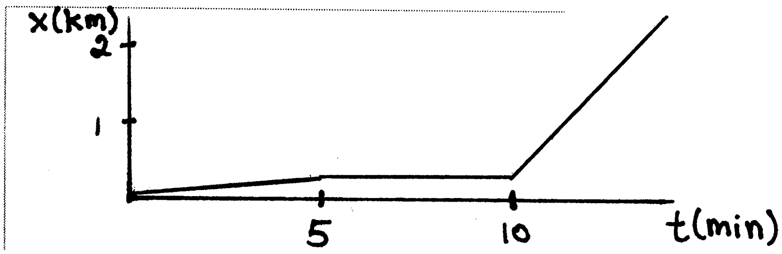
2 Motion in One Dimension

2.1 Describing Motion

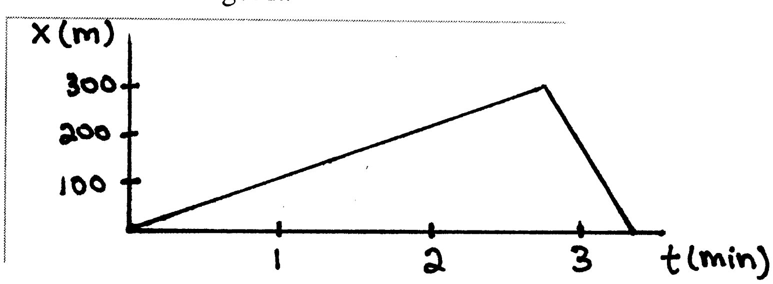
1. Sketch position-versus-time graphs for the following motions. Include a numerical scale on both axes with units that are *reasonable* for this motion. Some numerical information is given in the problem, but for other quantities, make reasonable estimates.

   Note: A *sketched* graph is hand-drawn, rather than laid out with a ruler. Even so, a sketch must be neat, accurate, and include axis labels.

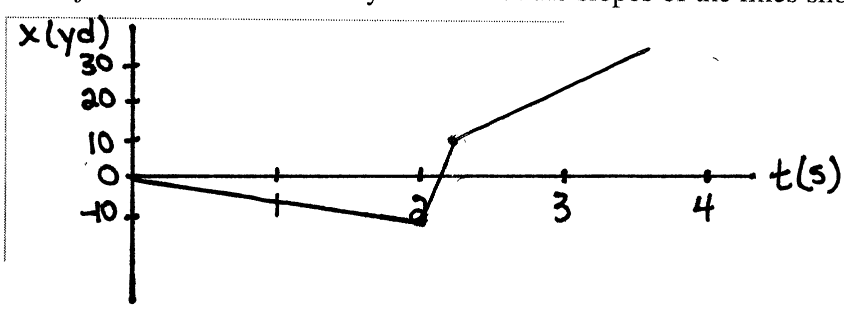
   a. A student walks to the bus stop, waits for the bus, and then rides to campus. Assume that all the motion is along a straight street.



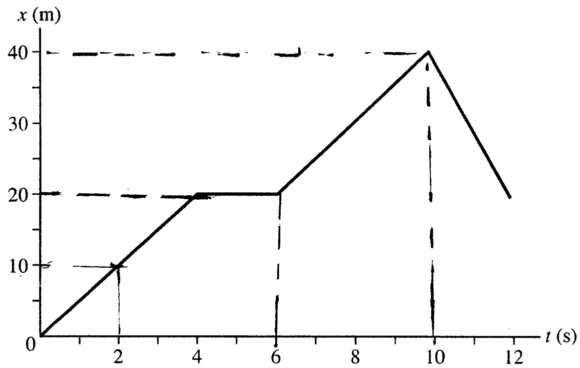
   b. A student walks slowly to the bus stop, realizes he forgot his paper that is due, and *quickly* walks home to get it.



   c. The quarterback drops back 10 yards from the line of scrimmage, and then throws a pass 20 yards to a receiver, who catches it and sprints 20 yards to the goal. Draw your graph for the *football*. Think carefully about what the slopes of the lines should be.



2. The position-versus-time graph below shows the position of an object moving in a straight line for 12 seconds.



a. What is the position of the object at 2 s, 6 s, and 10 s after the start of the motion?

At 2 s: 10 m

At 6 s: 20 m

At 10 s: 40 m

b. What is the object's velocity during the first 4 s of motion?

$5\,\frac{m}{s}$ $\left(= \frac{20\,m}{4\,s}\right)$

c. What is the object's velocity during the interval from $t = 4$ s to $t = 6$ s?

$0\,\frac{m}{s}$

d. What is the object's velocity during the four seconds from $t = 6$ s to $t = 10$ s?

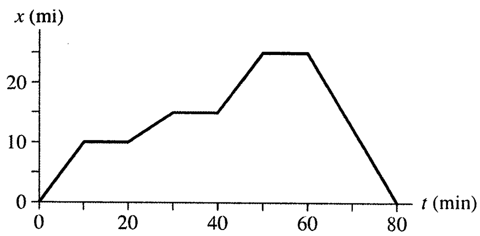
$5\,\frac{m}{s}$ $\left(= \frac{40\,m - 20\,m}{10\,s - 6\,s}\right)$

e. What is the object's velocity during the final two seconds from $t = 10$ s to $t = 12$ s?

$-10\,\frac{m}{s}$ $\left(= \frac{20\,m - 40\,m}{12\,s - 10\,s}\right)$

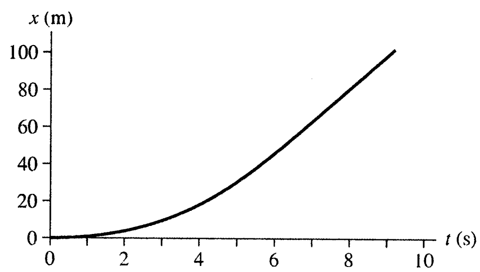
3. Interpret the following position-versus-time graphs by writing a very short "story" of what is happening. Be creative! Have characters and situations! Simply saying that "a car moves 100 meters to the right" doesn't qualify as a story. Your stories should make *specific reference* to information you obtain from the graphs, such as distances moved or time elapsed.

a. Moving car



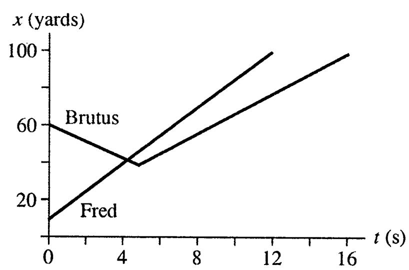
After driving 10 mph at 60 mph on the interstate, I stopped at a rest area for coffee. When I got back on the road 10 min later, I was slowed to 30 mph by a construction zone for 10 min. Finally, back up to cruising speed, I got off my exit 25 mi from home. After searching for 10 min I realized that I left my wallet at home so I drove back, without stops or construction delays, at 75 mph!

b. Sprinter



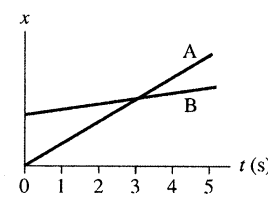
With a slow start out of the blocks, a super sprinter reached top speed in about 5 seconds, having gone only 30 meters. He still was able to finish his 100 meters in only just over 9 seconds by running a world record pace for the rest of the race.

c. Two football players



At the kickoff, Fred receives the ball on the 10 yd line and heads upfield at a sprint. Brutus runs towards Fred in an attempt to tackle him, but misses as Fred crosses the 50 yd line. Brutus vainly tries to catch up, but Fred scores. Foolishly, Brutus continues to run after Fred has already scored.

4. The figure shows a position-versus-time graph for the motion of objects A and B that are moving along the same axis.



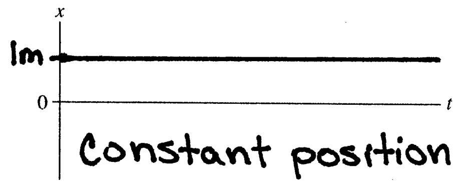
a. At the instant $t = 1$ s, is the speed of A greater than, less than, or equal to the speed of B? Explain.

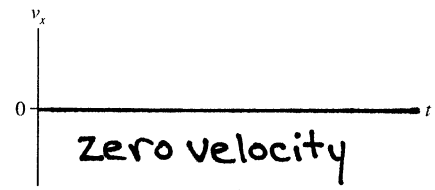
At $t = 1s$, the slope of the line for A is greater than that for object B. Therefore, object A's speed is greater. (Both are positive slopes.)

b. Do objects A and B ever have the *same* speed? If so, at what time or times? Explain.

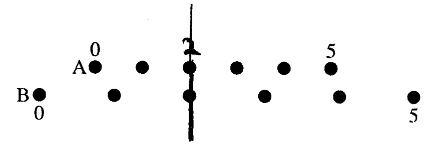
No, the speeds are never the same. Each has a constant speed (constant slope) and A's speed is always greater.

5. Draw both a position-versus-time graph *and* a velocity-versus-time graph for an object that is at rest at $x = 1$ m.

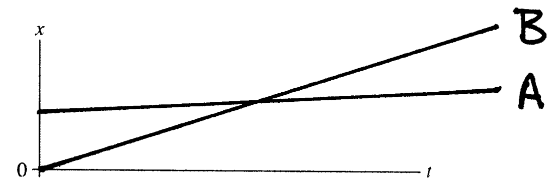


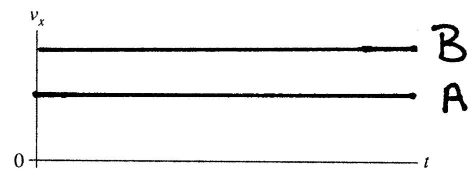


6. The figure shows six frames from the motion diagram of two moving cars, A and B.



a. Draw both a position-versus-time graph and a velocity-versus-time graph. Show the motion of *both* cars on each graph. Label them A and B.





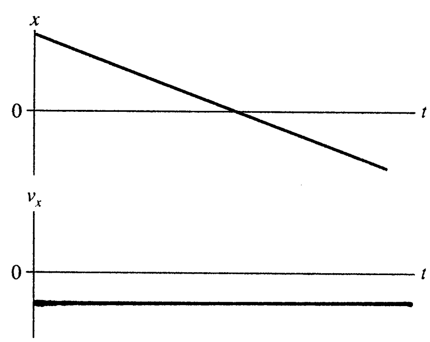
b. Do the two cars ever have the same position at one instant of time?

If so, in which frame number (or numbers)? yes, at 2

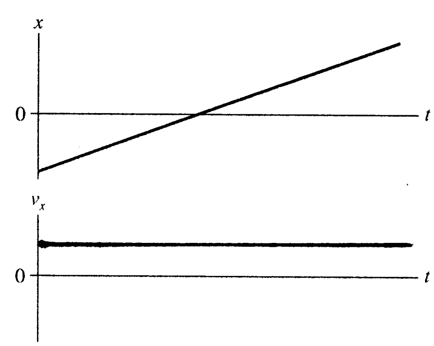
Draw a vertical line through your graphs of part a to indicate this instant of time.

7. Below are four position-versus-time graphs. For each, draw the corresponding velocity-versus-time graph directly below it. A vertical line drawn through both graphs should connect the velocity v_x at time t with the position x at the *same* time t. There are no numbers, but your graphs should correctly indicate the *relative* speeds.

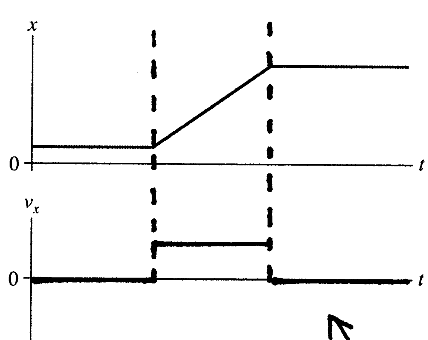
a.



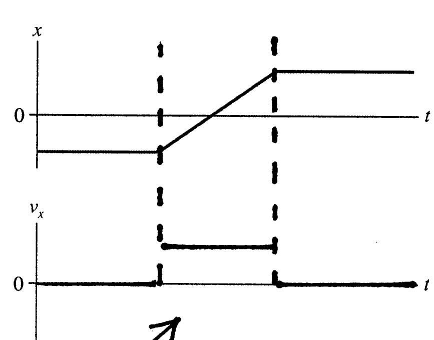
b.



c.



d.

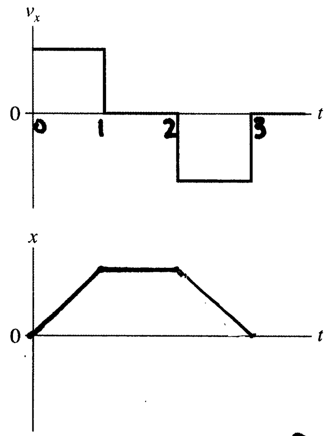


identical graphs

8. Below are two velocity-versus-time graphs. For each:
   - Draw the corresponding position-versus-time graph.
   - Give a written description of the motion.

   Assume that the motion takes place along a horizontal line and that $x_i = 0$.

a.

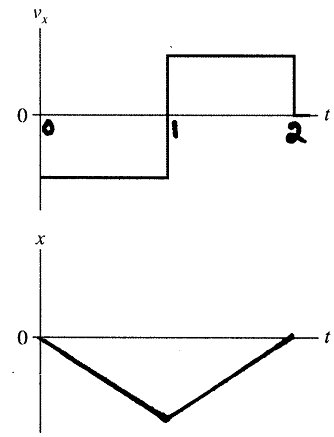


0 to 1 - moving forward at constant speed

1 to 2 - stationary

2 to 3 - moving backward at the same speed as from 0 to 1

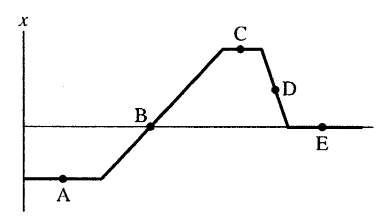
b.



0 to 1 - moving backward at constant speed

1 to 2 - moving forward at the same speed to return to the starting point

9. The figure shows a position-versus-time graph for a moving object. At which lettered point or points:



   a. Is the object moving the slowest without being at rest? B

   b. Is the object moving the fastest? D

   c. Is the object at rest? A, C, E

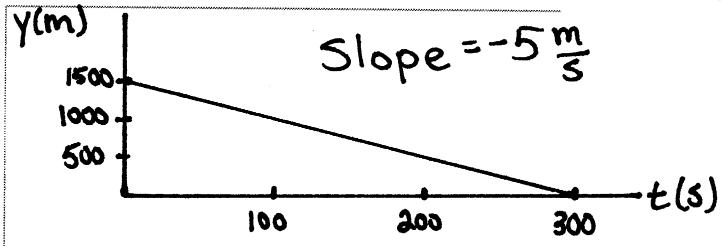
   d. Does the object have a constant nonzero velocity? B, D

   e. Is the object moving to the left? D

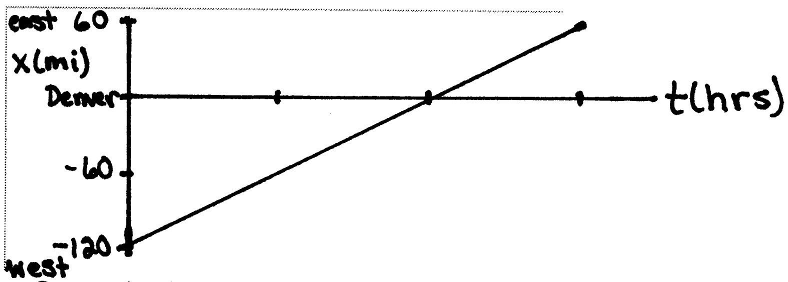
2.2 Uniform Motion

10. Sketch position-versus-time graphs for the following motions. Include appropriate numerical scales along both axes. A small amount of computation may be necessary.

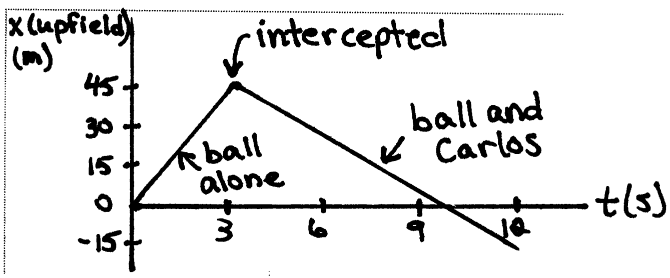
 a. A parachutist opens her parachute at an altitude of 1500 m. She then descends slowly to earth at a steady speed of 5 m/s. Start your graph as her parachute opens.



 b. Trucker Bob starts the day 120 miles west of Denver. He drives east for 3 hours at a steady 60 miles/hour before stopping for his coffee break. Let Denver be located at $x = 0$ mi and assume that the x-axis points to the east.



 c. Quarterback Bill throws the ball to the right at a speed of 15 m/s. It is intercepted 45 m away by Carlos, who is running to the left at 7.5 m/s. Carlos carries the ball 60 m to score. Let $x = 0$ m be the point where Bill throws the ball. Draw the graph for the *football*.



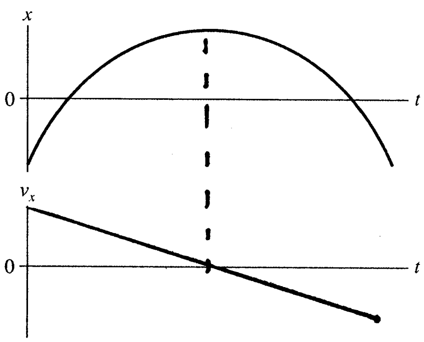
11. The height of a building is proportional to the number of stories it has. If a 25-story building is 260 ft tall, what is the height of an 80-story building? Answer this using ratios, not by calculating the height per story.

$$\frac{260 \text{ ft}}{25 \text{ st}} = \frac{h}{80 \text{ st}} \qquad h = \frac{80 \times 260}{25} = 832 \text{ ft.}$$

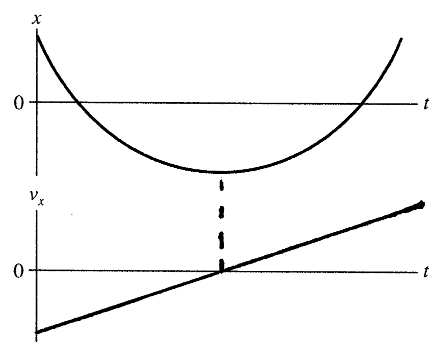
2.3 Instantaneous Velocity

12. Below are two position-versus-time graphs. For each, draw the corresponding velocity-versus-time graph directly below it. A vertical line drawn through both graphs should connect the velocity v_x at time t with the position x at the *same* time t. There are no numbers, but your graphs should correctly indicate the *relative* speeds.

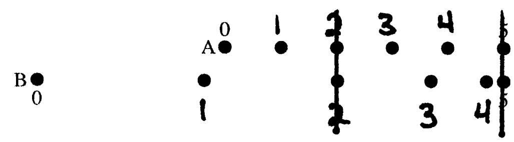
a.



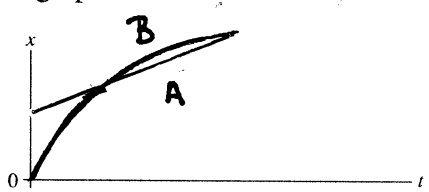
b.

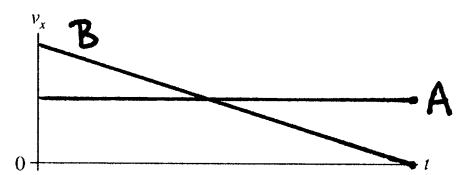


13. The figure shows six frames from the motion diagram of two moving cars, A and B.



a. Draw both a position-versus-time graph and a velocity-versus-time graph. Show *both* cars on each graph. Label them A and B.





b. Do the two cars ever have the same position at one instant of time? yes

If so, in which frame number (or numbers)? at 2 and 5

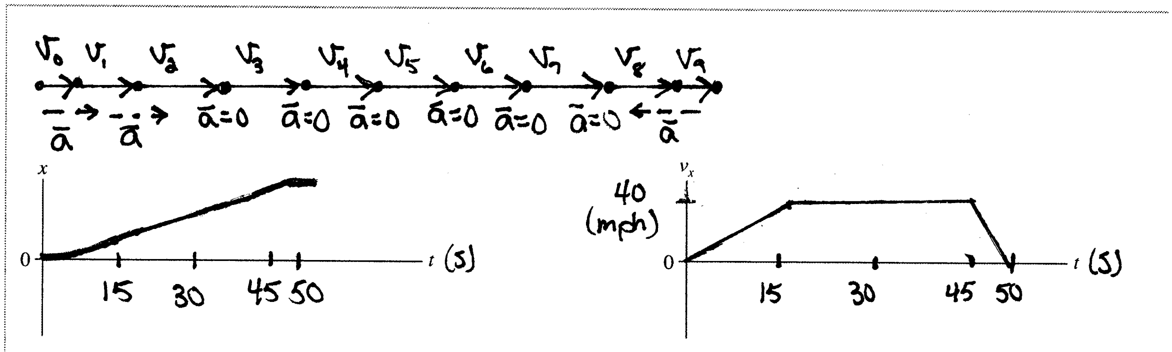
Draw a vertical line through your graphs of part a to indicate this instant of time.

c. Do the two cars ever have the same velocity at one instant of time? yes

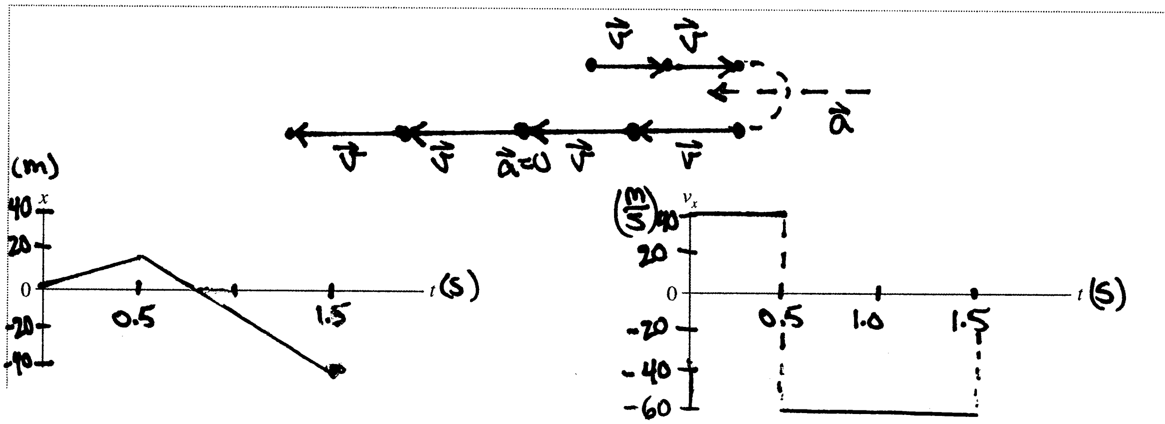
If so, between which two frames? from 3 to 4

14. For each of the following motions, draw
    - A motion diagram and
    - Both position and velocity graphs.

    a. A car starts from rest, steadily speeds up to 40 mph in 15 s, moves at a constant speed for 30 s, and then comes to a halt in 5 s.

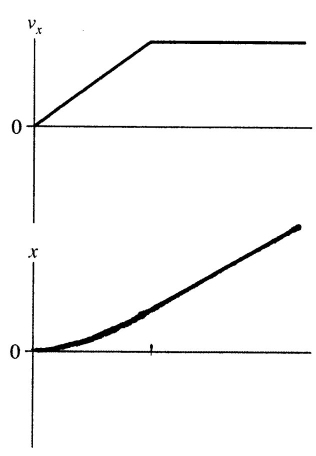


    b. A pitcher winds up and throws a baseball with a speed of 40 m/s. One-half second later, the batter hits a line drive with a speed of 60 m/s. The ball is caught 1 s after it is hit. From where you are sitting, the batter is to the right of the pitcher. Draw your motion diagram and graph for the *horizontal* motion of the ball.

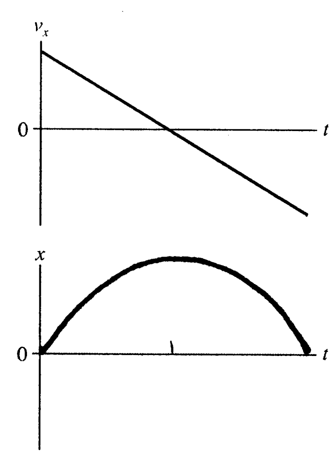


15. Below are three velocity-versus-time graphs. For each, draw the corresponding position-versus-time graph. Assume that the motion is horizontal and that $x_i = 0$.

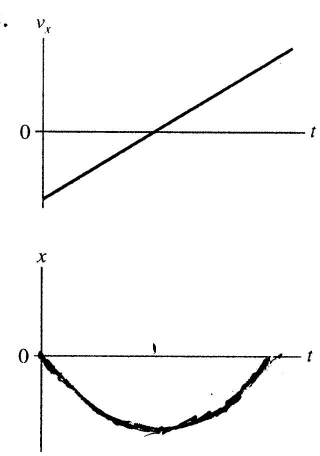
a.



b.

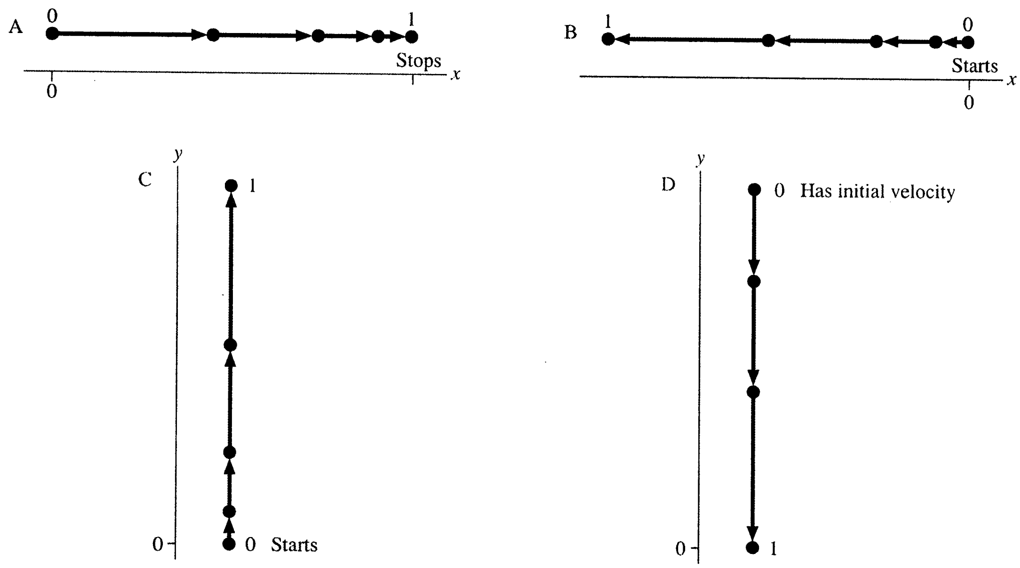


c.



2.4 Acceleration

16. The four motion diagrams below show an initial point 0 and a final point 1. A pictorial representation would define the five symbols: x_0, x_1, v_{0x}, v_{1x}, and a_x for horizontal motion and equivalent symbols with y for vertical motion. Determine whether each of these quantities is positive, negative, or zero. Give your answer by writing +, −, or 0 in the table below.



	A	B	C	D
x_0 or y_0	0	0	0	+
x_1 or y_1	+	−	+	0
v_{0x} or v_{0y}	+	0	0	−
v_{1x} or v_{1x}	0	−	+	−
a_x or a_y	−	−	+	−

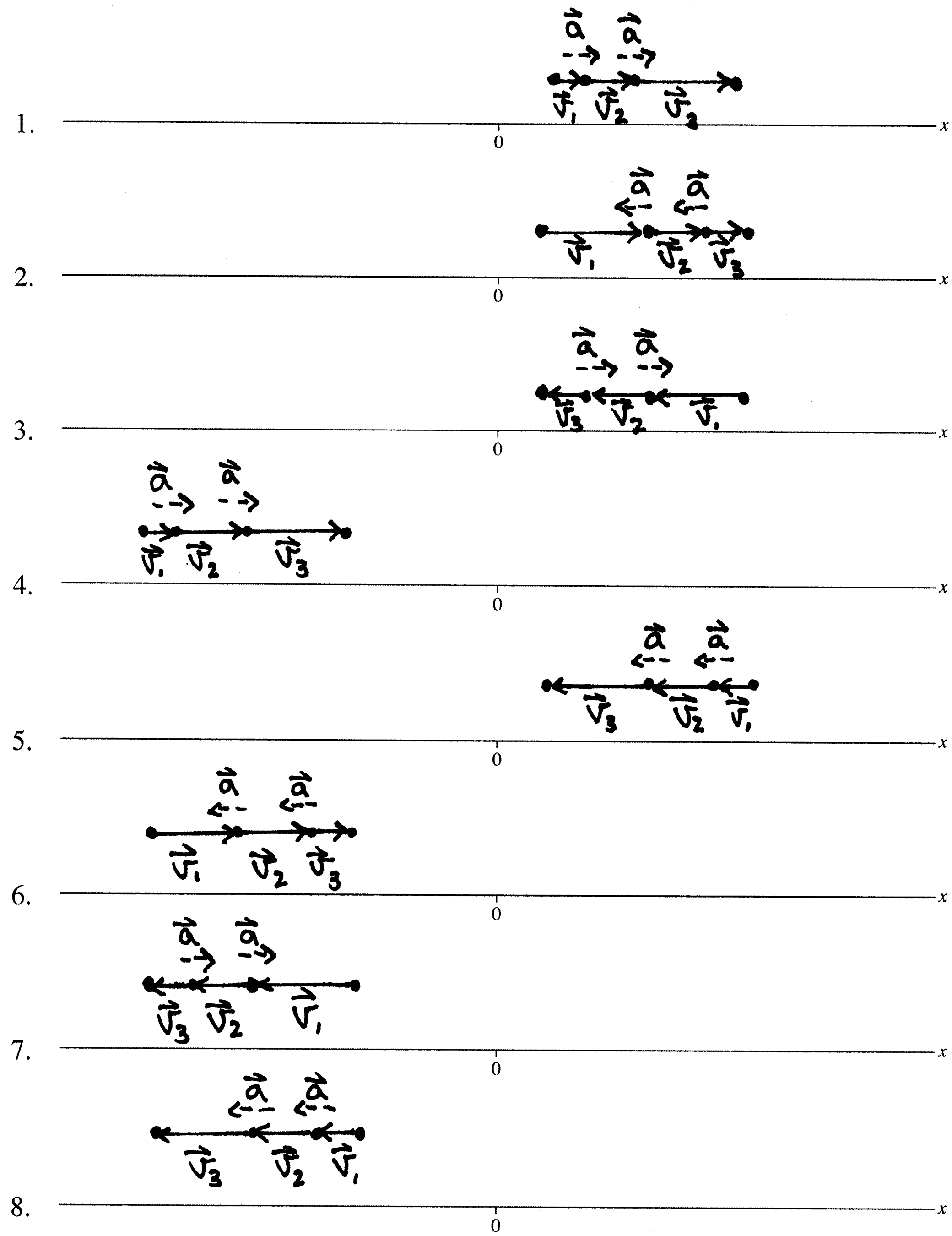
17. The three symbols x, v_x, and a_x have eight possible combinations of *signs*. For example, one combination is $(x, v_x, a_x) = (+, -, +)$.

a. List all eight combinations of signs for x, v_x, a_x.

1. + + +
2. + + −
3. + − +
4. − + +
5. + − −
6. − + −
7. − − +
8. − − −

b. For each of the eight combinations of signs you identified in part a:
- Draw a four-dot motion diagram of an object that has these signs for x, v_x, and a_x.
- Draw the diagram *above* the axis whose number corresponds to part a.
- Use **black** and **red** for your $\vec{v}$ and $\vec{a}$ vectors. Be sure to label the vectors.

—— Black
---- Red

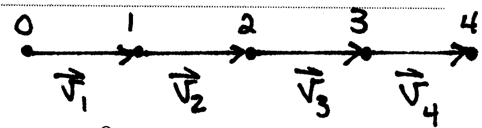


	x	v_x	a_x
1.	+	+	+
2.	+	+	−
3.	+	−	+
4.	−	+	+
5.	+	−	−
6.	−	+	−
7.	−	−	+
8.	−	−	−

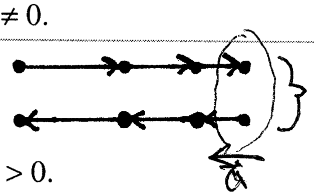
2.5 Motion with Constant Acceleration

18. Draw a motion diagram to illustrate each of the following situations.

a. $a_x = 0$ but $v_x \neq 0$.

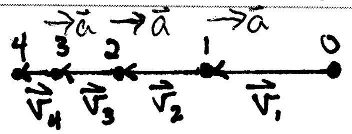


b. $v_x = 0$ but $a_x \neq 0$.



These two dots at the same horizontal location signify $v_x = 0$, $a_x < 0$

c. $v_x < 0$ and $a_x > 0$.



19. The quantity y is proportional to the square of x, and $y = 36$ when $x = 3$.

a. Find y if $x = 5$. 100

b. Find x if $y = 16$. 2

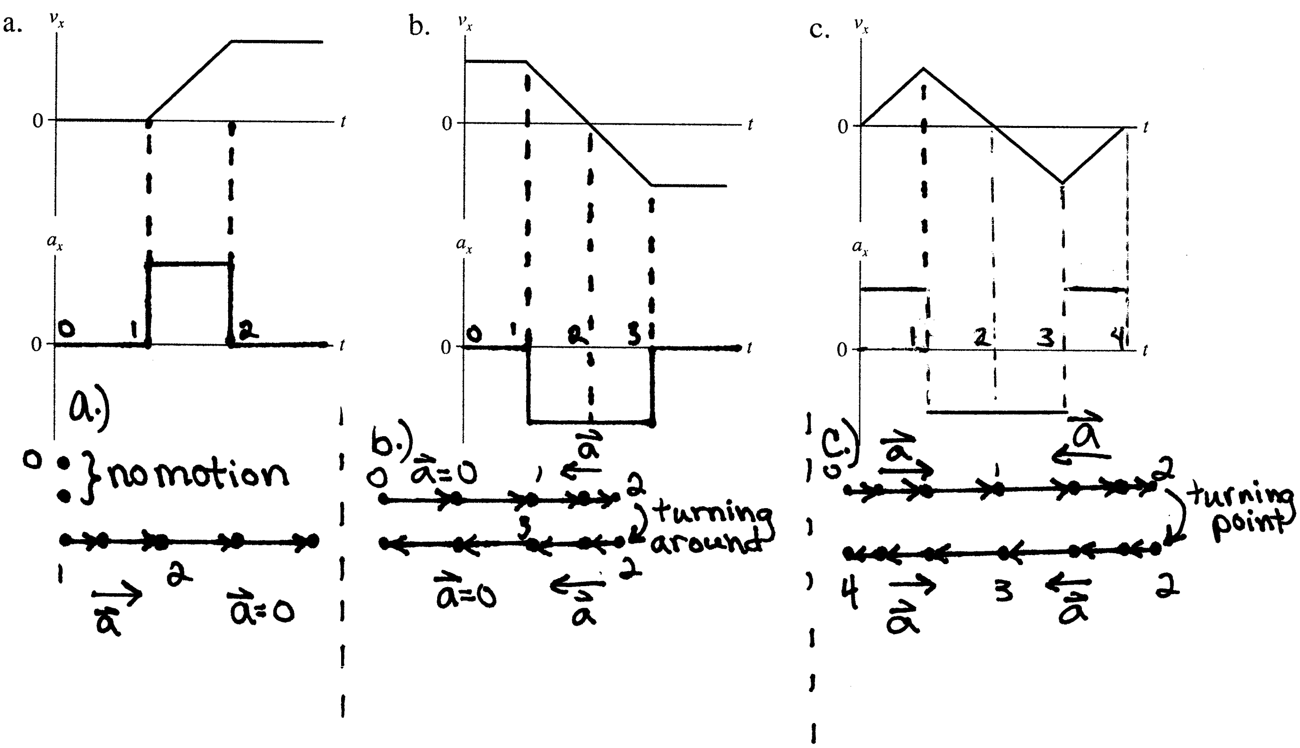
$36 = C(3)^2 \quad C = 4 \quad 4 \times 5^2 = 100 \quad 4 \times 2^2 = 16$

c. By what factor must x change for the value of y to double? $\sqrt{2}$

d. Consider the equation in your text relating Δx and Δt, for motion with constant acceleration a_x. Which of these three quantities plays the role of x in a quadratic relationship $y = Ax^2$? Which plays the role of y?

$x \rightarrow \Delta t$, $y \rightarrow \Delta x$

20. Below are three velocity-versus-time graphs. For each, draw the corresponding acceleration-versus-time graph and draw a motion diagram below the graphs.



2.6 Solving One-Dimensional Motion Problems

21. Draw a pictorial representation of each situation described below. That is, (i) sketch the situation, showing appropriate points in the motion, (ii) establish a coordinate system on your sketch, and (iii) define appropriate symbols for the known and unknown quantities. **Do not solve.** See textbook Figure 2.32 as an example.

a. A car traveling at 30 m/s screeches to a halt, leaving 55-m-long skid marks. What was the car's acceleration while braking?

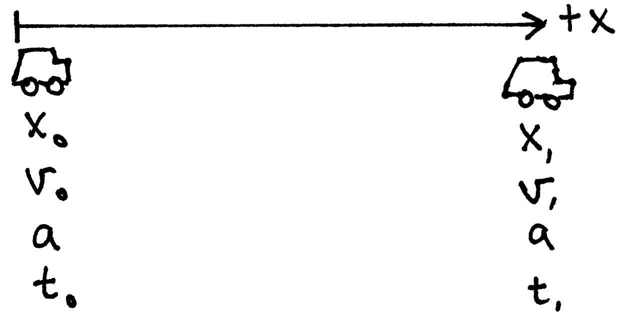
$x_0 = 0$ $x_1 = 55\text{m}$

$v_0 = 30\frac{\text{m}}{\text{s}}$

$v_1 = 0$

$a < 0 = \text{constant}$

Find a



b. A bicyclist starts from rest and accelerates at 4.0 m/s² for 3.0 s. The cyclist then travels for 20 s at a constant speed. How far does the cyclist travel?

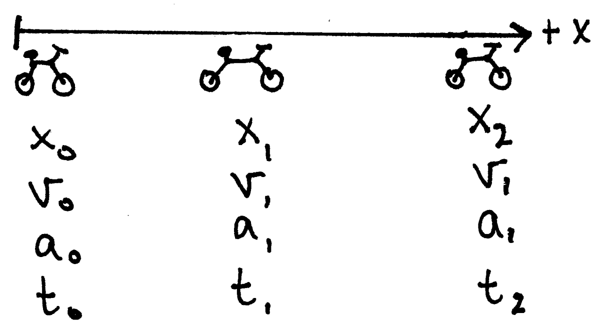
$x_0 = 0$

$v_0 = 0$ $v_1 = \text{constant}$

$a_0 = 4\frac{\text{m}}{\text{s}^2}$ $a_1 = 0$

$t_0 = 0$ $t_1 = 3\text{s}$ $t_2 - t_1 = 20\text{s}$

Find x_2



c. You are driving your car at 12 m/s when a deer jumps in front of your car. What is the shortest stopping distance for your car if your reaction time is 0.80 s and your car brakes at 6.0 m/s²?

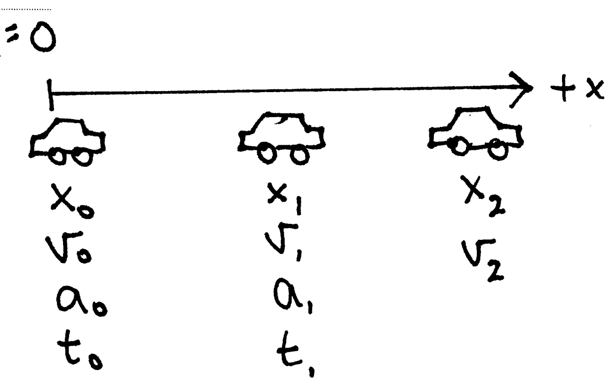
$v_0 = 12\frac{\text{m}}{\text{s}}$ $v_2 = 0$

$a_0 = 0$ $a_1 = -6.0\frac{\text{m}}{\text{s}^2}$

$x_0 = 0$

$t_0 = 0$ $t_1 = 0.80\text{s}$

Find x_2



2.7 Free Fall

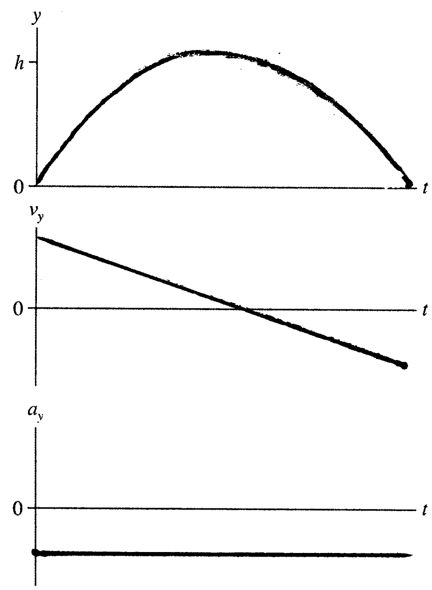
22. A ball is thrown straight up into the air. At each of the following instants, is the ball's acceleration g, $-g$, 0, $< g$, or $> g$?

    a. Just after leaving your hand? = $-g$

    b. At the very top (maximum height)? = $-g$

    c. Just before hitting the ground? = $-g$

23. A ball is thrown straight up into the air. It reaches height h, and then falls back down to the ground. On the axes below, graph the ball's position, velocity, and acceleration from an instant after it leaves the thrower's hand until an instant before it hits the ground. Indicate on your graphs the times during which the ball is moving upward, at its peak, and moving downward.



24. A rock is *thrown* (not dropped) straight down from a bridge into the river below.

    a. Immediately *after* being released, is the magnitude of the rock's acceleration greater than g, less than g, or equal to g? Explain.

    =$-g$ The initial velocity is independent of the acceleration.

    b. Immediately before hitting the water, is the magnitude of the rock's acceleration greater than g, less than g, or equal to g? Explain.

    =$-g$ The acceleration is the same throughout the motion.

25. A model rocket is launched straight up with constant acceleration a. It runs out of fuel at time t.
PSS 2.1 Suppose you need to determine the maximum height reached by the rocket. We'll assume that air resistance is negligible.

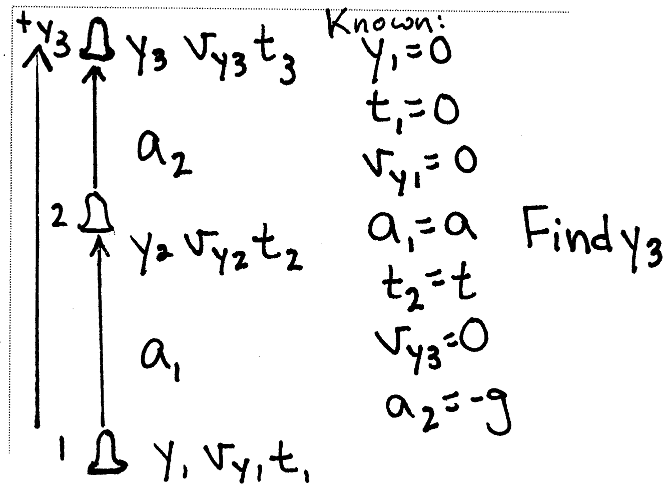
a. Is the rocket at maximum height the instant it runs out of fuel? No

b. Is there anything other than gravity acting on the rocket after it runs out of fuel? No

c. What is the name of motion under the influence of only gravity? projectile motion

d. Draw a pictorial representation for this problem. You should have three identified points in the motion: launch, out of fuel, maximum height. Call these points 1, 2, and 3.

- Using subscripts, define 11 quantities: y, v_y, and t at each of the three points, plus acceleration a_1 connecting points 1 and 2 and acceleration a_2 connecting points 2 and 3.
- Identify 7 of these quantities as Knowns, either 0 or given symbolically in terms of a, t, and g. Be careful with signs!
- Identify which one of the 4 unknown quantities you're trying to Find.



e. This is a two-part problem. Write two kinematic equations for the first part of the motion to determine—again symbolically—the two unknown quantities at point 2.

$$v_{y2} = at$$
$$y_2 = \frac{1}{2}at^2$$

f. Now write a kinematic equation for the second half of the motion that will allow you to find the desired unknown that will answer the question. Your equation should not contain the fourth unknown quantity. Just write the equation; don't yet solve it.

$$0 = (v_{y3})^2 = (v_{y2})^2 - 2g(y_3 - y_2)$$

g. Now, substitute what you learned in part e into your equation of part f, do the algebra to solve for the unknown, and simplify the result as much as possible.

$$v_{y2}^2 = 2g(y_3 - y_2)$$
$$a^2t^2 = 2g(y_3 - \tfrac{1}{2}at^2)$$
$$\text{or } \frac{a^2t^2}{2g} + \frac{1}{2}at^2 = y_3$$
$$\frac{(a^2+ga)t^2}{2g} = y_3$$