 dx = \frac{1}{2} \left(\frac{R}{H} \right)^2 \left(\frac{3M}{H^3} \right) \left[\frac{1}{5} x^5 \right]_0^H = \frac{3}{10} MR^2 \end{aligned}$$

Assess: This could have as easily been done with the cone oriented vertically along the y -axis. Note the location of the center of mass is independent of R and the moment of inertia is independent of H .

12.57. Model: Assume the woman is in equilibrium, so $\sum F = 0$ and $\sum \tau = 0$.

Visualize: Choose the axis to be the left end of the board.



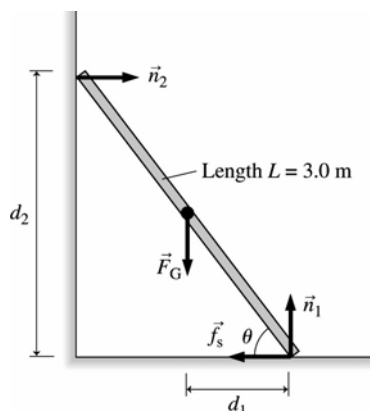
Solve: Use $\sum \tau = 0$.

$$\begin{aligned} \sum \tau &= L(\text{scalereading})g - d(m_w)g - \frac{L}{2}(m_b)g = 0 \text{ N} \cdot \text{m} \\ d &= \frac{L(\text{scalereading})g - \frac{L}{2}(m_b)g}{m_w g} = \frac{L[(\text{scalereading}) - \frac{1}{2}m_b]}{m_w} \\ &= \frac{(2.5 \text{ m})[(25 \text{ kg}) - \frac{1}{2}(6.1 \text{ kg})]}{60 \text{ kg}} = 0.91 \text{ m} \end{aligned}$$

Assess: This is a little more than halfway up the body of a woman of average height.

12.58. Model: The ladder is a rigid rod of length L . To not slip, it must be in both translational equilibrium ($\vec{F}_{\text{net}} = \vec{0} \text{ N}$) and rotational equilibrium ($\tau_{\text{net}} = 0 \text{ N} \cdot \text{m}$). We also apply the model of static friction.

Visualize:



Since the wall is frictionless, the only force from the wall on the ladder is the normal force $\vec{n}_2$. On the other hand, the floor exerts both the normal force $\vec{n}_1$ and the static frictional force $\vec{f}_s$. The gravitational force $\vec{F}_G$ on the ladder acts through the center of mass of the ladder.

Solve: The x - and y -components of $\vec{F}_{\text{net}} = \vec{0}$ N are

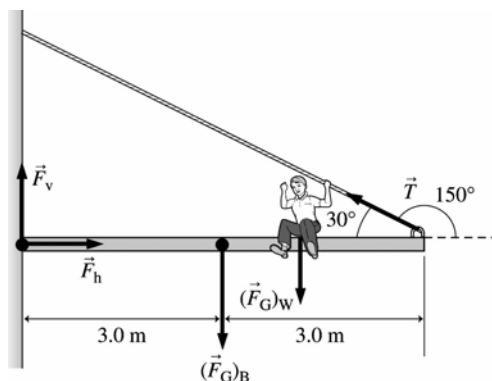
$$\Sigma F_x = n_2 - f_s = 0 \text{ N} \Rightarrow f_s = n_2 \quad \Sigma F_y = n_1 - F_G = 0 \text{ N} \Rightarrow n_1 = F_G$$

The minimum angle occurs when the static friction is at its maximum value $f_{s \text{ max}} = \mu_s n_1$. Thus we have $n_2 = f_s = \mu_s n_1 = \mu_s mg$. We choose the bottom corner of the ladder as a pivot point to obtain τ_{net} , because two forces pass through this point and have no torque about it. The net torque about the bottom corner is

$$\begin{aligned} \tau_{\text{net}} &= d_1 mg - d_2 n_2 = (0.5L \cos \theta_{\text{min}}) mg - (L \sin \theta_{\text{min}}) \mu_s mg = 0 \text{ N m} \\ \Rightarrow 0.5 \cos \theta_{\text{min}} &= \mu_s \sin \theta_{\text{min}} \Rightarrow \tan \theta_{\text{min}} = \frac{0.5}{\mu_s} = \frac{0.5}{0.4} = 1.25 \Rightarrow \theta_{\text{min}} = 51^\circ \end{aligned}$$

12.59. Model: The structure is a rigid body.

Visualize:



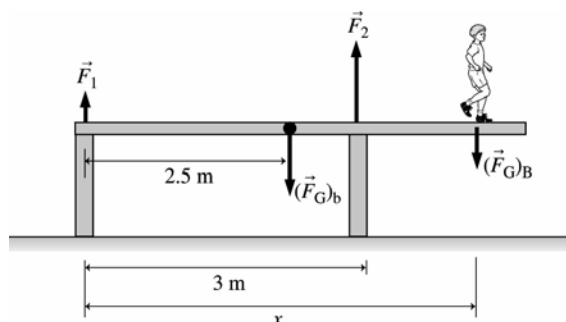
Solve: We pick the left end of the beam as our pivot point. We don't need to know the forces F_h and F_v because the pivot point passes through the line of application of F_h and F_v and therefore these forces do not exert a torque. For the beam to stay in equilibrium, the net torque about this point is zero. We can write

$$\tau_{\text{about left end}} = -(F_G)_B(3.0 \text{ m}) - (F_G)_W(4.0 \text{ m}) + (T \sin 150^\circ)(6.0 \text{ m}) = 0 \text{ N m}$$

Using $(F_G)_B = (1450 \text{ kg})(9.8 \text{ m/s}^2)$ and $(F_G)_W = (80 \text{ kg})(9.8 \text{ m/s}^2)$, the torque equation can be solved to yield $T = 15,300 \text{ N}$.

12.60. Model: Model the beam as a rigid body. For the beam not to fall over, it must be both in translational equilibrium ($\vec{F}_{\text{net}} = \vec{0}$ N) and rotational equilibrium ($\tau_{\text{net}} = 0$ Nm).

Visualize:



The boy walks along the beam a distance x , measured from the left end of the beam. There are four forces acting on the beam. F_1 and F_2 are from the two supports, $(\vec{F}_G)_b$ is the gravitational force on the beam, and $(\vec{F}_G)_B$ is the gravitational force on the boy.

Solve: We pick our pivot point on the left end through the first support. The equation for rotational equilibrium is

$$-(F_G)_b(2.5 \text{ m}) + F_2(3.0 \text{ m}) - (F_G)_B x = 0 \text{ N m}$$

$$-(40 \text{ kg})(9.80 \text{ m/s}^2)(2.5 \text{ m}) + F_2(3.0 \text{ m}) - (20 \text{ kg})(9.80 \text{ m/s}^2)x = 0 \text{ N m}$$

The equation for translation equilibrium is

$$\Sigma F_y = 0 \text{ N} = F_1 + F_2 - (F_G)_b - (F_G)_B$$

$$\Rightarrow F_1 + F_2 = (F_G)_b + (F_G)_B = (40 \text{ kg} + 20 \text{ kg})(9.8 \text{ m/s}^2) = 588 \text{ N}$$

Just when the boy is at the point where the beam tips, $F_1 = 0$ N. Thus $F_2 = 588$ N. With this value of F_2 , we can simplify the torque equation to:

$$-(40 \text{ kg})(9.80 \text{ m/s}^2)(2.5 \text{ m}) + (588 \text{ N})(3.0 \text{ m}) - (20 \text{ kg})(9.80 \text{ m/s}^2)x = 0 \text{ N m}$$

$$\Rightarrow x = 4.0 \text{ m}$$

Thus, the distance from the right end is $5.0 \text{ m} - 4.0 \text{ m} = 1.0 \text{ m}$.

12.61. Solve: The bricks are stable when the net gravitational torque on each individual brick or combination of bricks is zero. This is true as long as the center of gravity of each individual brick and any combination is over a base of support. To determine the relative positions of the bricks, work from the top down. The top brick can extend past the second brick by $L/2$. For maximum extension, their combined center of gravity will be at the edge of the third brick, and the combined center of gravity of the three upper bricks will be at the edge of the fourth brick. The combined center of gravity of all four bricks will be over the edge of the table.

Measuring from the left edge of brick 2, the center of gravity of the top two bricks is

$$(x_{12})_{\text{com}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2} = \frac{m\left(\frac{L}{2}\right) + mL}{2m} = \frac{3}{4}L$$

Thus the top two bricks can extend $L/4$ past the edge of the third brick. The top three bricks have a center of mass

$$(x_{123})_{\text{com}} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{3L}{4}\right) + m\left(\frac{5L}{4}\right)}{3m} = \frac{5}{6}L$$

Thus the top three bricks can extend past the edge of the fourth brick by $L/6$. Finally, the four bricks have a combined center of mass at

$$(x_{1234})_{\text{com}} = \frac{m\left(\frac{L}{2}\right) + m\left(\frac{4L}{6}\right) + m\left(\frac{11L}{12}\right) + m\left(\frac{17L}{12}\right)}{4m} = \frac{7}{8}L$$

The center of gravity of all four bricks combined is $7L/8$ from the left edge of the bottom brick, so brick 4 can extend $L/8$ past the table edge. Thus the maximum distance to the right edge of the top brick from the table edge is

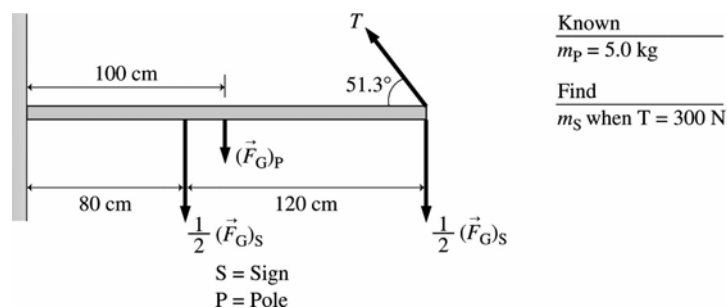
$$d_{\text{max}} = \frac{L}{8} + \frac{L}{6} + \frac{L}{4} + \frac{L}{2} = \frac{25}{24}L$$

Thus, yes, it is possible that no part of the top brick is directly over the table because $d_{\text{max}} > L$.

Assess: As crazy as this seems, the center of gravity of all four bricks is stably supported, so the net gravitational torque is zero, and the bricks do not fall over.

12.62. Model: The pole is a uniform rod. The sign is also uniform.

Visualize:



Solve: The geometry of the rod and cable give the angle that the cable makes with the rod.

$$\theta = \tan^{-1}\left(\frac{250}{200}\right) = 51.3^\circ$$

The rod is in rotational equilibrium about its left-hand end.

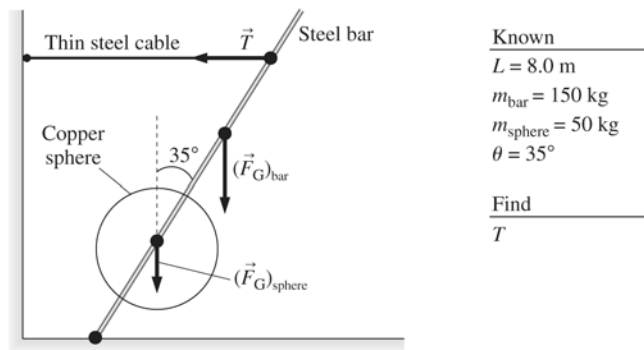
$$\begin{aligned}\tau_{\text{net}} = 0 &= -(100 \text{ cm})(F_G)_P - (80 \text{ cm})\left(\frac{1}{2}\right)(F_G)_S - (200 \text{ cm})\left(\frac{1}{2}\right)(F_G)_S + (200 \text{ cm})T \sin 51.3^\circ \\ &= -(100 \text{ cm})(5.0 \text{ kg})(9.8 \text{ m/s}^2) - m_S(9.8 \text{ m/s}^2)(140 \text{ cm}) + (156 \text{ cm})T\end{aligned}$$

With $T = 300 \text{ N}$, $m_S = 30.6 \text{ kg} \approx 31 \text{ kg}$.

Assess: A mass of 30.6 kg is reasonable for a sign.

12.63. Model: Assume the sculpture is in equilibrium. Choose the axis for the torques to be where the bar meets the floor. The distances from that axis are 2.0 m for the sphere, 4.0 m for the center of mass of the bar, and 6.0 m for the point where the cable attaches.

Visualize: Clockwise torques are negative and counterclockwise torques are positive.



Solve:

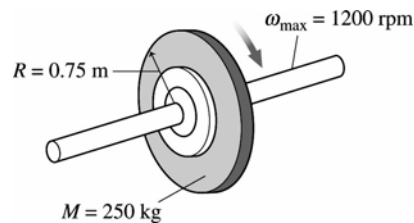
$$\Sigma \tau = (6.0 \text{ m} \cos 35^\circ)T - (4.0 \text{ m} \sin 35^\circ)(150 \text{ kg})(9.8 \text{ m/s}^2) - (2.0 \text{ m} \sin 35^\circ)(50 \text{ kg})(9.8 \text{ m/s}^2) = 0$$

$$T = \frac{(4.0 \text{ m} \sin 35^\circ)(150 \text{ kg})(9.8 \text{ m/s}^2) + (2.0 \text{ m} \sin 35^\circ)(50 \text{ kg})(9.8 \text{ m/s}^2)}{(6.0 \text{ m} \cos 35^\circ)} = 800 \text{ N}$$

Assess: This is a reasonable tension for a steel cable.

12.64. Model: The flywheel is a rigid body rotating about its central axis.

Visualize:



Solve: (a) The radius of the flywheel is $R = 0.75 \text{ m}$ and its mass is $M = 250 \text{ kg}$. The moment of inertia about the axis of rotation is that of a disk:

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(250 \text{ kg})(0.75 \text{ m})^2 = 70.31 \text{ kg m}^2$$

The angular acceleration is calculated as follows:

$$\tau_{\text{net}} = I\alpha \Rightarrow \alpha = \tau_{\text{net}}/I = (50 \text{ N m})/(70.31 \text{ kg m}^2) = 0.711 \text{ rad/s}^2$$

Using the kinematic equation for angular velocity gives

$$\omega_1 = \omega_0 + \alpha(t_1 - t_0) = 1200 \text{ rpm} = 40\pi \text{ rad/s} = 0 \text{ rad/s} + 0.711 \text{ rad/s}^2(t_1 - 0 \text{ s})$$

$$\Rightarrow t_1 = 177 \text{ s}$$

(b) The energy stored in the flywheel is rotational kinetic energy:

$$K_{\text{rot}} = \frac{1}{2}I\omega_1^2 = \frac{1}{2}(70.31 \text{ kg m}^2)(40\pi \text{ rad/s})^2 = 5.55 \times 10^5 \text{ J}$$

The energy stored is $5.6 \times 10^5 \text{ J}$.

(c) Average power delivered = $\frac{\text{energy delivered}}{\text{time interval}} = \frac{(5.55 \times 10^5 \text{ J})/2}{2.0 \text{ s}} = 1.39 \times 10^5 \text{ W} \approx 140 \text{ kW}$

(d) Because $\tau = I\alpha$, $\Rightarrow \tau = I \frac{\Delta\omega}{\Delta t} = I \left(\frac{\omega_{\text{full energy}} - \omega_{\text{half energy}}}{\Delta t} \right)$. $\omega_{\text{full energy}} = \omega_1$ (from part(a)) = 40π rad/s.

$\omega_{\text{half energy}}$ can be obtained as:

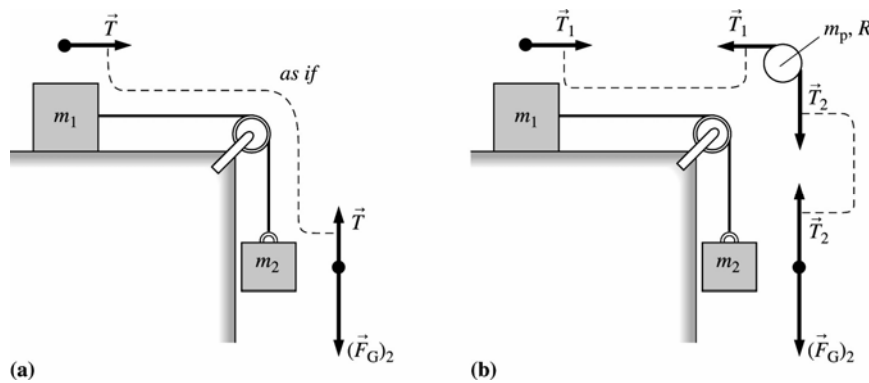
$$\frac{1}{2} I \omega_{\text{half energy}}^2 = \frac{1}{2} K_{\text{rot}} \Rightarrow \omega_{\text{half energy}} = \sqrt{\frac{K_{\text{rot}}}{I}} = \sqrt{\frac{5.55 \times 10^5 \text{ J}}{70.31 \text{ kg m}^2}} = 88.85 \text{ rad/s}$$

Thus

$$\tau = (70.31 \text{ kg m}^2) \left(\frac{40\pi \text{ rad/s} - 88.85 \text{ rad/s}}{2.0 \text{ s}} \right) = 1.30 \text{ kN m}$$

12.65. Model: Assume the string does not slip on the pulley.

Visualize:



The free-body diagrams for the two blocks and the pulley are shown. The tension in the string exerts an upward force on the block m_2 , but a downward force on the outer edge of the pulley. Similarly the string exerts a force on block m_1 to the right, but a leftward force on the outer edge of the pulley.

Solve: (a) Newton's second law for m_1 and m_2 is $T = m_1 a_1$ and $T - m_2 g = m_2 a_2$. Using the constraint $-a_2 = +a_1 = a$, we have $T = m_1 a$ and $-T + m_2 g = m_2 a$. Adding these equations, we get $m_2 g = (m_1 + m_2) a$, or

$$a = \frac{m_2 g}{m_1 + m_2} \Rightarrow T = m_1 a = \frac{m_1 m_2 g}{m_1 + m_2}$$

(b) When the pulley has mass m , the tensions (T_1 and T_2) in the upper and lower portions of the string are different. Newton's second law for m_1 and the pulley are:

$$T_1 = m_1 a \quad \text{and} \quad T_1 R - T_2 R = -I \alpha$$

We are using the minus sign with α because the pulley accelerates clockwise. Also, $a = R\alpha$. Thus, $T_1 = m_1 a$ and

$$T_2 - T_1 = \frac{I}{R} \frac{a}{R} = \frac{aI}{R^2}$$

Adding these two equations gives

$$T_2 = a \left(m_1 + \frac{I}{R^2} \right)$$

Newton's second law for m_2 is $T_2 - m_2g = m_2a_2 = -m_2a$. Using the above expression for T_2 ,

$$a\left(m_1 + \frac{I}{R^2}\right) + m_2a = m_2g \Rightarrow a = \frac{m_2g}{m_1 + m_2 + I/R^2}$$

Since $I = \frac{1}{2}m_pR^2$ for a disk about its center,

$$a = \frac{m_2g}{m_1 + m_2 + \frac{1}{2}m_p}$$

With this value for a we can now find T_1 and T_2 :

$$T_1 = m_1a = \frac{m_1m_2g}{m_1 + m_2 + \frac{1}{2}m_p} \quad T_2 = a(m_1 + I/R^2) = \frac{m_2g}{(m_1 + m_2 + \frac{1}{2}m_p)}\left(m_1 + \frac{1}{2}m_p\right) = \frac{m_2(m_1 + \frac{1}{2}m_p)g}{m_1 + m_2 + \frac{1}{2}m_p}$$

Assess: For $m = 0$ kg, the equations for a , T_1 and T_2 of part (b) simplify to

$$a = \frac{m_2g}{m_1 + m_2} \quad \text{and} \quad T_1 = \frac{m_1m_2g}{m_1 + m_2} \quad \text{and} \quad T_2 = \frac{m_1m_2g}{m_1 + m_2}$$

These agree with the results of part (a).

12.66. Model: The disk is a rigid spinning body.

Visualize: Please refer to Figure P12.66. The initial angular velocity is 300 rpm or $(300)(2\pi)/60 = 10\pi$ rad/s. After 3.0 s the disk stops.

Solve: Using the kinematic equation for angular velocity,

$$\omega_1 = \omega_0 + \alpha(t_1 - t_0) \Rightarrow \alpha = \frac{\omega_1 - \omega_0}{t_1 - t_0} = \frac{(0 \text{ rad/s} - 10\pi \text{ rad/s})}{(3.0 \text{ s} - 0 \text{ s})} = \frac{-10\pi}{3} \text{ rad/s}^2$$

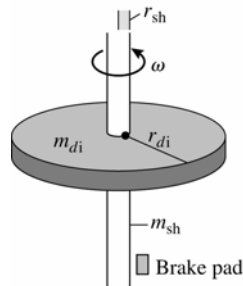
Thus, the torque due to the force of friction that brings the disk to rest is

$$\tau = I\alpha = -fR \Rightarrow f = -\frac{I\alpha}{R} = -\frac{(\frac{1}{2}mR^2)\alpha}{R} = -\frac{1}{2}(mR)\alpha = -\frac{1}{2}(2.0 \text{ kg})(0.15 \text{ m})\left(-10\frac{\pi}{3} \text{ rad/s}^2\right) = 1.57 \text{ N} \approx 1.6 \text{ N}$$

The minus sign with $\tau = -fR$ indicates that the torque due to friction acts clockwise.

12.67. Model: Assume there are no other forces or torques other than the brake pad.

Visualize: The friction force is tangent to the shaft. The initial angular momentum is $L_i = (I_{di} + I_{sh})\omega$.



Known

$$\begin{aligned} r_{di} &= 0.15 \text{ m} \\ r_{sh} &= 0.006 \text{ m} \\ m_{di} &= 1.2 \text{ kg} \\ m_{sh} &= 0.45 \text{ kg} \\ \Delta t &= 15 \text{ s} \\ \omega &= 33 \text{ rpm} = 3.46 \text{ rad/s} \end{aligned}$$

Find

$$\vec{f}_k$$

Solve:

$$\begin{aligned}\frac{\Delta \vec{L}}{\Delta t} &= \vec{\tau} = \vec{r} \times \vec{F} = \vec{r}_{\text{sh}} \times \vec{f}_k \Rightarrow f_k = \left| \frac{1}{r_{\text{sh}}} \frac{\Delta L}{\Delta t} \right| = \frac{1}{r_{\text{sh}}} \frac{L_i}{\Delta t} = \frac{1}{r_{\text{sh}}} \frac{(I_{\text{di}} + I_{\text{sh}})\omega}{\Delta t} = \frac{1}{r_{\text{sh}}} \frac{(\frac{1}{2}m_{\text{di}}r_{\text{di}}^2 + \frac{1}{2}m_{\text{sh}}r_{\text{sh}}^2)\omega}{\Delta t} \\ &= \frac{1}{0.006 \text{ m}} \frac{(\frac{1}{2}(1.2 \text{ kg})(0.15 \text{ m})^2 + \frac{1}{2}(0.45 \text{ kg})(0.006 \text{ m})^2)(3.46 \text{ rad/s})}{15 \text{ s}} = 0.52 \text{ N}\end{aligned}$$

Assess: This is not a large force, but it took 15 s to slow down the disk, so it is reasonable.

12.68. Model: Assume the turbine is a rigid rotating body.

Visualize: Start with Newton's second law: $\sum \tau = I\alpha$. The net torque is just the frictional torque we seek.

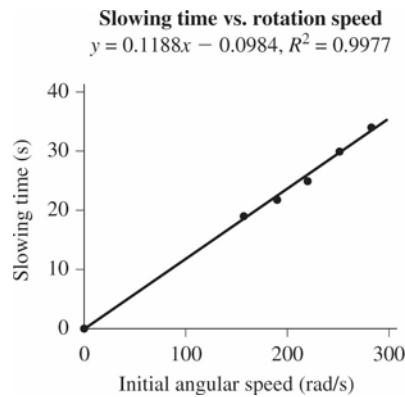
Solve: Apply the definition of α .

$$\tau = I\alpha = I \frac{\Delta \omega}{\Delta t}$$

When the turbine has reduced its rotation speed by 50% then $\Delta \omega = \frac{1}{2}\omega_i$.

$$\Delta t = \frac{I}{2\tau} \omega_i$$

This suggests that a graph of Δt vs. ω_i should be a straight line whose slope is $I/2\tau$.



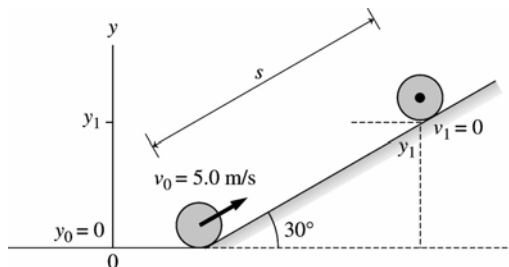
We see that the fit is quite good and that the slope is 0.1188 s^2 , so the frictional torque is

$$\tau = \frac{I}{2 \cdot \text{slope}} = \frac{2.6 \text{ kg} \cdot \text{m}^2}{2(0.1188 \text{ s}^2)} = 10.94 \text{ N} \cdot \text{m} \approx 11 \text{ N} \cdot \text{m}$$

Assess: Your boss wants the frictional torque to be as small as possible, but $11 \text{ N} \cdot \text{m}$ seems reasonable.

12.69. Model: Assume that the hollow sphere is a rigid rolling body and that the sphere rolls up the incline without slipping. We also assume that the coefficient of rolling friction is zero.

Visualize:



The initial kinetic energy, which is a combination of rotational and translational energy, is transformed in gravitational potential energy. We chose the bottom of the incline as the zero of the gravitational potential energy.

Solve: The conservation of energy equation $K_f + U_{gf} = K_i + U_{gi}$ is

$$\begin{aligned}\frac{1}{2}M(v_1)_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}(\omega_1)^2 + Mgy_1 &= \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}(\omega_0)^2 + Mgy_0 \\ 0 \text{ J} + 0 \text{ J} + Mgy_1 &= \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{2}\left(\frac{2}{3}MR^2\right)(\omega_0)_{\text{cm}}^2 + 0 \text{ J} \Rightarrow Mgy_1 = \frac{1}{2}M(v_0)_{\text{cm}}^2 + \frac{1}{3}MR^2 \frac{(v_0)_{\text{cm}}^2}{R^2} \\ \Rightarrow gy_1 &= \frac{5}{6}(v_0)_{\text{cm}}^2 \Rightarrow y_1 = \frac{\frac{5}{6}(v_0)_{\text{cm}}^2}{g} = \frac{5}{6} \frac{(5.0 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 2.126 \text{ m}\end{aligned}$$

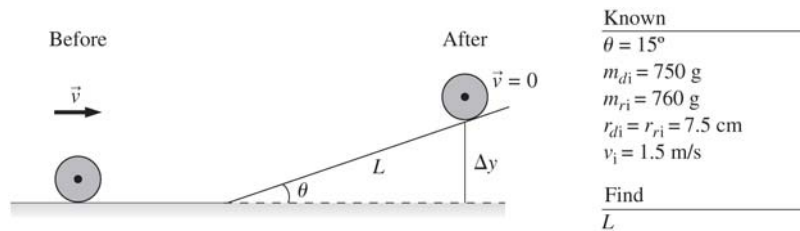
The distance traveled along the incline is

$$s = \frac{y_1}{\sin 30^\circ} = \frac{2.126 \text{ m}}{0.5} = 4.3 \text{ m}$$

Assess: This is a reasonable stopping distance for an object rolling up an incline when its speed at the bottom of the incline is approximately 10 mph.

12.70. Model: Assume there is no air resistance or friction.

Visualize: Use conservation of energy in each case. Initially each object has translational and rotational kinetic energy.



Solve:

$$\begin{aligned}K_i &= U_f \\ \frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2 &= mg\Delta y \\ \Rightarrow \Delta y &= \frac{\frac{1}{2}mv_i^2 + \frac{1}{2}I\omega_i^2}{mg} = \frac{\frac{1}{2}mv_i^2 + \frac{1}{2}I\left(\frac{v_i}{r}\right)^2}{mg}\end{aligned}$$

For the disk insert $I_{di} = \frac{1}{2}m_{di}r_{di}^2$:

$$\Delta y = \frac{\frac{1}{2}mv_i^2 + \frac{1}{2}\left(\frac{1}{2}m_{di}r_{di}^2\right)\left(\frac{v_i}{r_{di}}\right)^2}{mg} = \frac{\frac{1}{2}mv_i^2 + \frac{1}{4}m_{di}v_i^2}{mg} = \frac{\frac{3}{4}v_i^2}{g} = \frac{\frac{3}{4}(1.5 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 17.2 \text{ cm}$$

Now find how far up the slope the disk goes to achieve a height of 17.2 cm.

$$L = \frac{\Delta y}{\sin \theta} = \frac{17.2 \text{ cm}}{\sin 15^\circ} = 67 \text{ cm}$$

For the ring insert $I_{\text{ri}} = m_{\text{ri}} r_{\text{ri}}^2$:

$$\Delta y = \frac{\frac{1}{2} m v_i^2 + \frac{1}{2} (m_{\text{ri}} r_{\text{ri}}^2) \left(\frac{v_i}{r_{\text{ri}}} \right)^2}{mg} = \frac{\frac{1}{2} m v_i^2 + \frac{1}{2} m v_i^2}{mg} = \frac{v_i^2}{g} = \frac{(1.5 \text{ m/s})^2}{9.8 \text{ m/s}^2} = 23.0 \text{ cm}$$

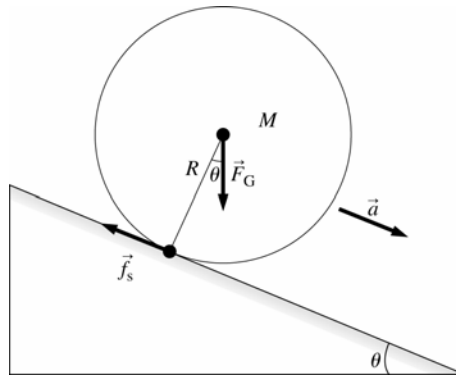
Now find how far up the slope the ring goes to achieve a height of 23 cm.

$$L = \frac{\Delta y}{\sin \theta} = \frac{23.0 \text{ cm}}{\sin 15^\circ} = 89 \text{ cm}$$

Assess: Because the ring has a greater moment of inertia, it has greater total kinetic energy than the disk moving at the same translational speed. Notice both answers are independent of the mass and the radius.

12.71. Model: Assume there is no air resistance. The cylinder rolls without slipping.

Visualize:



Solve: The static friction force creates a torque computed around the axis of the cylinder equal to $\tau = R f_s$.

Use the rotational version of Newton's second law with the acceleration when the cylinder is rolling without slipping: $\alpha = a/R$.

$$\tau = I \alpha \Rightarrow R f_s = I \frac{a}{R} \Rightarrow f_s = I \frac{a}{R^2} \Rightarrow \mu_s n = I \frac{a}{R^2}$$

A side calculation from forces will give us the value of n and then a .

$$\Sigma F_y = n - Mg \cos \theta = 0 \Rightarrow n = Mg \cos \theta$$

$$\Sigma F_x = Mg \sin \theta - f_s = Mg \sin \theta - \mu_s n = Mg \sin \theta - \mu_s Mg \cos \theta = Ma$$

$$\Rightarrow a = g(\sin \theta - \mu_s \cos \theta)$$

Plug these back into the torque equation.

$$\mu_s n = I \frac{a}{R^2} \Rightarrow \mu_s = \frac{I}{R^2} \frac{a}{n} = \frac{\left(\frac{1}{2} MR^2 \right)}{R^2} \frac{g(\sin \theta - \mu_s \cos \theta)}{Mg \cos \theta} = \frac{1}{2} \frac{(\sin \theta - \mu_s \cos \theta)}{\cos \theta}$$

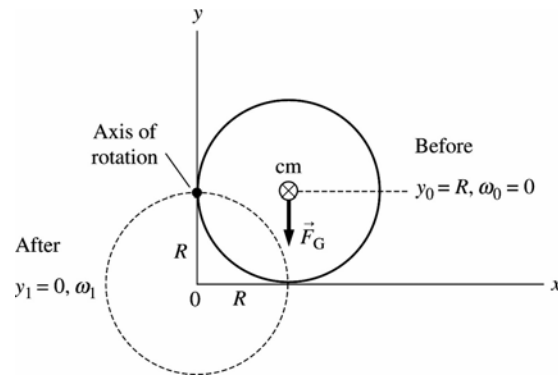
There are μ_s on both sides of the equation, but we rearrange terms, and isolate μ_s .

$$2\mu_s \cos \theta = \sin \theta - \mu_s \cos \theta \Rightarrow 3\mu_s \cos \theta = \sin \theta \Rightarrow \mu_s = \frac{1}{3} \tan \theta$$

Assess: It is reasonable that the coefficient of static friction depends on the angle in this way, but not on M , L , or R . Also, as another check, see that in the result for the acceleration if the friction is zero then the acceleration is what it was for a particle sliding down a frictionless slope.

12.72. Model: The disk is a rigid body rotating on an axle passing through one edge. The gravitational potential energy is transformed into rotational kinetic energy as the disk is released.

Visualize:



We placed the origin of the coordinate system at a distance R just below the axle. In the initial position, the center of mass of the disk is at the same level as the axle. The center of mass of the disk in the final position is coincident with the origin of the coordinate system.

Solve: (a) The torque is due to the gravitational force on the disk acting at the center of mass. Thus

$$\tau = (mg)R = (5.0 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m}) = 14.7 \text{ Nm}$$

The moment of inertia about the disk's edge is obtained using the parallel-axis theorem:

$$I = I_{\text{cm}} + mR^2 = \frac{1}{2}mR^2 + mR^2 = \frac{3}{2}mR^2 = \left(\frac{3}{2}\right)(5.0 \text{ kg})(0.30 \text{ m})^2 = 0.675 \text{ kg m}^2$$

$$\Rightarrow \alpha = \frac{\tau}{I} = \frac{14.7 \text{ Nm}}{0.675 \text{ kg m}^2} = 22 \text{ rad/s}^2$$

(b) The energy conservation equation $K_f + U_{\text{gf}} = K_i = U_{\text{gi}}$ is

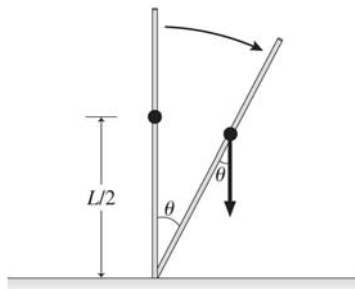
$$\frac{1}{2}I\omega_1^2 + mgy_1 = \frac{1}{2}I\omega_0^2 + mgy_0 \Rightarrow \frac{1}{2}I\omega_1^2 + 0 \text{ J} = 0 \text{ J} + mgR$$

$$\omega_1 = \sqrt{\frac{2mgR}{I}} = \sqrt{\frac{2(5.0 \text{ kg})(9.8 \text{ m/s}^2)(0.30 \text{ m})}{0.675 \text{ kg m}^2}} = 6.6 \text{ rad/s}$$

Assess: An angular velocity of 6.6 rad/s (or 1.05 revolutions/s) as the center of mass of the disk reaches below the axle is reasonable.

12.73. Model: Assume the rod has uniform density so the center of mass is at the center.

Visualize: Use conservation of energy.



Solve: (a) With friction so the bottom tip can't move, the potential energy is transformed into rotational kinetic energy.

$$U_i = K_f \Rightarrow Mg\left(\frac{L}{2}\right) = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\left(\frac{v}{L/2}\right)^2 \Rightarrow Lg = \frac{1}{3}\left(\frac{v}{1/2}\right)^2 \Rightarrow v = \sqrt{\frac{3}{4}Lg}$$

(b) If there is no friction, then the center of mass falls straight down and the potential energy is transformed into translational kinetic energy.

$$U_i = K_f \Rightarrow mg\left(\frac{L}{2}\right) = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{Lg}$$

Assess: The final speed is faster without friction, as we would expect. An alternate solution for part (a) is the gravitational torque around the point of contact is $\tau = (L/2)(\sin\theta)Mg$ which varies as the rod falls. Find the work done as it falls.

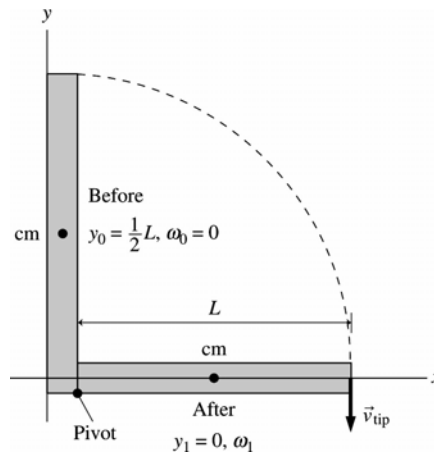
$$W = \int \tau d\theta = \frac{L}{2}Mg \int_{0^\circ}^{90^\circ} \sin\theta d\theta = \frac{L}{2}Mg[-\cos\theta]_{0^\circ}^{90^\circ} = \frac{L}{2}Mg$$

Use the work-kinetic energy theorem with $I_{\text{rod}} = \frac{1}{3}MR^2$ for an axis on one end.

$$W = \Delta K \Rightarrow \frac{L}{2}Mg = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{3}ML^2\right)\left(\frac{v}{L/2}\right)^2 \Rightarrow Lg = \frac{4}{3}v^2 \Rightarrow v = \sqrt{\frac{3}{4}Lg}$$

12.74. Model: The long, thin rod is a rigid body rotating about a frictionless pivot on the end of the rod. The gravitational torque on the rod causes it to rotate, transforming the gravitational potential energy of the rod's center of mass into rotational kinetic energy.

Visualize:



We placed the origin of the coordinate system at the pivot point. In the initial position, the center of mass is a distance $\frac{1}{2}L$ above the origin. In the final position, the center of mass is at $y = 0$ m and thus has zero gravitational potential energy.

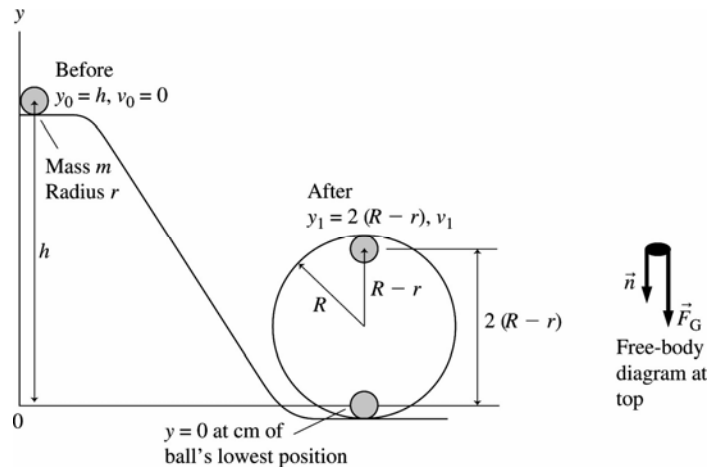
Solve: (a) The energy conservation equation for the rod $K_f + U_{gf} = K_i + U_{gi}$ is

$$\frac{1}{2}I\omega_1^2 + mgy_1 = \frac{1}{2}I\omega_0^2 + mgy_0 \quad \frac{1}{2}\left(\frac{1}{3}mL^2\right)\omega_1^2 + 0 = 0 + mg(L/2) \Rightarrow \omega_1 = \sqrt{3g/L}$$

(b) The speed at the tip of the rod is $v_{\text{tip}} = (\omega_1)L = \sqrt{3gL}$.

12.75. Model: Assume that the marble does not slip as it rolls down the track and around a loop-the-loop. The mechanical energy of the marble is conserved.

Visualize:



Solve: The marble's center of mass moves in a circle of radius $R - r$. The free-body diagram on the marble at its highest position shows that Newton's second law for the marble is

$$mg + n = \frac{mv_1^2}{R - r}$$

The minimum height (h) that the track must have for the marble to make it around the loop-the-loop occurs when the normal force of the track on the marble tends to zero. Then the weight will provide the centripetal acceleration needed for the circular motion. For $n \rightarrow 0$ N,

$$mg = \frac{mv^2}{(R - r)} \Rightarrow v_1^2 = g(R - r)$$

Since rolling motion requires $v_1^2 = r^2 \omega_1^2$, we have

$$\omega_1^2 r^2 = g(R - r) \Rightarrow \omega_1^2 = \frac{g(R - r)}{r^2}$$

The conservation of energy equation is

$$(K_f + U_{gf})_{\text{top of loop}} = (K_i + U_{gi})_{\text{initial}} \Rightarrow \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2 + mgy_1 = mgy_0 = mgh$$

Using the above expressions and $I = \frac{2}{5}mr^2$ the energy equation simplifies to

$$\frac{1}{2}mg(R - r) + \frac{1}{2}\left(\frac{2}{5}\right)mr^2\left(\frac{g(R - r)}{r^2}\right) + mg2(R - r) = mgh \Rightarrow h = 2.7(R - r)$$

12.76. Model: The sphere attached to a thin rod is a rigid body rotating about the rod. Assume the rod is vertical and the sphere solid.

Visualize: Please refer to Figure P12.76. The sphere rotates because the string wrapped around the rod exerts a torque τ .

Solve: The torque exerted by the string on the rod is $\tau = Tr$.

From the parallel-axis theorem, the moment of inertia of the sphere about the rod's axis is

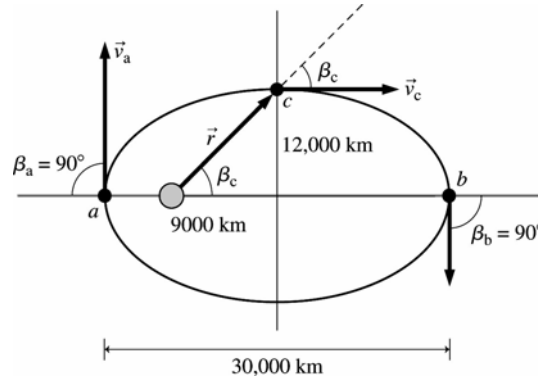
$$I_{\text{off center}} = I_{\text{cm}} + M\left(\frac{R}{2}\right)^2 = \frac{2}{5}MR^2 + \frac{MR^2}{4} = \frac{13}{20}MR^2$$

From Newton's second law,

$$\alpha = \frac{\tau}{I} = \frac{Tr}{(13MR^2/20)} = \frac{20Tr}{13MR^2}$$

12.77. Model: The angular momentum of the satellite in the elliptical orbit is a constant.

Visualize:



Solve: (a) Because the gravitational force is always along the same direction as the direction of the moment arm vector, the torque $\vec{\tau} = \vec{r} \times \vec{F}_g$ is zero at all points on the orbit.

(b) The angular momentum of the satellite at any point on the elliptical trajectory is conserved. The velocity is perpendicular to $\vec{r}$ at points a and b, so $\beta = 90^\circ$ and $L = mvr$. Thus

$$\begin{aligned} L_b &= L_a \Rightarrow mv_b r_b = mv_a r_a \Rightarrow v_b = \left(\frac{r_a}{r_b} \right) v_a \\ r_a &= \frac{30,000 \text{ km}}{2} - 9000 \text{ km} = 6000 \text{ km} \quad \text{and} \quad r_b = \frac{30,000 \text{ km}}{2} + 9000 \text{ km} = 24,000 \text{ km} \\ \Rightarrow v_b &= \left(\frac{6000 \text{ km}}{24,000 \text{ km}} \right) (8000 \text{ m/s}) = 2000 \text{ m/s} \end{aligned}$$

(c) Using the conservation of angular momentum $L_c = L_a$, we get

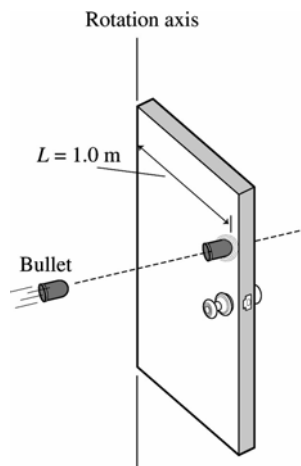
$$mv_c r_c \sin \beta_c = mv_a r_a \Rightarrow v_c = \left(\frac{r_a}{r_c} \right) v_a / \sin \beta_c \quad r_c = \sqrt{(9000 \text{ km})^2 + (12,000 \text{ km})^2} = 1.5 \times 10^7 \text{ m}$$

From the figure, we see that $\sin \beta_c = 12,000/15,000 = 0.80$. Thus

$$v_c = \left(\frac{6000 \text{ km}}{15,000 \text{ km}} \right) \frac{(8000 \text{ m/s})}{0.80} = 4000 \text{ m/s}$$

12.78. Model: For the (bullet + door) system, the angular momentum is conserved in the collision.

Visualize:



Solve: As the bullet hits the door, its velocity $\vec{v}$ is perpendicular to $\vec{r}$. Thus the initial angular momentum about the rotation axis, with $r = L$, is

$$L_i = m_B v_B L = (0.010 \text{ kg})(400 \text{ m/s})(1.0 \text{ m}) = 4.0 \text{ kg m}^2/\text{s}$$

After the collision, with the bullet in the door, the moment of inertia about the hinges is

$$I = I_{\text{door}} + I_{\text{bullet}} = \frac{1}{3} m_D L^2 + m_B L^2 = \frac{1}{3} (10.0 \text{ kg})(1.0 \text{ m})^2 + (0.010 \text{ kg})(1.0 \text{ m})^2 = 3.343 \text{ kg m}^2$$

Therefore, $L_f = I\omega = (3.343 \text{ kg m}^2)\omega$. Using the angular momentum conservation equation $L_f = L_i$, $(3.343 \text{ kg m}^2)\omega = 4.0 \text{ kg m}^2/\text{s}$ and thus $\omega = 1.2 \text{ rad/s}$.

12.79. Model: Model the turntable as a rigid disk rotating on frictionless bearings. For the (turntable + block) system, no external torques act as the block moves outward toward the outer edge. Angular momentum is thus conserved.

Visualize: The initial moment of inertia of the turntable is I_1 and the final moment of inertia is I_2 .

Solve: The initial moment of inertia is $I_1 = I_{\text{disk}} = \frac{1}{2} m R^2 = \frac{1}{2} (0.2 \text{ kg})(0.2 \text{ m})^2 = 0.0040 \text{ kg m}^2$. As the block reaches the outer edge, the final moment of inertia is

$$\begin{aligned} I_2 &= I_1 + m_B R^2 = 0.0040 \text{ kg m}^2 + (0.020 \text{ kg})(0.20 \text{ m})^2 \\ &= 0.0040 \text{ kg m}^2 + 0.0008 \text{ kg m}^2 = 0.0048 \text{ kg m}^2 \end{aligned}$$

Let ω_1 and ω_2 be the initial and final angular velocities, then the conservation of angular momentum equation is

$$L_f = L_i \Rightarrow \omega_2 I_2 = \omega_1 I_1 \Rightarrow \omega_2 = \frac{I_1 \omega_1}{I_2} = \frac{(0.0040 \text{ kg m}^2)(60 \text{ rpm})}{(0.0048 \text{ kg m}^2)} = 50 \text{ rpm}$$

Assess: A change of angular velocity from 60 rpm to 50 rpm with an increase in the value of the moment of inertia is reasonable.

12.80. Model: Assume the merry-go-round is a disk and Luc is a cylinder. Assume the frictional torque is negligible on the system of merry-go-round-plus-Luc. Assume the dumbbells contribute nothing to the moment of inertia when they are brought into Luc's chest.

Visualize: We are given $m_{\text{Luc}} = 96.9 \text{ kg}$, $m_{\text{mgr}} = 153 \text{ kg}$.

Solve: Use conservation of angular momentum because there is no net external torque on the system.

$$\begin{aligned}
 L_i &= L_f \Rightarrow (I_{\text{mgr}} + (I_{\text{Luc}})_i) \omega_i = (I_{\text{mgr}} + (I_{\text{Luc}})_f) \omega_f \\
 \omega_f &= \omega_i \frac{(I_{\text{mgr}} + (I_{\text{Luc}})_i)}{(I_{\text{mgr}} + (I_{\text{Luc}})_f)} \\
 &= (35 \text{ rpm}) \frac{\frac{1}{2}(153 \text{ kg})(2.0 \text{ m})^2 + \frac{1}{2}(96.9 \text{ kg})(0.20 \text{ m})^2 + 2(4.0 \text{ kg})(0.85 \text{ m})^2}{\frac{1}{2}(153 \text{ kg})(2.0 \text{ m})^2 + \frac{1}{2}(96.9 \text{ kg})(0.20 \text{ m})^2} = 36 \text{ rpm}
 \end{aligned}$$

Assess: The merry-go-round did not speed up much because its own moment of inertia dominated and the decrease in I was small compared to the original I . We did not need Luc's height.

12.81. Model: Model the merry-go-round as a rigid disk rotating on frictionless bearings about an axle in the center and John as a particle. For the (merry-go-round + John) system, no external torques act as John jumps on the merry-go-round. Angular momentum is thus conserved.

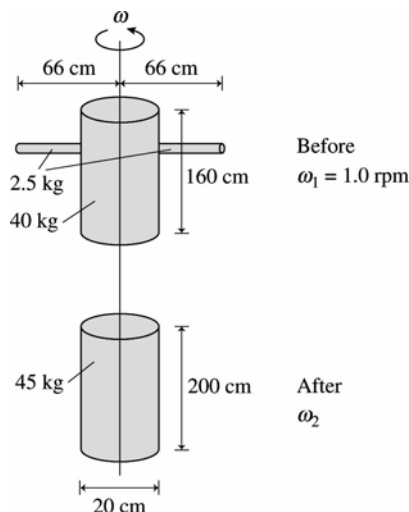
Visualize: The initial angular momentum is the sum of the angular momentum of the merry-go-round and the angular momentum of John. The final angular momentum as John jumps on the merry-go-round is equal to $I_{\text{final}} \omega_{\text{final}}$.

Solve: John's initial angular momentum is that of a particle: $L_J = m_J v_J R \sin \beta = m_J v_J R$. The angle $\beta = 90^\circ$ since John runs tangent to the disk. The conservation of angular momentum equation $L_f = L_i$ is

$$\begin{aligned}
 I_{\text{final}} \omega_{\text{final}} &= L_{\text{disk}} + L_J = \left(\frac{1}{2} M R^2 \right) \omega_i + m_J v_J R \\
 &= \left(\frac{1}{2} \right) (250 \text{ kg})(1.5 \text{ m})^2 (20 \text{ rpm}) \frac{2\pi}{60} \left(\frac{\text{rad}}{\text{rpm}} \right) + (30 \text{ kg})(5.0 \text{ m/s})(1.5 \text{ m}) = 814 \text{ kg m}^2/\text{s} \\
 \Rightarrow \omega_{\text{final}} &= \frac{814 \text{ kg m}^2/\text{s}}{I_{\text{final}}} \\
 I_{\text{final}} &= I_{\text{disk}} + I_J = \frac{1}{2} M R^2 + m_J R^2 = \frac{1}{2} (250 \text{ kg})(1.5 \text{ m})^2 + (30 \text{ kg})(1.5 \text{ m})^2 = 349 \text{ kg m}^2 \\
 \omega_{\text{final}} &= \frac{814 \text{ kg m}^2/\text{s}}{349 \text{ kg m}^2} = 2.33 \text{ rad/s} = 22 \text{ rpm}
 \end{aligned}$$

12.82. Model: Model the skater as a cylindrical torso with two rod-like arms that are perpendicular to the axis of the torso in the initial position and collapse into the torso in the final position.

Visualize:



Solve: For the initial position, the moment of inertia is $I_1 = I_{\text{Torso}} + 2I_{\text{Arm}}$. The moment of inertia of each arm is that of a 66-cm-long rod rotating about a point 10 cm from its end, and can be found using the parallel-axis theorem. In the final position, the moment of inertia is $I_2 = \frac{1}{2}MR^2$. The equation for the conservation of angular momentum $L_f = L_i$ can be written $I_2\omega_2 = I_1\omega_1 \Rightarrow \omega_2 = (I_1/I_2)\omega_1$. Calculating I_1 and I_2 ,

$$I_1 = \frac{1}{2}M_T R^2 + 2 \left[\frac{1}{12}M_A L_A^2 + M_A d^2 \right]$$

$$= \frac{1}{2}(40 \text{ kg})(0.10 \text{ m})^2 + 2 \left[\frac{1}{12}(2.5 \text{ kg})(0.66 \text{ m})^2 + (2.5 \text{ kg})(0.33 \text{ m} + 0.10 \text{ m})^2 \right] = 1.306 \text{ kg m}^2$$

$$I_2 = \frac{1}{2}MR^2 = \frac{1}{2}(45 \text{ kg})(0.10 \text{ m})^2 = 0.225 \text{ kg m}^2 \Rightarrow \omega_2 = \frac{(1.306 \text{ kg m}^2)}{(0.225 \text{ kg m}^2)}(1.0 \text{ rev/s}) = 5.8 \text{ rev/s}$$

reasonable.

12.83. Model: Because no external torque acts on the star during gravitational collapse, its angular momentum is conserved. Model the star as a solid rotating sphere.

Solve: (a) The equation for the conservation of angular momentum is

$$L_i = L_f \Rightarrow I_i \omega_i = I_f \omega_f \Rightarrow \left(\frac{2}{5} m R_i^2 \right) \omega_i = \left(\frac{2}{5} m R_f^2 \right) \omega_f$$

$$\Rightarrow R_f = R_i \sqrt{\frac{\omega_i}{\omega_f}}$$

The angular velocity is inversely proportional to the period T . We can write

$$R_f = R_i \sqrt{\frac{T_f}{T_i}} = (7.0 \times 10^8 \text{ m}) \sqrt{\frac{0.10 \text{ s}}{2.592 \times 10^6 \text{ s}}} = 1.3749 \times 10^5 \text{ m} = 137 \text{ km}$$

(b) A point on the equator rotates with $r = R_f$. Its speed is

$$v = \frac{2\pi R_f}{T} = \frac{2\pi(137,490 \text{ m})}{0.10 \text{ s}} = 8.6 \times 10^6 \text{ m/s}$$

12.84. Model: Assume the earth is a uniform sphere.

Visualize: Look up $m_E = 6.0 \times 10^{24} \text{ kg}$, $r_E = 6.37 \times 10^6 \text{ m}$. Calculate: $26000 \text{ yr} = 8.2 \times 10^{22} \text{ s}$, $1 \text{ d} = 24 \text{ h} = 86400 \text{ s}$.

Solve: Solve the equation for the torque.

$$\tau = \Omega I \omega = \Omega \left(\frac{2}{5} m_E r_E^2 \right) \omega = \left(\frac{2\pi}{26000 \text{ yr}} \right) \left(\frac{2}{5} (6.0 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 \right) \left(\frac{2\pi}{24 \text{ h}} \right)$$

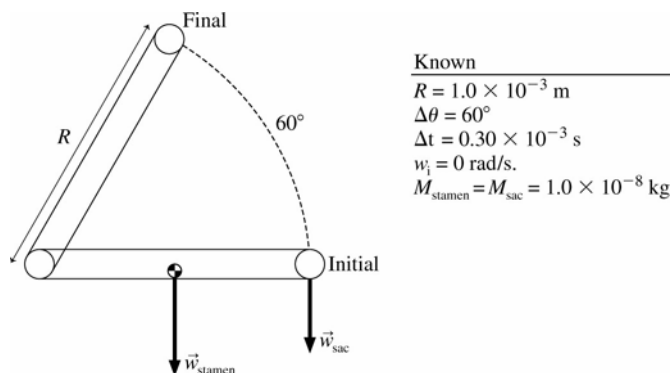
$$= \left(\frac{2\pi}{8.2 \times 10^{11} \text{ s}} \right) \left(\frac{2}{5} (6.0 \times 10^{24} \text{ kg})(6.37 \times 10^6 \text{ m})^2 \right) \left(\frac{2\pi}{86400 \text{ s}} \right) = 5.4 \times 10^{22} \text{ N} \cdot \text{m}$$

Assess: The torque from the moon and sun on the earth is huge, as it must be to change the angular momentum of the earth.

Challenge Problems

12.85. Model: The sacs are constrained by the stamens. The gravitational torque is 2000 times less than the straightening torque, so ignore it in part b.

Visualize: Refer to the diagram below.



Solve: (a) The moment of inertia of the stamen and sac can be calculated using kinematic equations.

$$I = m_{\text{stamen}} \left(\frac{R}{2} \right)^2 + m_{\text{sac}} R^2 = (1.0 \times 10^{-6} \text{ kg})(5.0 \times 10^{-4} \text{ m})^2 + (1.0 \times 10^{-6} \text{ kg})(1.0 \times 10^{-3} \text{ m})^2 = 1.25 \times 10^{-12} \text{ kg} \cdot \text{m}^2$$

An additional significant figure has been kept in this intermediate result.

The “straightening torque” is

$$\tau = I\alpha = (1.25 \times 10^{-12} \text{ kg} \cdot \text{m}^2)(2.32 \times 10^7 \text{ rad/s}^2) = 2.91 \times 10^{-5} \text{ N} \cdot \text{m} \approx 2.9 \times 10^{-5} \text{ N} \cdot \text{m}$$

(b) We can use the kinematic equations to find the angular acceleration of a sac, and then the corresponding tangential acceleration.

$$\alpha = \frac{2\Delta\theta}{\Delta t^2} = \frac{2(60^\circ) \left(\frac{2\pi \text{ rad}}{360^\circ} \right)}{(0.30 \times 10^{-3} \text{ s})^2} = 2.32 \times 10^7 \text{ rad/s}^2$$

The tangential acceleration is

$$a_t = \alpha R = (2.32 \times 10^7 \text{ rad/s}^2)(1.0 \times 10^{-3} \text{ m}) = 2.32 \times 10^4 \text{ m/s}^2$$

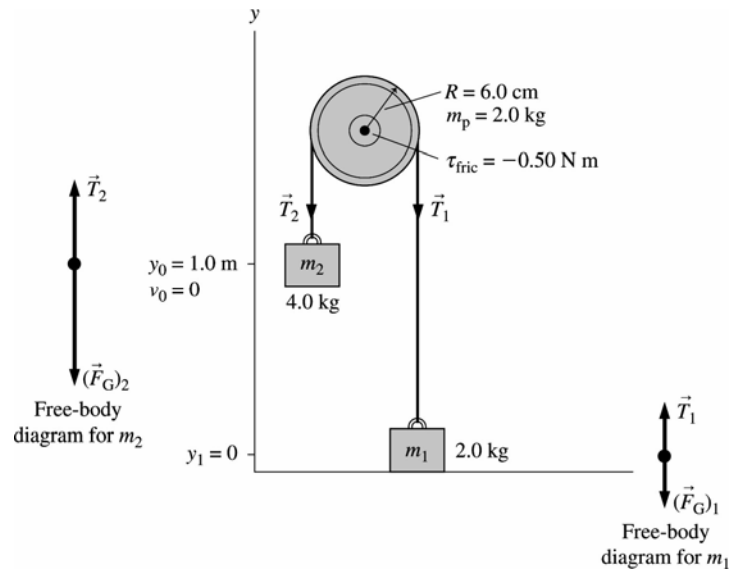
Using the kinematic equations with the result above gives

$$v_f = a_t \Delta t = (2.32 \times 10^4 \text{ m/s}^2)(0.30 \times 10^{-3} \text{ s}) = 7.0 \text{ m/s}$$

Assess: These results seem reasonable. The accelerations are huge, while the torque is relatively small. This is due to the relatively low moment of inertia of the structure.

12.86. Model: The pulley is a rigid rotating body. We also assume that the pulley has the mass distribution of a disk and that the string does not slip.

Visualize:



Because the pulley is not massless and frictionless, tension in the rope on both sides of the pulley is *not* the same.

Solve: Applying Newton's second law to m_1 , m_2 , and the pulley yields the three equations:

$$T_1 - (F_G)_1 = m_1 a_1 \quad -(F_G)_2 + T_2 = m_2 a_2 \quad T_2 R - T_1 R - 0.50 \text{ Nm} = I \alpha$$

Noting that $-a_2 = a_1 = a$, $I = \frac{1}{2} m_p R^2$, and $\alpha = a/R$, the above equations simplify to

$$T_1 - m_1 g = m_1 a \quad m_2 g - T_2 = m_2 a \quad T_2 - T_1 = \left(\frac{1}{2} m_p R^2 \right) \left(\frac{a}{R} \right) \frac{1}{R} + \frac{0.50 \text{ Nm}}{R} = \frac{1}{2} m_p a + \frac{0.50 \text{ Nm}}{0.060 \text{ m}}$$

Adding these three equations,

$$(m_2 - m_1)g = a \left(m_1 + m_2 + \frac{1}{2} m_p \right) + 8.333 \text{ N}$$

$$\Rightarrow a = \frac{(m_2 - m_1)g - 8.333 \text{ N}}{m_1 + m_2 + \frac{1}{2} m_p} = \frac{(4.0 \text{ kg} - 2.0 \text{ kg})(9.8 \text{ m/s}^2) - 8.333 \text{ N}}{2.0 \text{ kg} + 4.0 \text{ kg} + (2.0 \text{ kg}/2)} = 1.610 \text{ m/s}^2$$

We can now use kinematics to find the time taken by the 4.0 kg block to reach the floor:

$$y_1 = y_0 + v_0(t_1 - t_0) + \frac{1}{2} a_2(t_1 - t_0)^2 \Rightarrow 0 = 1.0 \text{ m} + 0 + \frac{1}{2} (-1.610 \text{ m/s}^2)(t_1 - 0 \text{ s})^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2(1.0 \text{ m})}{(1.610 \text{ m/s}^2)}} = 1.1 \text{ s}$$

12.87. Solve: (a) In S.I. units λ comes in kg/m. Since x comes in m, for the units of cx^2 to be kg/m it must be that c comes in kg/m³.

(b) Since the density is a function of x we integrate over the whole rod to get M . We have $dm = \lambda dx$.

$$M = \int dm = \int \lambda dx = 2 \int_0^{L/2} cx^2 dx = 2c \left[\frac{1}{3} x^3 \right]_0^{L/2} = \frac{2}{3} c \frac{L^3}{8} = \frac{1}{12} c L^3$$

$$\Rightarrow c = \frac{12M}{L^3}$$

(c) We'll use our value of c at the end.

$$I = \int r^2 dm = 2 \int_0^{L/2} x^2 (cx^2) dx = 2c \left[\frac{1}{5} x^5 \right]_0^{L/2} = \frac{2}{5} c \left(\frac{L}{2} \right)^5 = \frac{c}{80} L^5$$

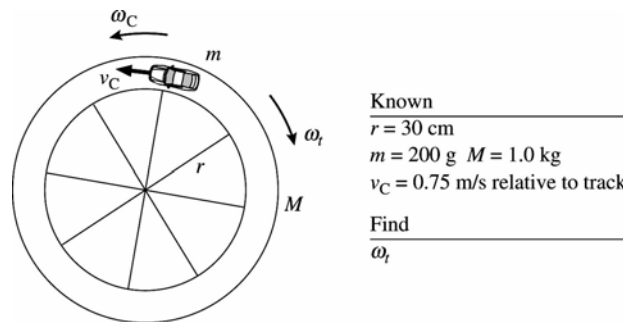
Now insert the value of c from part (b).

$$I = \frac{c}{80} L^5 = \frac{\left(\frac{12M}{L^3} \right)}{80} L^5 = \frac{3}{20} ML^2$$

Assess: The answer has the form of other moments of inertia.

12.88. Model: The toy car is a particle located at the rim of the track. The track is a cylindrical hoop rotating about its center, which is an axis of symmetry. No net torques are present on the track, so the angular momentum of the car and track is conserved.

Visualize:



Solve: The toy car's steady speed of 0.75 m/s relative to the track means that

$$v_c - v_t = 0.75 \text{ m/s} \Rightarrow v_c = v_t + 0.75 \text{ m/s},$$

where v_t is the velocity of a point on the track at the same radius as the car. Conservation of angular momentum implies that

$$L_i = L_f$$

$$0 = I_c \omega_c + I_t \omega_t = (mr^2) \omega_c + (Mr^2) \omega_t = m \omega_c + M \omega_t$$

The initial and final states refer to before and after the toy car was turned on. Table 12.2 was used for the track.

Since $\omega_c = \frac{v_c}{r}$, $\omega_t = \frac{v_t}{r}$, we have

$$0 = mv_c + Mv_t$$

$$\Rightarrow m(v_t + 0.75 \text{ m/s}) + Mv_t = 0$$

$$\Rightarrow v_t = -\frac{M}{m+M} (0.75 \text{ m/s}) = -\frac{(0.200 \text{ kg})}{(0.200 \text{ kg} + 1.0 \text{ kg})} (0.75 \text{ m/s}) = -0.125 \text{ m/s}$$

The minus sign indicates that the track is moving in the opposite direction of the car. The angular velocity of the track is

$$\omega_t = \frac{v_t}{r} = \frac{(0.125 \text{ m/s})}{0.30 \text{ m}} = 0.417 \text{ rad/s clockwise.}$$

In rpm,

$$\begin{aligned} \omega_t &= (0.417 \text{ rad/s}) \left(\frac{\text{rev}}{2\pi \text{ rad}} \right) \left(\frac{60 \text{ s}}{\text{min}} \right) \\ &= 4.0 \text{ rpm} \end{aligned}$$

Assess: The speed of the track is less than that of the car because it is more massive.

12.89. Model: Define the system as the rod and cube. Energy and angular momentum are conserved in a perfectly elastic collision in the absence of a net external torque. The rod is uniform.

Visualize: Please refer to Figure CP12.89.

Solve: Let the final speed of the cube be v_f , and the final angular velocity of the rod be ω . Energy is conserved, and angular momentum around the rod's pivot point is conserved.

$$E_i = E_f \Rightarrow \frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}I_{\text{rod}}\omega^2$$

$$L_i = L_f \Rightarrow mv_0\left(\frac{d}{2}\right) = mv_f\left(\frac{d}{2}\right) + I_{\text{rod}}\omega$$

This is two equations in the two unknowns v_f and ω . From Table 12.2,

$$I_{\text{rod}} = \frac{1}{12}Md^2 = \frac{1}{12}(2m)d^2 = \frac{1}{6}md^2$$

From the angular momentum equation,

$$v_0 = v_f + \frac{d}{3}\omega \Rightarrow \omega = \frac{3}{d}(v_0 - v_f)$$

Substituting into the energy equation,

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv_f^2 + \frac{1}{2}\left(\frac{1}{6}md^2\right)\left(\frac{9}{d^2}\right)(v_0 - v_f)^2$$

$$v_0^2 = v_f^2 + \frac{3}{2}(v_0 - v_f)^2$$

$$0 = v_f^2 - \frac{6}{5}v_0v_f + \frac{1}{5}v_0^2$$

This is a quadratic equation in v_f . The roots are

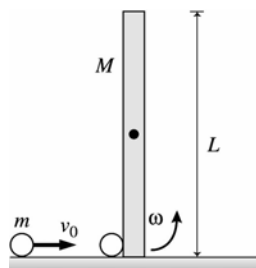
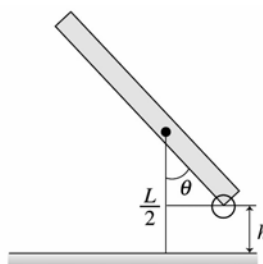
$$v_f = \frac{\frac{6}{5}v_0 \pm \sqrt{\left(\frac{6}{5}v_0\right)^2 - 4\left(\frac{1}{5}v_0^2\right)}}{2} = \frac{3}{5}v_0 \pm \frac{2}{5}v_0$$

$$= \begin{cases} \frac{1}{5}v_0 \\ v_0 \end{cases}$$

The answer $v_f = v_0$ means the ice cube missed the rod. So $v_f = \frac{1}{5}v_0$ to the right.

12.90. Model: The clay ball is a particle. The rod is a uniform thin rod rotating about its center. Angular momentum is conserved in the collision.

Visualize:



Known	
$L = 30 \text{ cm}$	
$M = 75 \text{ g}$	
$m = 10 \text{ g}$	
$V_0 = 2.5 \text{ m/s}$	
$h = \frac{L}{2}(1 - \cos\theta)$	
Find	
θ	

Solve: This is a two-part problem. Angular momentum is conserved in the collision, and energy is conserved as the ball rises like a pendulum. The angular momentum conservation equation about the rod's pivot point is

$$L_i = L_f \Rightarrow mv_0 r = (I_{\text{ball+rod}})\omega$$

Note $r = \frac{L}{2} = 15$ cm. The rod and ball are a composite object. From Table 12.2, $I_{\text{rod}} = \frac{1}{12}ML^2$, so

$$I_{\text{ball+rod}} = I_{\text{ball}} + I_{\text{rod}} = mr^2 + \frac{1}{12}ML^2 = m\frac{L^2}{4} + \frac{1}{12}ML^2 = \frac{L^2}{4}\left(m + \frac{M}{3}\right)$$

If v_f is the final velocity of the clay ball, $\omega = \frac{v_f}{r} = \frac{2v_f}{L}$ since the ball sticks to the rod. Thus

$$\begin{aligned} \frac{mv_0 L}{2} &= \frac{L^2}{4}\left(m + \frac{M}{3}\right)\left(\frac{2v_f}{L}\right) \\ \Rightarrow v_f &= \frac{mv_0}{m + \frac{M}{3}} = \frac{(0.010 \text{ kg})(2.5 \text{ m/s})}{(0.010 \text{ kg}) + \frac{(0.075 \text{ kg})}{3}} = 0.714 \text{ m/s} \end{aligned}$$

Energy is conserved as the clay ball rises. Compare the energy of the ball-rod system just after the collision to when the ball reaches the maximum height. Note that the center of mass of the rod does not change position.

$$E_i = E_f \Rightarrow \frac{1}{2}(I_{\text{rod+ball}})\omega^2 = mgh$$

Thus

$$\begin{aligned} \frac{1}{2}\frac{L^2}{4}\left(m + \frac{M}{3}\right)\left(\frac{2v_f}{L}\right)^2 &= mgh \Rightarrow v_f^2\left(m + \frac{M}{3}\right) = mgL(1 - \cos\theta) \\ \Rightarrow 1 - \frac{v_f^2}{mgL}\left(m + \frac{M}{3}\right) &= \cos\theta \end{aligned}$$

Using the various values, $\cos\theta = 0.393 \Rightarrow \theta = 67^\circ$.

Assess: The clay ball rises $h = \frac{L}{2}(1 - \cos\theta) = 9.1$ cm. This is about 2/3 of the height of the pivot point, and is reasonable.