11

OSCILLATIONS AND WAVES

Responses to Questions

1. The acceleration of a simple harmonic oscillator is momentarily zero as the mass passes through the equilibrium point. At this point, there is no force on the mass and therefore no acceleration.

2. Since the real spring has mass, the mass that is moving is greater than the mass at the end of the spring. Since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$, a larger mass means a smaller frequency. Thus the true frequency will be smaller than the massless spring approximation. And since the true frequency is smaller, the true period will be larger than the massless spring approximation. About one-third of the mass of the spring contributes to the total mass value.

3. The maximum speed is given by $\upsilon_{\max} = A\sqrt{k/m}$. Various combinations of changing *A*, *k*, and/or *m* can result in a doubling of the maximum speed. For example, if *k* and *m* are kept constant, then doubling the amplitude will double the maximum speed. Or if *A* and *k* are kept constant, then reducing the mass to one-fourth of its original value will double the maximum speed. Note that changing either *k* or *m* will also change the frequency of the oscillator, since $f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$. So doubling the frequency (no matter how it is done) will also double the maximum speed.

4. The period of a pendulum clock is inversely proportional to the square root of *g*, by Eq. 11–11a, $T = 2\pi\sqrt{\ell/g}$. When taken to high altitude, *g* will decrease (by a small amount), which means that the period will increase. If the period is too long, then the clock is running slow, so will lose time.

5. The tire swing is a good approximation of a simple pendulum. Pull the tire back a short distance and release it so that it oscillates as a pendulum in simple harmonic motion with a small amplitude. Measure the period of the oscillations and calculate the length of the pendulum from the expression $T = 2\pi\sqrt{\ell/g} \;\rightarrow\; \ell = \frac{gT^2}{4\pi^2}$. The length ℓ is the distance from the center of the tire to the branch. The height of the branch is ℓ plus the height of the center of the tire above the ground.

6. The displacement and velocity vectors are in the same direction while the oscillator is moving away from its equilibrium position. The displacement and acceleration vectors are never in the same direction.

7. The two masses reach the equilibrium point simultaneously. The period of oscillation is independent of amplitude and will be the same for both systems.

8. When walking at a normal pace, the period of a walking step is about 1 second. The faster you walk, the shorter the period. The shorter your legs, the shorter the period. If you approximate your leg as a pendulum of length 1 m, then the period would be $T = 2\pi\sqrt{\ell/g} = 2$ seconds.

9. When you rise to a standing position, you raise your center of mass and effectively shorten the length of the swing. The period of the swing will decrease, and the frequency will increase.

10. To make the water slosh, you must shake the water (and the pan) at the natural frequency for water waves in the pan. The water then is in resonance, or in a standing wave pattern, and the amplitude of oscillation gets large. That natural frequency is determined in part by the size of the pan—smaller pans will slosh at higher frequencies, corresponding to shorter wavelengths for the standing waves. The period of the shaking must be the same as the time it takes a water wave to make a "round trip" in the pan.

11. The frequency of a simple periodic wave is equal to the frequency of its source. The wave is created by the source moving the wave medium that is in contact with the source. If you have one end of a taut string in your hand, and you move your hand with a frequency of 2 Hz, then the end of the string in your hand will be moving at 2 Hz, because it is in contact with your hand. Then those parts of the medium that you are moving exert forces on adjacent parts of the medium and cause them to oscillate. Since those two portions of the medium stay in contact with each other, they also must be moving with the same frequency. That can be repeated all along the medium, so the entire wave throughout the medium has the same frequency as the source.

12. The speed of the transverse wave is the speed at which the wave disturbance moves along the cord. For a uniform cord, that speed is constant and depends on the tension in the cord and the mass density of the cord. The speed of a tiny piece of the cord is how fast the piece of cord moves perpendicularly to the cord as the disturbance passes by. That speed is not constant—if a sinusoidal wave is traveling on the cord, the speed of each piece of the cord will be given by the speed relationship of a simple harmonic oscillator (Eq. 11–9), which depends on the amplitude of the wave, the frequency of the wave, and the specific time of observation.

$\boxed{13.}$ (*a*) Striking the rod vertically from above will displace particles in a direction perpendicular to the rod and will set up primarily transverse waves.

(*b*) Striking the rod horizontally parallel to its length will give the particles an initial displacement parallel to the rod and will set up primarily longitudinal waves.

14. From Eq. 11–14b, the speed of waves in a gas is given by $v = \sqrt{B/\rho}$. A decrease in the density due to a temperature increase therefore leads to a higher speed of sound. We expect the speed of sound to increase as temperature increases.

15. For a rope with a fixed end, the reflected pulse is inverted relative to the incoming pulse. For a rope with a free end, the reflected pulse is not inverted. See Fig. 11–33 for an illustration. For the fixed end, the rope puts a force on the connecting point as the pulse reaches the connecting point. The connecting point puts an equal and opposite force on the rope. This force is what generates the inverted reflected

pulse. The point of connection is a node—a point of no motion. For a free end, the incoming pulse "whips" the end of the rope in a direction transverse to the wave motion, stretching it upward. As that whipped end is pulled back toward the equilibrium position, this whipping motion generates a wave much in the same way that the pulse was originally created and thus creates a wave that is not reflected.

16. Although both longitudinal and transverse waves can travel through solids, only longitudinal waves can travel through liquids. Since longitudinal waves, but no transverse waves, are detected on the Earth diametrically opposite the location of an earthquake, there must be some liquid as part of the Earth's interior.

17. The speed of a longitudinal wave is given in general by $\upsilon = \sqrt{\text{elastic force factor/inertia factor}}$. Even though the density of solids is 1000 to 10,000 times greater than that of air, the elastic force factor (bulk modulus) of most solids is at least 10^6 times as great as the bulk modulus of air. This difference overcomes the larger density of most solids and accounts for the speed of sound in most solids being greater than in air.

18. (*a*) Similar to the discussion in Section 11–9 for spherical waves, as a circular wave expands, the circumference of the wave increases. For the energy in the wave to be conserved, as the circumference increases, the intensity has to decrease. The intensity of the wave is proportional to the square of the amplitude.
    (*b*) The water waves will decrease in amplitude due to dissipation of energy from viscosity in the water (dissipative or frictional energy loss).

$\boxed{19.}$ Assuming the two waves are in the same medium, then they will both have the same speed. Since $\upsilon = f\lambda$, the wave with the smaller wavelength will have twice the frequency of the other wave. From Eq. 11–18, the intensity of a wave is proportional to the square of the frequency of the wave. Thus, the wave with the shorter wavelength will transmit four times as much energy as the other wave.

20. The frequency must stay the same because the media is continuous—the end of one section of cord is physically tied to the other section of cord. If the end of the first section of cord is vibrating up and down with a given frequency, then since it is attached to the other section of cord, the other section must vibrate at the same frequency. If the two pieces of cord did not move at the same frequency, then they would not stay connected, and the waves would not pass from one section to another.

21. Assuming that there are no dissipative processes, then yes, the energy is conserved. The particles in the medium, which are set into motion by the wave, have both kinetic and potential energy. At the instant in which two waves interfere destructively, the displacement of the medium may be zero, but the particles of the medium will have velocity and therefore kinetic energy.

$\boxed{22.}$ Yes. If you touch the string at any node you will not disturb the motion. There will be nodes at each end as well as at the points one-third and two-thirds of the distance along the length of the string.

23. From Eq. 11–13, the speed of waves on the string is $\upsilon = \sqrt{F_T/\mu}$. Equation 11–9b can be used to find the fundamental frequency of oscillation for a string with both ends fixed, $f_1 = \dfrac{\upsilon}{2\ell}$. Combining these two relationships gives $f_1 = \dfrac{1}{2\ell}\sqrt{\dfrac{F_T}{\mu}}$. By wrapping the string with wire, the mass per unit length (μ) of the string can be greatly increased without changing the length or the tension of the string. This gives the string a low fundamental frequency.

24. The energy of a wave is not localized at one point, because the wave is not localized at one point, so referring to the energy "at a node" being zero is not a meaningful statement. Due to the interference of the waves, the total energy of the medium particles at the nodes is zero, but the energy of the medium is not zero at points of the medium that are not nodes. In fact, the antinodes have more energy than they would have if only one of the two waves were present.

25. Yes. A standing wave is an example of a resonance phenomenon, caused by constructive interference between a traveling wave and its reflection. The wave energy is distributed around the antinodes, which exhibit large-amplitude oscillations, even when the generating oscillations from the hand are small.

26. This description of waves works well initially for both descriptions, but the waves continue after the initial motion. When the center is struck, a wave does move from the center to the rim, but then it reflects from the rim back to the center. Likewise, when the rim is struck, a wave does move from the rim to the center, but the wave does not "stop" at the center. Once reaching the center, it then spreads out again to the rim. The amplitude of the waves also changes as the waves travel. As the radius increases, the amplitude decreases, and as the radius decreases, the amplitude increases.

27. AM radio waves have a much longer wavelength than FM radio waves. How much waves bend, or diffract, around obstacles depends on the wavelength of the wave compared with the size of the obstacle. A hill is much larger than the wavelength of FM waves, so there will be a "shadow" region behind the hill. However, the hill is not large compared with the wavelength of AM signals, so the AM radio waves will bend around the hill.

Responses to MisConceptual Questions

1. (*e*) At $x = \pm A$, the velocity is zero, but at these points the acceleration is a maximum. At $x = 0$, the acceleration is zero, but the velocity is a maximum. For all other points, both the velocity and acceleration are nonzero. Thus there are no points where the acceleration and velocity are simultaneously zero.

2. (*a*, *c*, *d*) At the turning points in the oscillation $(x = \pm A)$, the velocity is zero and the acceleration is a maximum, so (*a*) is true. At the center of the oscillation $(x = 0)$, the acceleration is zero and the velocity is a maximum, so (*c*) is true. Since the velocity is only zero at the turning points where the acceleration is a maximum, there is no point where both the velocity and acceleration are zero, so (*b*) is not true. At all other points besides $x = \pm A$ and $x = 0$, both the acceleration and velocity are nonzero values, so (*d*) is also true.

3. (*c*) Students may believe that the period is proportional to the mass and therefore think that doubling the mass will double the period. However, the period is proportional to the square root of the mass, as seen in Eq. 11–6a. Therefore, the mass must be quadrupled (to 4*M*) for the period to double.

4. (*b*) A common misconception is that the amplitude of oscillation affects the frequency. Eq. 11–6b shows that the frequency can be increased by increasing *k* or decreasing *m*. The frequency does not depend upon the amplitude.

5. (*a*) The small angle approximation is valid only in units of radians because the angle in radians is equal to the ratio of the arc length to the radius. At small angles the arc length can be approximated as a straight line, being the opposite leg of a right triangle with hypotenuse equal

to the radius of the circle. This ratio is equal to the sine of the angle, so for small angles the angle in radians is equal to the sine of the angle.

6. (*e*) A common misconception is that the starting angle, or amplitude of oscillation, affects the period of a pendulum. Eq. 11–11a shows that the period of a small-amplitude pendulum is determined by the length of the string and the acceleration of gravity, not the amplitude. Both oscillations will then have the same period.

7. (*c*) Students may erroneously believe that the mass of the child will affect the period of oscillation. However, Eq. 11–11a shows that the period is determined by the length of the swing cords and the acceleration of gravity. It does not depend upon the weight of the child. Since the swings are identical, they should oscillate with the same period.

8. (*a*) To speed up the pendulum, the period of the oscillation must be decreased. Equation 11–11a shows that the period is proportional to the square root of the length, so shortening the string will decrease the period. The period does not depend upon the mass of the bob, so changing the mass will not affect the period.

9. (*e*) Students frequently have trouble distinguishing between the motion of a point on a cord and the motion of a wave on the cord. As a wave travels down the cord, a point on the cord will move vertically between the lowest point of the wave and the highest point on the wave. The wave and the point have the same amplitude. The point on the cord completes one up and down oscillation as each wavelength passes that point. Therefore, the motion of the point on the cord has the same frequency as the wave. The speed of the wave on the string is determined by the wavelength and frequency. It is constant in time. The point on the cord moves perpendicular to the wave with a speed that varies with time. The maximum speed of the point is proportional to the wave amplitude and the wave frequency. Changing the amplitude will change the maximum speed of the point on the cord, but it does not change the wave speed. The wave speed and string speed therefore are not equal.

10. (*a*) A common misconception is that the waves are objects that can collide. Waves obey the superposition principle such that at any point on the rope the total displacement is the sum of the displacements from each wave. The waves pass through each other unaffected.

11. (*d*) Equation 11–13 shows that the wave speed on a cord is related to the tension in the cord and the mass per unit length of the cord. The wave speed does not depend upon the amplitude, frequency, or wavelength. Stretching the cord increases the tension and decreases the mass per unit length, both of which increase the speed of the wave on the cord.

12. (*d*) The point on the string does not move horizontally, so answers (*a*) and (*b*) cannot be correct. The string has zero velocity only at the turning points (top and bottom), so (*e*) cannot be correct. Examining the graph shows that as the wave moves to the right the crest is approaching point B, so the string at B is traveling upward at this instant.

13. (*d*) Students frequently confuse the medium (an object) with the wave motion and answer that the waves will collide and bounce off of each other. The waves obey the superposition principle such that at any point in the lake the amplitude of the wave is the sum of the individual amplitudes of each wave. This produces the various patterns when they overlap. The waves, however, will pass through each other and continue in their same pattern after they pass.

14. (*c*) The speed of the wave along the Slinky depends upon the mass of the Slinky and the tension caused by stretching it. Since this has not changed, the wave speed remains constant. The wave

speed can also be written as the product of the wavelength and frequency. Therefore, as the frequency is increased, the wavelength must decrease.

15. (*a*) A common misconception is that a wave transports matter as well as energy. However, as shown by a transverse wave on a horizontal string, the wave transports the disturbance down the string, but each part of the string stays at its initial horizontal position.

Solutions to Problems

1. The particle would travel four times the amplitude: from $x = A$ to $x = 0$ to $x = -A$ to $x = 0$ to $x = A.$ So the total distance $= 4A = 4(0.21\text{ m}) = \boxed{0.84\text{ m}}$.

2. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(66\text{ kg})(9.80\text{ m/s}^2)}{5.0\times10^{-3}\text{ m}} = 1.294\times10^5\text{ N/m}$$

The frequency of oscillation is found from the total mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.294\times10^5\text{ N/m}}{1766\text{ kg}}} = 1.362\text{ Hz} \approx \boxed{1.4\text{ Hz}}$$

3. The spring constant is the ratio of external applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{210\text{ N} - 75\text{ N}}{0.85\text{ m} - 0.61\text{ m}} = \frac{135\text{ N}}{0.24\text{ m}} = 562.5\text{ N/m} \approx \boxed{560\text{ N/m}}$$

4. The period is 2.0 seconds, and the mass is 32 kg. The spring constant is calculated from Eq. 11–6a.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad\rightarrow\quad T^2 = 4\pi^2\frac{m}{k} \quad\rightarrow\quad k = 4\pi^2\frac{m}{T^2} = 4\pi^2\frac{32\text{ kg}}{(2.0\text{ s})^2} = 315.8\text{ N/m} \approx \boxed{320\text{ N/m}}$$

5. (*a*) The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(2.4\text{ kg})(9.80\text{ m/s}^2)}{0.036\text{ m}} = 653\text{ N/m} \approx \boxed{650\text{ N/m}}$$

(*b*) The amplitude is the distance pulled down from equilibrium, so $A = \boxed{2.1\text{ cm}}$.
The frequency of oscillation is found from the oscillating mass and the spring constant.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{653\text{ N/m}}{2.4\text{ kg}}} = 2.625\text{ Hz} \approx \boxed{2.6\text{ Hz}}$$

$\boxed{6.}$ The relationship between frequency, mass, and spring constant is Eq. 11–6b, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$.

(*a*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad\rightarrow\quad k = 4\pi^2 f^2 m = 4\pi^2(4.0\text{ Hz})^2(2.2\times10^{-4}\text{ kg}) = 0.1390\text{ N/m} \approx \boxed{0.14\text{ N/m}}$$

(*b*) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{0.1390\text{ N/m}}{4.4\times10^{-4}\text{ kg}}} = 2.828\text{ Hz} \approx \boxed{2.8\text{ Hz}}$$

7. The spring constant is the same regardless of what mass is attached to the spring.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow \quad \frac{k}{4\pi^2} = mf^2 = \text{constant} \quad \rightarrow \quad m_1 f_1^2 = m_2 f_1^2 \quad \rightarrow$$

$$(m\ \text{kg})(0.83\ \text{Hz})^2 = (m\ \text{kg} + 0.78\ \text{kg})(0.60\ \text{Hz})^2 \quad \rightarrow \quad m = \frac{(0.78\ \text{kg})(0.60\ \text{Hz})^2}{(0.83\ \text{Hz})^2 - (0.60\ \text{Hz})^2} = \boxed{0.85\ \text{kg}}$$

8. We assume that downward is the positive direction of motion. For this motion, we have $k = 305$ N/m, $A = 0.280$ m, $m = 0.235$ kg, and $\omega = \sqrt{k/m} = \sqrt{(305\ \text{N/m})/0.235\ \text{kg}} = 36.026$ rad/s.

(*a*) Since the mass has a zero displacement and a positive velocity at $t = 0$, the equation is a sine function.

$$\boxed{y(t) = (0.280\ \text{m})\sin[(36.0\ \text{rad/s})t]}$$

(*b*) The period of oscillation is given by $T = \frac{2\pi}{\omega} = \frac{2\pi}{36.026\ \text{rad/s}} = 0.17441$ s. The spring will have its maximum extension at times given by the following:

$$t_{\text{max}} = \frac{T}{4} + nT = \boxed{4.36\times10^{-2}\ \text{s} + n(0.174\ \text{s}),\ n = 0, 1, 2, \cdots}$$

The spring will have its minimum extension at times given by the following:

$$t_{\text{min}} = \frac{3T}{4} + nT = \boxed{1.31\times10^{-1}\ \text{s} + n(0.174\ \text{s}),\ n = 0, 1, 2, \cdots}$$

9. (*a*) For A, the amplitude is $A_{\text{A}} = \boxed{2.5\ \text{m}}$. For B, the amplitude is $A_{\text{B}} = \boxed{3.5\ \text{m}}$.

(*b*) For A, the frequency is 1 cycle every 4.0 seconds, so $f_{\text{A}} = \boxed{0.25\ \text{Hz}}$. For B, the frequency is 1 cycle every 2.0 seconds, so $f_{\text{B}} = \boxed{0.50\ \text{Hz}}$.

(*c*) For A, the period is $T_{\text{A}} = \boxed{4.0\ \text{s}}$. For B, the period is $T_{\text{B}} = \boxed{2.0\ \text{s}}$.

10. (*a*) We find the effective spring constant from the mass and the frequency of oscillation, using Eq. 11–6b.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k = 4\pi^2 mf^2 = 4\pi^2(0.052\ \text{kg})(3.0\ \text{Hz})^2 = 18.476\ \text{N/m} \approx \boxed{18\ \text{N/m}}$$

(*b*) Since the objects are the same size and shape, we anticipate that the spring constant is the same.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{18.476\ \text{N/m}}{0.28\ \text{kg}}} = 1.293\ \text{Hz} \approx \boxed{1.3\ \text{Hz}}$$

11. If the energy of the SHO is half potential and half kinetic, then the potential energy is half the total energy. The total energy is the potential energy when the displacement has the value of the amplitude.

$$E_{\text{pot}} = \tfrac{1}{2}E_{\text{tot}} \quad \rightarrow \quad \tfrac{1}{2}kx^2 = \tfrac{1}{2}\left(\tfrac{1}{2}kA^2\right) \quad \rightarrow \quad \boxed{x = \pm\frac{1}{\sqrt{2}}A \approx \pm 0.707A}$$

12. When the object is at rest, the magnitude of the spring force is equal to the force of gravity. This determines the spring constant. The period can then be found.

$$\sum F_{\text{vertical}} = kx_0 - mg \quad \rightarrow \quad k = mg/x_0$$

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{m}{(mg/x_0)}} = 2\pi\sqrt{\frac{x_0}{g}} = 2\pi\sqrt{\frac{0.14\ \text{m}}{9.80\ \text{m/s}^2}} = \boxed{0.75\ \text{s}}$$

13. The spring constant can be found from the stretch distance corresponding to the weight suspended on the spring.

$$k = \frac{F_{\text{ext}}}{x} = \frac{mg}{x} = \frac{(1.65\ \text{kg})(9.80\ \text{m/s}^2)}{0.215\ \text{m}} = 75.209\ \text{N/m}$$

After being stretched farther and released, the mass will oscillate. It takes one-quarter of a period for the mass to move from the maximum displacement to the equilibrium position, independent of the amplitude.

$$\tfrac{1}{4}T = \tfrac{1}{4}2\pi\sqrt{m/k} = \frac{\pi}{2}\sqrt{\frac{1.65\ \text{kg}}{75.209\ \text{N/m}}} = \boxed{0.233\ \text{s}}$$

14. The general form of the motion is $x = A\cos\omega t = 0.650\cos 8.40t$.

(*a*) The amplitude is $A = x_{\text{max}} = \boxed{0.650\ \text{m}}$.

(*b*) The frequency is found by $\omega = 2\pi f = 8.40\ \text{s}^{-1} \quad \rightarrow \quad f = \dfrac{8.40\ \text{s}^{-1}}{2\pi} = 1.337\ \text{Hz} \approx \boxed{1.34\ \text{Hz}}$

(*c*) The total energy is given by the kinetic energy at the maximum speed.

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(1.15\ \text{kg})[(8.40\ \text{s}^{-1})(0.650\ \text{m})]^2 = 17.142\ \text{J} \approx \boxed{17.1\ \text{J}}$$

(*d*) The potential energy is given by

$$E_{\text{potential}} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(1.15\ \text{kg})(8.40\ \text{s}^{-1})^2(0.360\ \text{m})^2 = 5.258\ \text{J} \approx \boxed{5.26\ \text{J}}$$

The kinetic energy is given by

$$E_{\text{kinetic}} = E_{\text{total}} - E_{\text{potential}} = 17.142\ \text{J} - 5.258\ \text{J} = 11.884\ \text{J} \approx \boxed{11.9\ \text{J}}$$

15. (*a*) At equilibrium, the velocity is its maximum, as given in Eq. 11–7.

$$v_{\text{max}} = \sqrt{\frac{k}{m}}A = \omega A = 2\pi fA = 2\pi(2.2\ \text{Hz})(0.15\ \text{m}) = 2.073\ \text{m/s} \approx \boxed{2.1\ \text{m/s}}$$

(*b*) From Eq. 11–5b, we find the speed at any position.

$$v = v_{\text{max}}\sqrt{1 - \frac{x^2}{A^2}} = (2.073\ \text{m/s})\sqrt{1 - \frac{(0.10\ \text{m})^2}{(0.15\ \text{m})^2}} = 1.545\ \text{m/s} \approx \boxed{1.5\ \text{m/s}}$$

(*c*) $E_{\text{total}} = \tfrac{1}{2}mv_{\text{max}}^2 = \tfrac{1}{2}(0.25\ \text{kg})(2.073\ \text{m/s})^2 = 0.5372\ \text{J} \approx \boxed{0.54\ \text{J}}$

(*d*) Since the object has a maximum displacement at $t = 0$, the position will be described by the cosine function.

$$x = (0.15\ \text{m})\cos(2\pi(2.2\ \text{Hz})t) \quad \rightarrow \quad \boxed{x = (0.15\ \text{m})\cos(4.4\pi t)}$$

16. The spring constant is found from the ratio of applied force to displacement.

$$k = \frac{F}{x} = \frac{91.0\text{ N}}{0.175\text{ m}} = 520\text{ N/m}$$

Assuming that there are no dissipative forces acting on the ball, the elastic potential energy in the loaded position will become kinetic energy of the ball.

$$E_{\text{i}} = E_{\text{f}} \quad \rightarrow \quad \tfrac{1}{2}kx_{\max}^2 = \tfrac{1}{2}mv_{\max}^2 \quad \rightarrow \quad v_{\max} = x_{\max}\sqrt{\frac{k}{m}} = (0.175\text{ m})\sqrt{\frac{520\text{ N/m}}{0.160\text{ kg}}} = \boxed{9.98\text{ m/s}}$$

17. To compare the total energies, we can compare the maximum potential energies. Since the frequencies and the masses are the same, the spring constants are the same.

$$\frac{E_{\substack{\text{high}\\\text{energy}}}}{E_{\substack{\text{low}\\\text{energy}}}} = \frac{\tfrac{1}{2}kA^2_{\substack{\text{high}\\\text{energy}}}}{\tfrac{1}{2}kA^2_{\substack{\text{low}\\\text{energy}}}} = \frac{A^2_{\substack{\text{high}\\\text{energy}}}}{A^2_{\substack{\text{low}\\\text{energy}}}} = 3 \quad \rightarrow \quad \boxed{\frac{A_{\substack{\text{high}\\\text{energy}}}}{A_{\substack{\text{low}\\\text{energy}}}} = \sqrt{3}}$$

18. (*a*) The spring constant can be found from the mass and the frequency of oscillation.

$$\omega = \sqrt{\frac{k}{m}} = 2\pi f \quad \rightarrow \quad k = 4\pi^2 f^2 m = 4\pi^2(2.5\text{ Hz})^2(0.24\text{ kg}) = 59.22\text{ N/m} \approx \boxed{59\text{ N/m}}$$

(*b*) The energy can be found from the maximum potential energy.

$$E = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(59.22\text{ N/m})(0.045\text{ m})^2 = 5.996\times10^{-2}\text{ J} \approx \boxed{0.060\text{ J}}$$

19. (*a*) The work done to compress a spring is stored as potential energy.

$$W = \tfrac{1}{2}kx^2 \quad \rightarrow \quad k = \frac{2W}{x^2} = \frac{2(3.6\text{ J})}{(0.13\text{ m})^2} = 426.0\text{ N/m} \approx \boxed{430\text{ N/m}}$$

(*b*) The distance that the spring was compressed becomes the amplitude of its motion. The maximum acceleration occurs at the maximum displacement and is given by $a_{\max} = \frac{k}{m}A.$ Solve this for the mass.

$$a_{\max} = \frac{k}{m}A \quad \rightarrow \quad m = \frac{k}{a_{\max}}A = \left(\frac{426\text{ N/m}}{12\text{ m/s}^2}\right)(0.13\text{ m}) = 4.615\text{ kg} \approx \boxed{4.6\text{ kg}}$$

$\boxed{20.}$ (*a*) The total energy of an object in SHM is constant. When the position is at the amplitude, the speed is zero. Use that relationship to find the amplitude.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \quad \rightarrow$$

$$A = \sqrt{\frac{m}{k}v^2 + x^2} = \sqrt{\frac{2.7\text{ kg}}{310\text{ N/m}}(0.55\text{ m/s})^2 + (0.020\text{ m})^2} = 5.509\times10^{-2}\text{ m} \approx \boxed{5.5\times10^{-2}\text{ m}}$$

(*b*) Again use conservation of energy. The energy is all kinetic energy when the object has its maximum velocity.

$$E_{\text{tot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 = \tfrac{1}{2}mv_{\max}^2 \quad \rightarrow$$

$$v_{\max} = A\sqrt{\frac{k}{m}} = (5.509\times10^{-2}\text{ m})\sqrt{\frac{310\text{ N/m}}{2.7\text{ kg}}} = 0.5903\text{ m/s} \approx \boxed{0.59\text{ m/s}}$$

21. (*a*) Find the period and frequency from the mass and the spring constant.

$$T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.885\text{ kg}/(184\text{ N/m})} = 0.4358\text{ s} \approx \boxed{0.436\text{ s}}$$

$$f = 1/T = 1/(0.4358\text{ s}) = \boxed{2.29\text{ Hz}}$$

(*b*) The initial speed is the maximum speed, and that can be used to find the amplitude.

$$\upsilon_{\text{max}} = A\sqrt{k/m} \quad \rightarrow$$

$$A = \upsilon_{\text{max}}\sqrt{m/k} = (2.26\text{ m/s})\sqrt{0.885\text{ kg}/(184\text{ N/m})} = 0.1567\text{ m} \approx \boxed{0.157\text{ m}}$$

(*c*) The maximum acceleration can be found from the mass, spring constant, and amplitude.

$$a_{\text{max}} = Ak/m = (0.1567\text{ m})(184\text{ N/m})/(0.885\text{ kg}) = \boxed{32.6\text{ m/s}^2}$$

(*d*) The maximum energy is the kinetic energy that the object has when at the equilibrium position.

$$E = \tfrac{1}{2}m\upsilon_{\text{max}}^2 = \tfrac{1}{2}(0.885\text{ kg})(2.26\text{ m/s})^2 = 2.2601\text{ J} \approx \boxed{2.26\text{ J}}$$

(*e*) Use conservation of mechanical energy for the oscillator, noting that we found total energy in part (*d*) and that another expression for the total energy is $E_{\text{total}} = \tfrac{1}{2}kA^2$.

$$E = \tfrac{1}{2}kx^2 + \tfrac{1}{2}m\upsilon^2 = \tfrac{1}{2}kA^2 \quad \rightarrow \quad \tfrac{1}{2}k(0.40\ A)^2 + \text{KE} = E_{\text{total}} = \tfrac{1}{2}kA^2 \quad \rightarrow$$

$$\text{KE} = \tfrac{1}{2}kA^2 - \tfrac{1}{2}k(0.40A)^2 = \tfrac{1}{2}kA^2(1 - 0.40^2) = E_{\text{total}}(0.84) = (2.2601\text{ J})(0.84) = \boxed{1.90\text{ J}}$$

$\boxed{22.}$ We assume that the collision of the bullet and block is so quick that there is no significant motion of the large mass or spring during the collision. Linear momentum is conserved in this collision. The speed that the combination has right after the collision is the maximum speed of the oscillating system. Then the kinetic energy that the combination has right after the collision is stored in the spring when it is fully compressed, at the amplitude of its motion.

$$p_{\text{before}} = p_{\text{after}} \quad \rightarrow \quad m\upsilon_0 = (m+M)\upsilon_{\text{max}} \quad \rightarrow \quad \upsilon_{\text{max}} = \frac{m}{m+M}\upsilon_0$$

$$\tfrac{1}{2}(m+M)\upsilon_{\text{max}}^2 = \tfrac{1}{2}kA^2 \quad \rightarrow \quad \tfrac{1}{2}(m+M)\left(\frac{m}{m+M}\upsilon_0\right)^2 = \tfrac{1}{2}kA^2 \quad \rightarrow$$

$$\upsilon_0 = \frac{A}{m}\sqrt{k(m+M)} = \frac{(9.460\times10^{-2}\text{ m})}{(7.870\times10^{-3}\text{ kg})}\sqrt{(162.7\text{ N/m})(7.870\times10^{-3}\text{ kg} + 4.148\text{ kg})}$$

$$= \boxed{312.6\text{ m/s}}$$

23. The period of the jumper's motion is $T = \dfrac{43.0\text{ s}}{7\text{ cycles}} = 6.143\text{ s}$. The spring constant can then be found from the period and the jumper's mass.

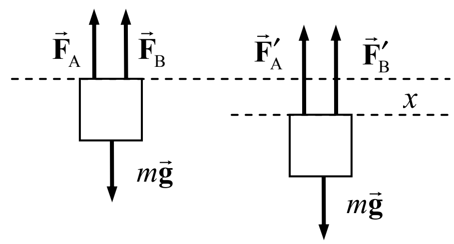
$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(65.0\text{ kg})}{(6.143\text{ s})^2} = 68.004\text{ N/m} \approx \boxed{68.0\text{ N/m}}$$

The stretch of the bungee cord needs to provide a force equal to the weight of the jumper when he is at the equilibrium point.

$$k\Delta x = mg \quad \rightarrow \quad \Delta x = \frac{mg}{k} = \frac{(65.0\text{ kg})(9.80\text{ m/s}^2)}{68.004\text{ N/m}} = 9.37\text{ m}$$

Thus the unstretched bungee cord must be $25.0\text{ m} - 9.37\text{ m} = \boxed{15.6\text{ m}}$.

24. Consider the first free-body diagram for the block while it is at equilibrium, so that the net force is zero. Newton's second law for vertical forces, with up as positive, gives the following:



$$\sum F_y = F_A + F_B - mg = 0 \quad \rightarrow \quad F_A + F_B = mg$$

Now consider the second free-body diagram, in which the block is displaced a distance x from the equilibrium point. Each upward force will have increased by an amount $-kx$, since $x < 0$. Again write Newton's second law for vertical forces.

$$\sum F_y = F_{net} = F'_A + F'_B - mg = F_A - kx + F_B - kx - mg = -2kx + (F_A + F_B - mg) = -2kx$$

This is the general form of a restoring force that produces SHM, with an effective spring constant of $2k$. Thus the frequency of vibration is as follows:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{effective}}}{m}} = \boxed{\frac{1}{2\pi}\sqrt{\frac{2k}{m}}}$$

25. (*a*) The object starts at the maximum displacement in the positive direction so will be represented by a cosine function. The mass, period, and amplitude are given.

$$A = 0.16 \text{ m}; \ \omega = \frac{2\pi}{T} = \frac{2\pi}{0.45 \text{ s}} = 13.96 \text{ rad/s} \quad \rightarrow \quad \boxed{y = (0.16 \text{ m}) \cos (14t)}$$

(*b*) The time to reach the equilibrium is one-quarter of a period, so $\frac{1}{4}(0.45 \text{ s}) = \boxed{0.11 \text{ s}}$.

(*c*) The maximum speed is given by Eq. 11–7a.

$$v_{\max} = 2\pi f A = \omega A = (13.96 \text{ rad/s})(0.16 \text{ m}) = \boxed{2.2 \text{ m/s}}$$

(*d*) The maximum acceleration is found from Eq. 11–10.

$$a_{\max} = \frac{kA}{m} = \omega^2 A = (13.96 \text{ rad/s})^2 (0.16 \text{ m}) = \boxed{31 \text{ m/s}^2}$$

The maximum acceleration occurs at the endpoints of the motion and is first attained at the $\boxed{\text{release point}}$.

26. Each object will pass through the origin at the times when the argument of its sine function is a multiple of π.

A: $4.0t_A = n_A\pi \quad \rightarrow \quad t_A = \frac{1}{4}n_A\pi, n_A = 1,2,3,\ldots$ so $t_A = \frac{1}{4}\pi, \frac{1}{2}\pi, \frac{3}{4}\pi, \pi, \frac{5}{4}\pi, \frac{3}{2}\pi, \frac{7}{4}\pi, 2\pi, \frac{9}{4}\pi, \ \ldots$

B: $3.0t_B = n_B\pi \quad \rightarrow \quad t_B = \frac{1}{3}n_B\pi, n_B = 1,2,3,\ldots$ so $t_B = \frac{1}{3}\pi, \frac{2}{3}\pi, \pi, \frac{4}{3}\pi, \frac{5}{3}\pi, 2\pi, \frac{7}{3}\pi, \frac{8}{3}\pi, 3\pi, \ldots$

Thus we see the first three times are $\boxed{\pi \text{ s}, 2\pi \text{ s, and } 3\pi \text{ s}}$ or $\boxed{3.1 \text{ s}, 6.3 \text{ s, and } 9.4 \text{ s}}$.

27. The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$. The length is assumed to be the same for the pendulum both on Mars and on Earth.

$$T = 2\pi\sqrt{\ell/g} \quad \rightarrow \quad \frac{T_{\text{Mars}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{\ell/g_{\text{Mars}}}}{2\pi\sqrt{\ell/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} \quad \rightarrow$$

$$T_{\text{Mars}} = T_{\text{Earth}}\sqrt{\frac{g_{\text{Earth}}}{g_{\text{Mars}}}} = (1.85 \text{ s})\sqrt{\frac{1}{0.37}} = \boxed{3.0 \text{ s}}$$

28. The period of a pendulum is given by $T = 2\pi\sqrt{\ell/g}$. Solve for the length using a period of 2.0 seconds.

$$T = 2\pi\sqrt{\ell/g} \quad\rightarrow\quad \ell = \frac{T^2 g}{4\pi^2} = \frac{(2.0\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = \boxed{0.99\text{ m}}$$

29. (*a*) The period is given by $T = 50\text{ s}/28\text{ cycles} = \boxed{1.8\text{ s}}$.

(*b*) The frequency is given by $f = 28\text{ cycles}/50\text{ s} = \boxed{0.56\text{ Hz}}$.

30. The period of a pendulum is given by Eq. 11–11a, $T = 2\pi\sqrt{\ell/g}$.

(*a*) $T = 2\pi\sqrt{\ell/g} = 2\pi\sqrt{\dfrac{0.47\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.4\text{ s}}$

(*b*) If the pendulum is in free fall, there is no tension in the string supporting the pendulum bob and no restoring force to cause oscillations. Thus there will be no period—the pendulum will not oscillate, so no period can be defined.

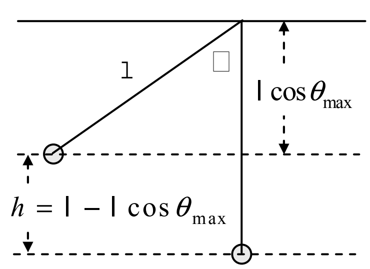
31. There are $(24\text{ h})(60\text{ min/h})(60\text{ s/min}) = 86{,}400\text{ s}$ in a day. The clock should make one cycle in exactly two seconds (a "tick" and a "tock"), so the clock should make 43,200 cycles per day. After one day, the clock in question is 21 seconds slow, which means that it has made 10.5 fewer cycles than are required for precise timekeeping. Thus the clock is only making 43,189.5 cycles in a day. Accordingly, the period of the clock must be decreased by a factor of $\dfrac{43{,}189.5}{43{,}200}$.

$$T_{\text{new}} = \frac{43{,}189.5}{43{,}200}T_{\text{old}} \quad\rightarrow\quad 2\pi\sqrt{\ell_{\text{new}}/g} = \left(\frac{43{,}189.5}{43{,}200}\right)2\pi\sqrt{\ell_{\text{old}}/g} \quad\rightarrow$$

$$\ell_{\text{new}} = \left(\frac{43{,}189.5}{43{,}200}\right)^2 \ell_{\text{old}} = \left(\frac{43{,}189.5}{43{,}200}\right)^2 (0.9930\text{ m}) = 0.9925\text{ m}$$

Thus the pendulum should be $\boxed{\text{shortened by 0.5 mm}}$.

32. Use energy conservation to relate the potential energy at the maximum height of the pendulum to the kinetic energy at the lowest point of the swing. Take the lowest point to be the zero location for gravitational potential energy. See the diagram.



$$E_{\text{top}} = E_{\text{bottom}} \quad\rightarrow\quad \text{KE}_{\text{top}} + \text{PE}_{\text{top}} = \text{KE}_{\text{bottom}} + \text{PE}_{\text{bottom}} \quad\rightarrow$$

$$0 + mgh = \tfrac{1}{2}mv_{\text{max}}^2 \quad\rightarrow\quad v_{\text{max}} = \sqrt{2gh} = \boxed{\sqrt{2g\ell(1-\cos\theta_{\text{max}})}}$$

33. If we consider the pendulum as starting from its maximum displacement, then the equation of motion can be written as $\theta = \theta_0 \cos\omega t = \theta_0 \cos\dfrac{2\pi t}{T}$. Solve for the time for the position to decrease to half of the amplitude.

$$\theta_{1/2} = \tfrac{1}{2}\theta_0 = \theta_0 \cos\frac{2\pi t_{1/2}}{T} \quad\rightarrow\quad \frac{2\pi t_{1/2}}{T} = \cos^{-1}\tfrac{1}{2} = \frac{\pi}{3} \quad\rightarrow\quad t_{1/2} = \tfrac{1}{6}T$$

It takes $\frac{1}{6}T$ for the position to change from $+10°$ to $+5°$. It takes $\frac{1}{4}T$ for the position to change from $+10°$ to 0º. Thus it takes $\frac{1}{4}T - \frac{1}{6}T = \frac{1}{12}T$ for the position to change from $+5°$ to 0º. Due to the symmetric nature of the cosine function, it will also take $\frac{1}{12}T$ for the position to change from 0º to $-5°$, so going from $+5°$ to $-5°$ takes $\frac{1}{6}T$. The second half of the cycle will be identical to the first, so the total time spent between $+5°$ and $-5°$ is $\frac{1}{3}T$. So the pendulum spends $\boxed{\text{one-third}}$ of its time between $+5°$ and $-5°$.

34. The equation of motion for an object in SHM that has the maximum displacement at $t = 0$ is given by $x = A\cos(2\pi ft)$. We let x be the position of the pendulum bob in terms of arc length measured relative to the lowest point of the pendulum. For a pendulum of length ℓ, the arc length is given by $x = \ell\theta$, so $x_{\max} = A = \ell\theta_{\max}$, where θ must be measured in radians. The equation for the pendulum's angular displacement is then found from the equation for the arc length.

$$\ell\theta = \ell\theta_{\max}\cos(2\pi ft) \quad \rightarrow \quad \theta = \theta_{\max}\cos(2\pi ft)$$

If both sides of the equation are multiplied by $180°/\pi$ rad, then the angles can be measured in degrees. Thus the angular displacement of the pendulum can be written as below. Note that the argument of the cosine function is still in radians.

$$\theta = \theta_{\max}\cos(2\pi ft) = 12°\cos(5.0\pi t) = (\pi/15)\cos(5.0\pi t)$$

(*a*) $\theta(t = 0.25\text{ s}) = (\pi/15)\cos(5.0\pi(0.25)) = \boxed{-0.15\text{ rad}}$

(*b*) $\theta(t = 1.60\text{ s}) = (\pi/15)\cos(5.0\pi(1.60)) = \boxed{\pi/15\text{ rad}}$ (The time is exactly 4 periods.)

(*c*) $\theta(t = 500\text{ s}) = (\pi/15)\cos(5.0\pi(500)) = \boxed{\pi/15\text{ rad}}$ (The time is exactly 1250 periods.)

35. The wave speed is given by $v = \lambda f$. The period is 3.0 seconds, and the wavelength is 8.0 m.

$$v = \lambda f = \lambda/T = (7.0\text{ m})/(3.0\text{ s}) = \boxed{2.3\text{ m/s}}$$

36. The distance between wave crests is the wavelength of the wave.

$$\lambda = v/f = (343\text{ m/s})/282\text{ Hz} = \boxed{1.22\text{ m}}$$

37. The elastic and bulk moduli are taken from Table 9–1 in Chapter 9. The densities are taken from Table 10–1 in Chapter 10.

(*a*) For water: $v = \sqrt{B/\rho} = \sqrt{\dfrac{2.0\times10^9\text{ N/m}^2}{1.00\times10^3\text{ kg/m}^3}} = \boxed{1400\text{ m/s}}$

(*b*) For granite: $v = \sqrt{E/\rho} = \sqrt{\dfrac{45\times10^9\text{ N/m}^2}{2.7\times10^3\text{ kg/m}^3}} = \boxed{4100\text{ m/s}}$

(*c*) For steel: $v = \sqrt{E/\rho} = \sqrt{\dfrac{200\times10^9\text{ N/m}^2}{7.8\times10^3\text{ kg/m}^3}} = \boxed{5100\text{ m/s}}$

38. To find the wavelength, use $\lambda = \upsilon/f$.

AM:

$$\lambda_1 = \frac{\upsilon}{f_1} = \frac{3.00\times10^8\ \text{m/s}}{550\times10^3\ \text{Hz}} = 545\ \text{m} \quad \lambda_2 = \frac{\upsilon}{f_2} = \frac{3.00\times10^8\ \text{m/s}}{1600\times10^3\ \text{Hz}} = 188\ \text{m} \quad \boxed{\text{AM: 190 m to 550 m}}$$

FM: $$\lambda_1 = \frac{\upsilon}{f_1} = \frac{3.00\times10^8\ \text{m/s}}{88\times10^6\ \text{Hz}} = 3.41\ \text{m} \quad \lambda_2 = \frac{\upsilon}{f_2} = \frac{3.00\times10^8\ \text{m/s}}{108\times10^6\ \text{Hz}} = 2.78\ \text{m} \quad \boxed{\text{FM: 2.8 m to 3.4 m}}$$

39. (*a*) Both waves travel the same distance, so $\Delta x = \upsilon_1 t_1 = \upsilon_2 t_2$. We let the smaller speed be υ_1 and the larger speed be υ_2. The slower wave will take longer to arrive, so t_1 is greater than t_2.

$$t_1 = t_2 + 1.5\ \text{min} = t_2 + 90\ \text{s} \quad \rightarrow \quad \upsilon_1(t_2 + 102\ \text{s}) = \upsilon_2 t_2 \quad \rightarrow$$

$$t_2 = \frac{\upsilon_1}{\upsilon_2 - \upsilon_1}(90\ \text{s}) = \frac{5.5\ \text{km/s}}{8.5\ \text{km/s} - 5.5\ \text{km/s}}(90\ \text{s}) = 165\ \text{s}$$

$$\Delta x = \upsilon_2 t_2 = (8.5\ \text{km/s})(165\ \text{s}) = \boxed{1400\ \text{km}}$$

(*b*) This is not enough information to determine the epicenter. All that is known is the distance of the epicenter from the seismic station. The direction is not known, so the epicenter lies on a circle of radius $1400\,\text{km}$ from the seismic station. Readings from at least two other seismic stations are needed to determine the epicenter's position.

40. To find the time for a pulse to travel from one end of the cord to the other, the velocity of the pulse on the cord must be known. For a cord under tension, Eq. 11–13 gives $\upsilon = \sqrt{F_\text{T}/\mu}$.

$$\upsilon = \frac{\Delta x}{\Delta t} = \sqrt{\frac{F_\text{T}}{\mu}} = \sqrt{\frac{F_\text{T}}{m/L}} \quad \rightarrow \quad \Delta t = \frac{\Delta x}{\sqrt{\dfrac{F_\text{T}}{m/L}}} = \frac{8.0\ \text{m}}{\sqrt{\dfrac{120\ \text{N}}{(0.65\ \text{kg})/(8.0\ \text{m})}}} = \boxed{0.21\,\text{s}}$$

41. For a cord under tension, we have from Eq. 11–13 that $\upsilon = \sqrt{F_\text{T}/\mu}$. The speed is also the distance traveled by the wave divided by the elapsed time, $\upsilon = \dfrac{\Delta x}{\Delta t}$. The distance traveled is the length of the cord.

$$\upsilon = \sqrt{\frac{F_T}{\mu}} = \frac{\Delta x}{\Delta t} \quad \rightarrow \quad F_T = \mu\upsilon^2 = \mu\frac{\ell^2}{(\Delta t)^2} = \frac{m}{\ell}\frac{\ell^2}{(\Delta t)^2} = \frac{m\ell}{(\Delta t)^2} = \frac{(0.40\ \text{kg})(8.7\ \text{m})}{(0.85\ \text{s})^2} = \boxed{4.8\ \text{N}}$$

$\boxed{42.}$ The speed of the water wave is given by Eq. 11–14b, $\upsilon = \sqrt{B/\rho}$, where B is the bulk modulus of water, from Table 9–1, and ρ is the density of seawater, from Table 10–1. The wave travels twice the depth of the ocean during the elapsed time.

$$\upsilon = \frac{2\ell}{t} \quad \rightarrow \quad \ell = \frac{\upsilon t}{2} = \frac{t}{2}\sqrt{\frac{B}{\rho}} = \frac{2.4\ \text{s}}{2}\sqrt{\frac{2.0\times10^9\ \text{N/m}^2}{1.025\times10^3\ \text{kg/m}^3}} = \boxed{1700\ \text{m}}$$

43. The speed of the waves on the cord can be found from Eq. 11–13, $\upsilon = \sqrt{F_\text{T}/\mu}$. The distance between the children is the wave speed times the elapsed time.

$$\Delta x = \upsilon\Delta t = \Delta t\sqrt{\frac{F_\text{T}}{\mu}} = \Delta t\sqrt{\frac{F_\text{T}}{m/\Delta x}} \quad \rightarrow \quad \Delta x = (\Delta t)^2\frac{F_\text{T}}{m} = (0.55\ \text{s})^2\frac{35\ \text{N}}{0.50\ \text{kg}} = \boxed{21\ \text{m}}$$

44. (*a*) Assume that the earthquake waves spread out spherically from the source. Under those conditions, Eq. 11–16b applies, which states that intensity is inversely proportional to the square of the distance from the source of the wave.

$$I_{45\text{ km}}/I_{15\text{ km}} = (15\text{ km})^2/(45\text{ km})^2 = \boxed{0.11}$$

(*b*) The intensity is proportional to the square of the amplitude, so the amplitude is inversely proportional to the distance from the source of the wave.

$$A_{45\text{ km}}/A_{15\text{ km}} = 15\text{ km}/45\text{ km} = \boxed{0.33}$$

45. (*a*) Assuming spherically symmetric waves, the intensity will be inversely proportional to the square of the distance from the source, as given by Eq. 11–16b. Thus Ir^2 will be constant.

$$I_{\text{near}}r_{\text{near}}^2 = I_{\text{far}}r_{\text{far}}^2 \quad \rightarrow$$

$$I_{\text{near}} = I_{\text{far}}\frac{r_{\text{far}}^2}{r_{\text{near}}^2} = (3.0\times10^6\text{ W/m}^2)\frac{(54\text{ km})^2}{(1.0\text{ km})^2} = 8.748\times10^9\text{ W/m}^2 \approx \boxed{8.7\times10^9\text{ W/m}^2}$$

(*b*) The power passing through an area is the intensity times the area.

$$P = IA = (8.748\times10^9\text{ W/m}^2)(2.0\text{ m}^2) = \boxed{1.7\times10^{10}\text{ W}}$$

46. The bug moves in SHM as the wave passes. The maximum KE of a particle in SHM is the total energy, which is given by $E_{\text{total}} = \frac{1}{2}kA^2$. Compare the two KE maxima.

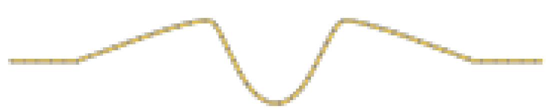
$$\frac{\text{KE}_2}{\text{KE}_1} = \frac{\frac{1}{2}kA_2^2}{\frac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{2.25\text{ cm}}{3.5\text{ cm}}\right)^2 = \boxed{0.41}$$

47. From Eq. 11–18, if the speed, medium density, and frequency of the two waves are the same, then the intensity is proportional to the square of the amplitude.

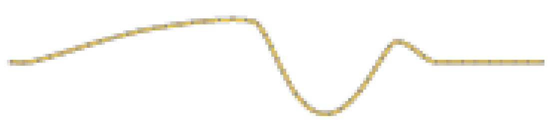
$$I_2/I_1 = E_2/E_1 = A_2^2/A_1^2 = 5.0 \quad \rightarrow \quad A_2/A_1 = \sqrt{5.0} = \boxed{2.2}$$

The more energetic wave has the larger amplitude.

48. (*a*)



(*b*)



(*c*) The energy is $\boxed{\text{all kinetic energy}}$ at the moment when the string has no displacement. There is no elastic potential energy at that moment. Each piece of the string has speed but no displacement.

49. The frequencies of the harmonics of a string that is fixed at both ends are given by $f_n = nf_1$, so the first four harmonics are $\boxed{f_1 = 440\text{ Hz}, f_2 = 880\text{ Hz}, f_3 = 1320\text{ Hz, and } f_4 = 1760\text{ Hz}}$.

50. The fundamental frequency of the full string is given by Eq. 11–19b, $f_{\text{unfingered}} = \frac{v}{2\ell} = 294$ Hz. If the length is reduced to two-thirds of its current value, and the velocity of waves on the string is not changed, then the new frequency will be

$$f_{\text{fingered}} = \frac{v}{2\left(\frac{2}{3}\ell\right)} = \frac{3}{2}\frac{v}{2\ell} = \left(\frac{3}{2}\right)f_{\text{unfingered}} = \left(\frac{3}{2}\right)294\text{ Hz} = \boxed{441\text{ Hz}}$$

51. Four loops is the standing wave pattern for the fourth harmonic, with a frequency given by $f_4 = 4f_1 = 240$ Hz. Thus, $\boxed{f_1 = 60\text{ Hz}, f_2 = 120\text{ Hz}, f_3 = 180\text{ Hz, and } f_5 = 300\text{ Hz}}$ are all other resonant frequencies, where f_1 is the fundamental or first harmonic, f_2 is the first overtone or second harmonic, f_3 is the second overtone or third harmonic, and f_5 is the fourth overtone or fifth harmonic.

52. Adjacent nodes are separated by a half-wavelength, as examination of Fig. 11–41b will show.

$$\lambda = \frac{v}{f} \quad \rightarrow \quad \Delta x_{\text{node}} = \tfrac{1}{2}\lambda = \frac{v}{2f} = \frac{97\text{ m/s}}{2(475\text{ Hz})} = 0.10211\text{ m} \approx \boxed{0.10\text{ m}}$$

53. Since $f_n = nf_1$, two successive overtones differ by the fundamental frequency, as shown below.

$$\Delta f = f_{n+1} - f_n = (n+1)\,f_1 - nf_1 = f_1 = 350\text{ Hz} - 280\text{ Hz} = \boxed{70\text{ Hz}}$$

54. The speed of waves on the string is given by Eq. 11–13, $v = \sqrt{F_T/\mu}$. The resonant frequencies of a string with both ends fixed are given by Eq. 11–19b, $f_n = \frac{nv}{2\ell_{\text{vib}}}$, where ℓ_{vib} is the length of the portion that is actually vibrating. Combining these relationships allows the frequencies to be calculated.

$$f_n = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{\mu}} = \frac{n}{2\ell_{\text{vib}}}\sqrt{\frac{F_T}{m/\ell}} \qquad f_1 = \frac{1}{2(0.62\text{ m})}\sqrt{\frac{520\text{ N}}{(3.4\times10^{-3}\text{ kg})/(0.92\text{ m})}} = 302.51\text{ Hz}$$

$$f_2 = 2f_1 = 605.01\text{ Hz} \qquad f_3 = 3f_1 = 907.52\text{ Hz}$$

So the three frequencies are $\boxed{300\text{ Hz, 610 Hz, and 910 Hz}}$, to 2 significant figures.

$\boxed{55.}$ The string must oscillate in a standing wave pattern to have a certain number of loops. The frequencies of the standing waves will all be 60.0 Hz, the same as the oscillator. That frequency is also expressed by Eq. 11–19b, $f_n = \frac{nv}{2\ell}$. The speed of waves on the string is given by Eq. 11–13, $v = \sqrt{F_T/\mu}$. The tension in the string will be the same as the weight of the masses hung from the end of the string, $F_T = mg$. Combining these relationships gives an expression for the masses hung from the end of the string.

(*a*) $f_n = \dfrac{n\upsilon}{2\ell} = \dfrac{n}{2\ell}\sqrt{\dfrac{F_T}{\mu}} = \dfrac{n}{2\ell}\sqrt{\dfrac{mg}{\mu}} \rightarrow m = \dfrac{4\ell^2 f_n^2 \mu}{n^2 g}$

$$m_1 = \frac{4(1.50\text{ m})^2(60.0\text{ Hz})^2(3.5\times10^{-4}\text{ kg/m})}{1^2(9.80\text{ m/s}^2)} = 1.157\text{ kg} \approx \boxed{1.2\text{ kg}}$$

(*b*) $m_2 = \dfrac{m_1}{2^2} = \dfrac{1.157\text{ kg}}{4} = \boxed{0.29\text{ kg}}$

(*c*) $m_5 = \dfrac{m_1}{5^2} = \dfrac{1.157\text{ kg}}{25} = \boxed{4.6\times10^{-2}\text{ kg}}$

56. The tension in the string is the weight of the hanging mass, $F_T = mg$. The speed of waves on the string can be found by $\upsilon = \sqrt{\dfrac{F_T}{\mu}} = \sqrt{\dfrac{mg}{\mu}}$, and the frequency is given as $f = 60$ Hz. The wavelength of waves created on the string will thus be given by

$$\lambda = \frac{\upsilon}{f} = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{1}{60.0\text{ Hz}}\sqrt{\frac{(0.080\text{ kg})(9.80\text{ m/s}^2)}{(3.5\times10^{-4}\text{ kg/m})}} = 0.7888\text{ m}$$

The length of the string must be an integer multiple of half of the wavelength for there to be nodes at both ends and thus form a standing wave. Thus, $\ell = \lambda/2, \lambda, 3\lambda/2, \cdots$. This gives $\ell = 0.39\text{ m}, 0.79\text{ m}, 1.18\text{ m}, 1.58\text{ m}, \cdots$ as the possible lengths, so there are $\boxed{\text{three}}$ standing wave patterns that may be achieved.

57. From the description of the water's behavior, there is an antinode at each end of the tub and a node in the middle. Thus, one wavelength is twice the tub length.

$$\upsilon = \lambda f = (2\ell_{\text{tub}})f = 2(0.75\text{ m})(0.85\text{ Hz}) = 1.275\text{ m/s} \approx \boxed{1.3\text{ m/s}}$$

58. The speed in the second medium can be found from the law of refraction, Eq. 11–20.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} \rightarrow \upsilon_2 = \upsilon_1\frac{\sin\theta_2}{\sin\theta_1} = (8.0\text{ km/s})\left(\frac{\sin 33^\circ}{\sin 44^\circ}\right) = \boxed{6.3\text{ km/s}}$$

59. The angle of refraction can be found from the law of refraction, Eq. 11–20. The relative velocities can be found from the relationship given in the problem.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} = \frac{331+0.60T_2}{331+0.60T_1} \rightarrow \sin\theta_2 = \sin 25^\circ\left(\frac{331+0.60(-15)}{331+0.60(15)}\right) = \sin 25^\circ\left(\frac{322}{340}\right) = 0.4002$$

$$\theta_2 = \sin^{-1}0.4002 = 23.59^\circ \approx \boxed{24^\circ}$$

60. The angle of refraction can be found from the law of refraction, Eq. 11–20. The relative velocities can be found from Eq. 11–14a.

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{\upsilon_2}{\upsilon_1} = \frac{\sqrt{E/\rho_2}}{\sqrt{E/\rho_1}} = \sqrt{\frac{\rho_1}{\rho_2}} = \sqrt{\frac{\text{SG}_1\rho_{\text{water}}}{\text{SG}_2\rho_{\text{water}}}} = \sqrt{\frac{\text{SG}_1}{\text{SG}_2}}$$

$$\sin\theta_2 = \sin\theta_1\sqrt{\frac{\text{SG}_1}{\text{SG}_2}} = \sin 38^\circ\sqrt{\frac{3.6}{2.5}} = 0.7388 \rightarrow \theta_2 = \sin^{-1}0.7388 = \boxed{48^\circ}$$

61. The wavelength is to be 1.0 m. Use Eq. 11–12.

$$\upsilon = f\lambda \quad \rightarrow \quad f = \frac{\upsilon}{\lambda} = \frac{344 \text{ m/s}}{0.75 \text{ m}} = 458.7 \text{ Hz} \approx \boxed{460 \text{ Hz}}$$

There will be significant diffraction only for wavelengths larger than the width of the window, so waves with frequencies $\boxed{\text{lower than 460 Hz}}$ would diffract when passing through this window.

62. Consider the conservation of energy for the person. Call the unstretched position of the fire net the zero location for both elastic potential energy and gravitational potential energy. We can measure both the amount of stretch of the fire net and the vertical displacement for gravitational potential energy by the variable y, measured positively for the upward direction. Calculate the spring constant by conserving energy between the window height $(y_{\text{top}} = 20.0 \text{ m})$ and the lowest location of the person $(y_{\text{bottom}} = -1.4 \text{ m})$. The person has no kinetic energy at either location.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}ky_{\text{bottom}}^2$$

$$k = 2mg\frac{(y_{\text{top}} - y_{\text{bottom}})}{y_{\text{bottom}}^2} = 2(62 \text{ kg})(9.80 \text{ m/s}^2)\frac{[20.0 \text{ m} - (-1.4 \text{ m})]}{(-1.4 \text{ m})^2} = 1.3268\times10^4 \text{ N/m}$$

(a) If the person were to lie on the fire net, the person would stretch the net an amount such that the upward force of the net would be equal to their weight.

$$F_{\text{ext}} = k|y| = mg \quad \rightarrow \quad |y| = \frac{mg}{k} = \frac{(62 \text{ kg})(9.80 \text{ m/s}^2)}{1.3268\times10^4 \text{ N/m}} = \boxed{4.6\times10^{-2} \text{ m}}$$

(b) To find the amount of stretch given a starting height of 38 m, again use conservation of energy. Note that there is no kinetic energy at the top or bottom positions.

$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad mgy_{\text{top}} = mgy_{\text{bottom}} + \tfrac{1}{2}ky_{\text{bottom}}^2 \quad \rightarrow \quad y_{\text{bottom}}^2 + \frac{2mg}{k}y_{\text{bottom}} - \frac{2mg}{k}y_{\text{top}} = 0$$

$$y_{\text{bottom}}^2 + 2\frac{(62 \text{ kg})(9.80 \text{ m/s}^2)}{1.3268\times10^4 \text{ N/m}}y_{\text{bottom}} - 2\frac{(62 \text{ kg})(9.80 \text{ m/s}^2)}{1.3268\times10^4 \text{ N/m}}(38 \text{ m}) = 0 \quad \rightarrow$$

$$y_{\text{bottom}}^2 + 0.091589\, y_{\text{bottom}} - 3.4804 = 0 \quad \rightarrow \quad y_{\text{bottom}} = -1.9119 \text{ m},\ 1.8204 \text{ m}$$

This is a quadratic equation. The solution is the negative root, since the net must be below the unstretched position. The result is that it stretches $\boxed{1.9 \text{ m}}$ down if the person jumps from 38 m.

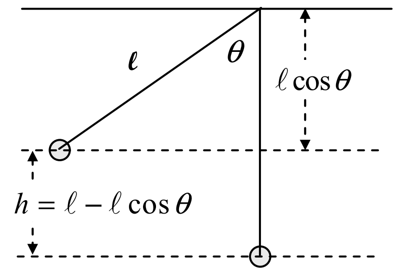
63. Apply conservation of mechanical energy to the car, calling condition 1 to be before the collision and condition 2 to be after the collision. Assume that all of the kinetic energy of the car is converted to potential energy stored in the bumper. We know that $x_1 = 0$ and $\upsilon_2 = 0$.

$$E_1 = E_2 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}m\upsilon_2^2 + \tfrac{1}{2}kx_2^2 \quad \rightarrow \quad \tfrac{1}{2}m\upsilon_1^2 = \tfrac{1}{2}kx_2^2 \quad \rightarrow$$

$$x_2 = \sqrt{\frac{m}{k}}\,\upsilon_1 = \sqrt{\frac{1300 \text{ kg}}{410\times10^3 \text{ N/m}}}(2.0 \text{ m/s}) = 0.1126 \text{ m} \approx \boxed{0.11 \text{ m}}$$

64. (a) The frequency can be found from the length of the pendulum and the acceleration due to gravity.

$$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{9.80 \text{ m/s}^2}{0.72 \text{ m}}} = 0.5872 \text{ Hz} \approx \boxed{0.59 \text{ Hz}}$$



(*b*) To find the speed at the lowest point, use the conservation of energy relating the lowest point to the release point of the pendulum. Take the lowest point to be the zero level of gravitational potential energy.

$$E_{\text{top}} = E_{\text{bottom}} \quad\rightarrow\quad \text{KE}_{\text{top}} + \text{PE}_{\text{top}} = \text{KE}_{\text{bottom}} + \text{PE}_{\text{bottom}}$$

$$0 + mg(\ell - \ell\cos\theta) = \tfrac{1}{2}mv_{\text{bottom}}^2 + 0$$

$$v_{\text{bottom}} = \sqrt{2g\ell(1-\cos\theta)} = \sqrt{2(9.80\text{ m/s}^2)(0.72\text{ m})(1-\cos 12^\circ)} = 0.5553\text{ m/s} \approx \boxed{0.56\text{ m/s}}$$

(*c*) The total energy can be found from the kinetic energy at the bottom of the motion.

$$E_{\text{total}} = \tfrac{1}{2}mv_{\text{bottom}}^2 = \tfrac{1}{2}(0.295\text{ kg})(0.5553\text{ m/s})^2 = \boxed{4.5\times10^{-2}\text{ J}}$$

65. For the penny to stay on the block at all times means that there will be a normal force on the penny from the block, exerted upward. If down is taken to be the positive direction, then the net force on the penny is $F_{\text{net}} = mg - F_{\text{N}} = ma.$ Solving for the magnitude of the normal force gives $F_{\text{N}} = mg - ma.$ This expression is always positive if the acceleration is upward $(a < 0),$ so there is no possibility of the penny losing contact while accelerating upward. But if a downward acceleration were to be larger than *g*, then the normal force would go to zero, since the normal force cannot switch directions $(F_{\text{N}} > 0).$ Thus the limiting condition is $a_{\text{down}} = g.$ This is the maximum value for the acceleration. For SHM, we also know that $a_{\max} = \omega^2 A = \dfrac{k}{M+m}A \approx \dfrac{k}{M}A.$ Equate these two values for the acceleration.

$$a_{\max} = \frac{k}{M}A = g \quad\rightarrow\quad \boxed{A = \frac{Mg}{k}}$$

66. Block *m* stays on top of block *M* (executing SHM relative to the ground) without slipping due to static friction. The maximum static frictional force on *m* is $F_{\text{fr}_{\max}} = \mu_{\text{s}}mg.$ This frictional force causes block *m* to accelerate, so $ma_{\max} = \mu_{\text{s}}mg \quad\rightarrow\quad a_{\max} = \mu_{\text{s}}g.$ Thus, for the blocks to stay in contact without slipping, the maximum acceleration of block *M* is also $a_{\max} = \mu_{\text{s}}g.$ But an object in SHM has a maximum acceleration given by $a_{\max} = \omega^2 A = \dfrac{k}{M_{\text{total}}}A.$ Equate these two expressions for the maximum acceleration.

$$a_{\max} = \frac{k}{M_{\text{total}}}A = \mu_{\text{s}}g \quad\rightarrow\quad A = \frac{\mu_{\text{s}}g}{k}(M+m) = \frac{(0.30)(9.80\text{ m/s}^2)}{130\text{ N/m}}(7.25\text{ kg}) = \boxed{0.16\text{ m}}$$

67. The frequency of a simple pendulum is given by Eq. 11–11b, $f = \dfrac{1}{2\pi}\sqrt{\dfrac{g}{\ell}}.$ The pendulum is accelerating vertically which is equivalent to increasing (or decreasing) the acceleration due to gravity by the acceleration of the pendulum.

(*a*) $$f_{\text{new}} = \frac{1}{2\pi}\sqrt{\frac{g+a}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{g+0.35\,g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{1.35\,g}{\ell}} = \sqrt{1.35}\,\frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \sqrt{1.35}\,f = \boxed{1.16\,f}$$

(b) $$f_{new} = \frac{1}{2\pi}\sqrt{\frac{g+a}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{g-0.35\,g}{\ell}} = \frac{1}{2\pi}\sqrt{\frac{0.65\,g}{\ell}} = \sqrt{0.65}\,\frac{1}{2\pi}\sqrt{\frac{g}{\ell}} = \sqrt{0.65}\,f = \boxed{0.81\,f}$$

68. The equation of motion is $x = 0.25\sin\,(4.70\,t) = A\sin\omega t$.

(a) The amplitude is $A = x_{max} = \boxed{0.25\text{ m}}$.

(b) The frequency is found by $\omega = 2\pi f = 4.70\text{ rad/s} \rightarrow f = \frac{4.70\text{ s}^{-1}}{2\pi} = \boxed{0.748\text{ Hz}}$.

(c) The period is the reciprocal of the frequency. $T = 1/f = \frac{2\pi}{4.70\text{ s}^{-1}} = \boxed{1.34\text{ s}}$.

(d) The total energy is given by the following:

$$E_{total} = \tfrac{1}{2}mv_{max}^2 = \tfrac{1}{2}m(\omega A)^2 = \tfrac{1}{2}(0.650\text{ kg})[(4.70\text{ s}^{-1})(0.25\text{ m})]^2 = 0.4487\text{ J} \approx \boxed{0.45\text{ J}}$$

(e) The potential energy is given by

$$\text{PE} = \tfrac{1}{2}kx^2 = \tfrac{1}{2}m\omega^2x^2 = \tfrac{1}{2}(0.650\text{ kg})(4.70\text{ s}^{-1})^2(0.15\text{ m})^2 = 0.1615\text{ J} \approx \boxed{0.16\text{ J}}$$

The kinetic energy is given by

$$\text{KE} = E_{total} - \text{PE} = 0.4487\text{ J} - 0.1615\text{ J} = 0.2872\text{ J} \approx \boxed{0.29\text{ J}}$$

69. The spring constant does not change, but the mass does, so the frequency will change. Use Eq. 11–6b to relate the spring constant, the mass, and the frequency for oxygen (O) and sulfur (S).

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow \frac{k}{4\pi^2} = f^2m = \text{constant} \rightarrow f_O^2m_O = f_S^2m_S \rightarrow$$

$$f_S = f_O\sqrt{\frac{m_O}{m_S}} = (3.7\times10^{13}\text{ Hz})\sqrt{\frac{16.0}{32.0}} = \boxed{2.6\times10^{13}\text{ Hz}}$$

70. Assume the block has a cross-sectional area of A. In the equilibrium position, the net force on the block is zero, so $F_{buoy} = mg$. When the block is pushed into the water (downward) an additional distance Δx, there is an increase in the buoyancy force (F_{extra}) equal to the weight of the additional water displaced. The weight of the extra water displaced is the density of water times the volume displaced.

$$F_{extra} = m_{\substack{extra\\water}}\,g = \rho_{water}V_{\substack{extra\\water}}\,g = \rho_{water}gA\Delta x = (\rho_{water}gA)\Delta x$$

This is the net force on the displaced block. Note that if the block is pushed down, the additional force is upward. And if the block were to be displaced upward by a distance Δx, the buoyancy force would actually be less than the weight of the block by the amount F_{extra}, so there would be a net force downward of magnitude F_{extra}. In both upward and downward displacement, there is a net force of magnitude $(\rho_{water}gA)\Delta x$ but opposite to the direction of displacement. So we can write $F_{net} = -(\rho_{water}gA)\Delta x$, indicating that the direction of the force is opposite to the direction of the displacement. This is the equation of simple harmonic motion, with a “spring constant” of $\boxed{k = \rho_{water}gA}$.

71. The force of the man’s weight causes the raft to sink, and that causes the water to put a larger upward force on the raft. This extra buoyant force is a restoring force, because it is in the opposite direction of

the force put on the raft by the man. This is analogous to pulling down on a mass–spring system that is in equilibrium by applying an extra force. Then when the man steps off, the restoring force pushes upward on the raft, and thus the raft–water system acts like a spring, with a spring constant found as follows:

$$k = \frac{F_{\text{ext}}}{x} = \frac{(68\text{ kg})(9.80\text{ m/s}^2)}{3.5\times10^{-2}\text{ m}} = 1.904\times10^4\text{ N/m}$$

(*a*) The frequency of vibration is determined by the "spring constant" and the mass of the raft.

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1.904\times10^4\text{ N/m}}{320\text{ kg}}} = 1.228\text{ Hz} \approx \boxed{1.2\text{ Hz}}$$

(*b*) As explained in the text, for a vertical spring the gravitational potential energy can be ignored if the displacement is measured from the oscillator's equilibrium position. The total energy is thus all elastic potential energy.

$$E_{\text{total}} = \tfrac{1}{2}kA^2 = \tfrac{1}{2}(1.904\times10^4\text{ N/m})(3.5\times10^{-2}\text{ m})^2 = 11.66\text{ J} \approx \boxed{12\text{ J}}$$

72. The pebble losing contact with the board means that there is no normal force of the board on the pebble. If there is no normal force on the pebble, then the only force on the pebble is the force of gravity, and the acceleration of the pebble will be *g* downward, the acceleration due to gravity. This is the maximum downward acceleration that the pebble can have. Thus, if the board's downward acceleration exceeds *g*, then the pebble will lose contact. The maximum acceleration and the amplitude are related by $a_{\max} = \frac{kA}{m} = \omega^2 A = 4\pi^2 f^2 A.$

$$a_{\max} = 4\pi^2 f^2 A \le g \quad\rightarrow\quad A \le \frac{g}{4\pi^2 f^2} \le \frac{9.80\text{ m/s}^2}{4\pi^2(2.8\text{ Hz})^2} \le \boxed{3.2\times10^{-2}\text{ m}}$$

73. (*a*) From conservation of energy, the initial kinetic energy of the car will all be changed into elastic potential energy by compressing the spring.

$$E_1 = E_2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 + \tfrac{1}{2}kx_1^2 = \tfrac{1}{2}mv_2^2 + \tfrac{1}{2}kx_2^2 \quad\rightarrow\quad \tfrac{1}{2}mv_1^2 = \tfrac{1}{2}kx_2^2 \quad\rightarrow$$

$$k = m\frac{v_1^2}{x_2^2} = (950\text{ kg})\frac{(25\text{ m/s})^2}{(4.0\text{ m})^2} = 3.711\times10^4\text{ N/m} \approx \boxed{3.7\times10^4\text{ N/m}}$$

(*b*) The car will be in contact with the spring for half a period, as it moves from the equilibrium location to maximum displacement and back to equilibrium.

$$\tfrac{1}{2}T = \tfrac{1}{2}2\pi\sqrt{\frac{m}{k}} = \pi\sqrt{\frac{950\text{ kg}}{3.711\times10^4\text{ N/m}}} = \boxed{0.50\text{ s}}$$

74. (*a*) The relationship between the velocity and the position of a SHO is given by Eq. 11–5b. Set that expression equal to half the maximum speed, and solve for the displacement.

$$v = \pm v_{\max}\sqrt{1 - x^2/A^2} = \tfrac{1}{2}v_{\max} \quad\rightarrow\quad \pm\sqrt{1 - x^2/A^2} = \tfrac{1}{2} \quad\rightarrow\quad 1 - x^2/A^2 = \tfrac{1}{4} \quad\rightarrow$$

$$x^2/A^2 = \tfrac{3}{4} \quad\rightarrow\quad \boxed{x = \pm\sqrt{3}A/2 \approx \pm\,0.866A}$$

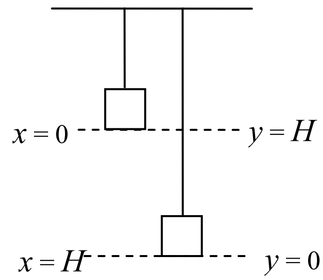
(*b*) Since $F = -kx = ma$ for an object attached to a spring, the acceleration is proportional to the displacement (although in the opposite direction), as $a = -xk/m.$ Thus the acceleration will have half its maximum value where the displacement has half its maximum value, at $\boxed{\pm\tfrac{1}{2}x_0}$.

75. The effective spring constant is determined by the frequency of vibration and the mass of the oscillator. Use Eq. 11–6b.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \quad \rightarrow$$

$$k = 4\pi^2 f^2 m = 4\pi^2 (2.83\times10^{13}\text{ Hz})(16.00\text{ u})\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = \boxed{8.40\times10^2\text{ N/m}}$$

76. Consider energy conservation for the mass over the range of motion from letting go (the highest point) to the lowest point. The mass falls the same distance that the spring is stretched, and has no KE at either endpoint. Call the lowest point the zero of gravitational potential energy. The variable x represents the amount that the spring is stretched from the equilibrium position.



$$E_{\text{top}} = E_{\text{bottom}} \quad \rightarrow \quad \tfrac{1}{2}mv_{\text{top}}^2 + mgy_{\text{top}} + \tfrac{1}{2}kx_{\text{top}}^2 = \tfrac{1}{2}mv_{\text{bottom}}^2 + mgy_{\text{bottom}} + \tfrac{1}{2}kx_{\text{bottom}}^2$$

$$0 + mgH + 0 = 0 + 0 + \tfrac{1}{2}kH^2 \quad \rightarrow \quad \frac{k}{m} = \frac{2g}{H} = \omega^2 \quad \rightarrow \quad \omega = \sqrt{\frac{2g}{H}}$$

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2g}{H}} = \frac{1}{2\pi}\sqrt{\frac{2(9.80\text{ m/s}^2)}{0.270\text{ m}}} = \boxed{1.36\text{ Hz}}$$

77. The maximum velocity is given by Eq. 11–7.

$$v_{\max} = \frac{2\pi A}{T} = \frac{2\pi(0.15\text{ m})}{7.0\text{ s}} = \boxed{0.13\text{ m/s}}$$

The maximum acceleration is given right after Eq. 11–10.

$$a_{\max} = \frac{kA}{m} = \omega^2 A = \frac{4\pi^2 A}{T^2} = \frac{4\pi^2(0.15\text{ m})}{(7.0\text{ s})^2} = 0.1209\text{ m/s}^2 \approx \boxed{0.12\text{ m/s}^2}$$

$$\frac{a_{\max}}{g} = \frac{0.1209\text{ m/s}^2}{9.80\text{ m/s}^2} = 1.2\times10^{-2} = \boxed{1.2\%}$$

78. The frequency at which the water is being shaken is about 1 Hz. The sloshing coffee is in a standing wave mode, with antinodes at each edge of the cup. The cup diameter is thus a half-wavelength, or $\lambda = 16$ cm. The wave speed can be calculated from the frequency and the wavelength.

$$v = \lambda f = (16\text{ cm})(1\text{ Hz}) = 16\text{ cm/s} \approx \boxed{20\text{ cm/s}}$$

79. (*a*) The amplitude is half the peak-to-peak distance, so $\boxed{0.06\text{ m}}$.

(*b*) The maximum kinetic energy of a particle in simple harmonic motion is the total energy, which is given by $E_{\text{total}} = \tfrac{1}{2}kA^2$. Compare the two kinetic energy maxima.

$$\frac{\text{KE}_{2\max}}{\text{KE}_{1\max}} = \frac{\tfrac{1}{2}kA_2^2}{\tfrac{1}{2}kA_1^2} = \left(\frac{A_2}{A_1}\right)^2 = \left(\frac{0.16\text{ m}}{0.06\text{ m}}\right)^2 = \boxed{7.1}$$

80. We assume that the earthquake wave is moving the ground vertically, since it is a transverse wave. An object sitting on the ground will then be moving with SHM, due to the two forces on it—the normal force upward from the ground and the weight downward due to gravity. If the object loses contact with the ground, then the normal force will be zero, and the only force on the object will be its weight. If the only force is the weight, then the object will have an acceleration of g downward. Thus, the limiting condition for beginning to lose contact with the ground is when the maximum acceleration caused by the wave is greater than g. Any larger downward acceleration, and the ground would "fall" quicker than the object. The maximum acceleration is related to the amplitude and the frequency as given after Eq. 11–10.

$$a_{\max} = \frac{kA}{m} = \omega^2 A > g \quad \rightarrow \quad A > \frac{g}{\omega^2} = \frac{g}{4\pi^2 f^2} = \frac{9.80\ \mathrm{m/s^2}}{4\pi^2 (0.60\ \mathrm{Hz})^2} = \boxed{0.69\ \mathrm{m}}$$

81. (*a*) The overtones are given by $f_n = nf_1,\ n = 2,3,4,\ldots.$

$$\text{G:}\quad f_2 = 2(392\ \mathrm{Hz}) = \boxed{784\ \mathrm{Hz}} \qquad f_3 = 3(392\ \mathrm{Hz}) = 1176\ \mathrm{Hz} \approx \boxed{1180\ \mathrm{Hz}}$$

$$\text{B:}\quad f_2 = 2(494\ \mathrm{Hz}) = \boxed{988\ \mathrm{Hz}} \qquad f_3 = 3(440\ \mathrm{Hz}) = 1482\ \mathrm{Hz} \approx \boxed{1480\ \mathrm{Hz}}$$

(*b*) If the two strings have the same length, they have the same wavelength. The frequency difference is then due to a difference in wave speed caused by different masses for the strings. The mass enters the problem through the mass per unit length.

$$\frac{f_G}{f_A} = \frac{\upsilon_G/\lambda}{\upsilon_A/\lambda} = \frac{\upsilon_G}{\upsilon_A} = \frac{\sqrt{\dfrac{F_T}{\mu_G}}}{\sqrt{\dfrac{F_T}{\mu_A}}} = \frac{\sqrt{\dfrac{F_T}{m_G/\ell}}}{\sqrt{\dfrac{F_T}{m_A/\ell}}} = \sqrt{\frac{m_A}{m_G}} \quad \rightarrow \quad \frac{m_G}{m_A} = \left(\frac{f_A}{f_G}\right)^2 = \left(\frac{494}{392}\right)^2 = \boxed{1.59}$$

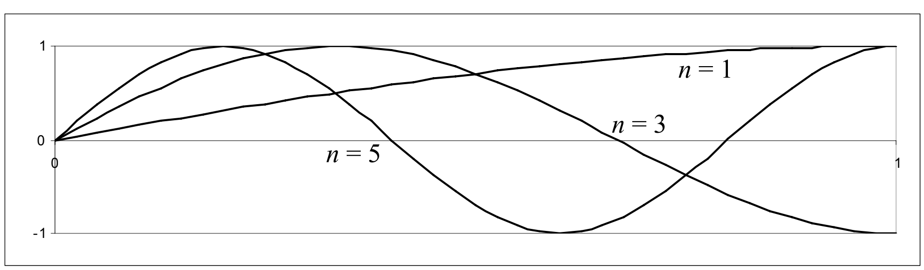
(*c*) If the two strings have the same mass per unit length and the same tension, then the wave speed on both strings is the same. The frequency difference is then due to a difference in wavelength. For the fundamental, the wavelength is twice the length of the string.

$$\frac{f_G}{f_B} = \frac{\upsilon/\lambda_G}{\upsilon/\lambda_B} = \frac{\lambda_B}{\lambda_G} = \frac{2\ell_B}{2\ell_G} \quad \rightarrow \quad \frac{\ell_G}{\ell_B} = \frac{f_B}{f_G} = \frac{494}{392} = \boxed{1.26}$$

(*d*) If the two strings have the same length, they have the same wavelength. They also have the same mass per unit length. The frequency difference is then due to a difference in wave speed caused by different tensions for the strings.

$$\frac{f_B}{f_A} = \frac{\upsilon_B/\lambda}{\upsilon_A/\lambda} = \frac{\upsilon_B}{\upsilon_A} = \frac{\sqrt{\dfrac{F_{TB}}{\mu}}}{\sqrt{\dfrac{F_{TA}}{\mu}}} = \sqrt{\frac{F_{TB}}{F_{TA}}} \quad \rightarrow \quad \frac{F_{TB}}{F_{TA}} = \left(\frac{f_B}{f_A}\right)^2 = \left(\frac{392}{494}\right)^2 = \boxed{0.630}$$

82.



For a resonant condition, the free end of the string will be an antinode, and the fixed end of the string will be a node. The minimum distance from a node to an antinode is $\lambda/4$. Other wave patterns that fit the boundary conditions of a node at one end and an antinode at the other end include $3\lambda/4, 5\lambda/4, \cdots$. See the diagrams. The general relationship is $\ell = (2n-1)\lambda/4,\ n = 1,2,3,\cdots$. Solving for the wavelength gives the following:

$$\boxed{\lambda = \frac{4\ell}{2n-1},\ n = 1, 2, 3, \cdots}$$

83. Relative to the fixed needle position, the ripples are moving with a linear velocity given by $v = \omega r$.

$$v = \left(33\tfrac{1}{3}\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)(0.102\text{ m}) = 0.3560\text{ m/s}$$

This speed is the speed of the ripple waves moving past the needle. The frequency of the waves is

$$f = \frac{v}{\lambda} = \frac{0.3560\text{ m/s}}{1.55\times10^{-3}\text{ m}} = \boxed{230\text{ Hz}}\ \text{(3 significant figures)}$$

84. The wave speed is given by Eq. 11–12, $v = \lambda f$, while the maximum speed of particles on the cord is given by Eq. 11–7, $v_{\text{max}} = 2\pi Af$. We equate the two expressions.

$$\lambda f = 2\pi Af \quad\rightarrow\quad A = \frac{\lambda}{2\pi} = \frac{10.0\text{ cm}}{2\pi} = \boxed{1.59\text{ cm}}$$

85. From the given data, $A = 0.50\text{ m}$ and $v = 2.5\text{ m}/4.0\text{ s} = 0.625\text{ m/s}$. We use Eq. 11–17b for the average power, with the density of seawater from Table 10–1. We estimate the area of the chest as $(0.30\text{ m})^2$. Answers may vary according to the approximation used for the area of the chest.

$$\overline{P} = 2\pi^2\rho Svf^2A^2 = 2\pi^2(1025\text{ kg/m}^3)(0.30\text{ m})^2(0.625\text{ m/s})(0.25\text{ Hz})^2(0.50\text{ m})^2$$
$$= \boxed{18\text{ W}}$$

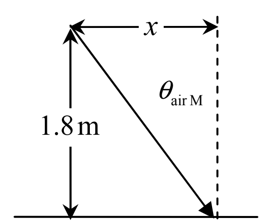
86. The unusual decrease of water corresponds to a trough in Fig. 11–24. The crest or peak of the wave is then one-half wavelength distant, traveling at 550 km/h.

$$\Delta x = vt = \tfrac{1}{2}\lambda \quad\rightarrow\quad t = \frac{\lambda}{2v} = \frac{235\text{ km}}{2(550\text{ km/h})}\left(\frac{60\text{ min}}{1\text{ h}}\right) = \boxed{13\text{ min}}$$

87. (*a*) Equation 11–20 gives the relationship between the angles and the speed of sound in the two media. For total internal reflection (for no sound to enter the water), $\theta_{\text{water}} = 90°$ or $\sin\theta_{\text{water}} = 1$. The air is the "incident" media. Thus the incident angle is given by the following:

$$\frac{\sin\theta_{\text{air}}}{\sin\theta_{\text{water}}} = \frac{v_{\text{air}}}{v_{\text{water}}};\ \theta_{\text{air}} = \theta_{\text{i}} = \sin^{-1}\left[\sin\theta_{\text{water}}\frac{v_{\text{air}}}{v_{\text{water}}}\right] \quad\rightarrow\quad \boxed{\theta_{\text{iM}} = \sin^{-1}\left[\frac{v_{\text{air}}}{v_{\text{water}}}\right] = \sin^{-1}\left[\frac{v_{\text{i}}}{v_{\text{r}}}\right]}$$

(*b*) From the angle of incidence, the distance is found. See the diagram.



$$\theta_{\text{air M}} = \sin^{-1}\frac{v_{\text{air}}}{v_{\text{water}}} = \sin^{-1}\frac{343\text{ m/s}}{1440\text{ m/s}} = 13.8°$$

$$\tan\theta_{\text{air M}} = \frac{x}{1.8\text{ m}} \quad\rightarrow\quad x = (1.8\text{ m})\tan 13.8° = \boxed{0.44\text{ m}}$$

Solutions to Search and Learn Problems

1. For an underdamped oscillation, the period is approximately equal to the period of an undamped oscillation. The period is related to the mass and spring constant as in Eq. 11–6a. To measure the period of oscillation, you can push down on the bumpers of the car and time the oscillation period. Then solving Eq. 11–6a gives the effective spring constant.

$$T = 2\pi\sqrt{\frac{m}{k}} \quad \rightarrow \quad k = m\left(\frac{2\pi}{T}\right)^2$$

This spring constant is for the four springs acting together. To determine the spring constant of each individual spring, you would divide the mass of the car by four.

$$k_1 = \frac{m}{4}\left(\frac{2\pi}{T}\right)^2 = \frac{m\pi^2}{T^2}$$

2. The resonant angular frequency of the spring is equal to the angular frequency of the tires when traveling at 90.0 km/h.

$$\omega = \frac{v}{r} = \frac{90.0\text{ km/h}}{\frac{1}{2}(0.58\text{ m})}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 86.21\text{ rad/s}$$

The spring constant can be calculated from this angular frequency and the mass of the tire/wheel combination.

$$\omega^2 = \frac{k}{m} \quad \rightarrow \quad k = \omega^2 m = (86.21\text{ rad/s})^2(17.0\text{ kg}) = 1.263\times10^5\text{ N/m}$$

Hooke's law (Eq. 6–8) can then be used to determine the distance the spring is compressed when the mass is added. We use the absolute value of the force and the distance, so that there is no negative sign in the equation.

$$F = kx \quad \rightarrow \quad x = \frac{F}{k} = \frac{mg}{k} = \frac{\frac{1}{4}(280\text{ kg})(9.80\text{ m/s}^2)}{1.263\times10^5\text{ N/m}} = 5.432\times10^{-3}\text{ m} \approx \boxed{5.4\times10^{-3}\text{ m}}$$

3. Forced resonance and underdamping are factors in the pronounced rattle and vibration. Forced resonance becomes a factor because hitting the bumps in the road is "forcing" action on the car. Assuming that the bumps are regularly spaced and the car has a constant speed, then that forcing will have a particular frequency. The forcing becomes "forced resonance" when the frequency of hitting the road bumps matches a resonant frequency of some part in the car. For example, if the frequency at which the car hits the bumps in the road is in resonance with the natural frequency of the springs, the car will have a large amplitude of oscillation and will shake strongly. If those springs are underdamped, then the oscillation resulting from hitting one bump is still occurring when you hit the next bump, so the amplitude increases with each bump. A significant oscillation amplitude can be built up. If the springs were critically damped or overdamped, the oscillation would not build up—it would dampen before the next bump was encountered.

4. One wavelength, or one full oscillation, corresponds to 360°—a full cycle of the sinusoidal oscillation that is creating the wave disturbance. Therefore, a half of a wavelength ($\lambda/2$) corresponds to a half oscillation, or 180°.

5. We must make several assumptions. Consider a static displacement of the trampoline, by someone sitting on the trampoline mat. The upward elastic force of the trampoline must equal the downward

force of gravity. We estimate that a 75-kg person will depress the trampoline about 25 cm at its midpoint. Hooke's law (Eq. 6–8) can then be solved for the spring constant.

$$kx = mg \quad \to \quad k = \frac{mg}{x} = \frac{(75\text{ kg})(9.80\text{ m/s}^2)}{0.25\text{ m}} = 2940\text{ N/m} \approx \boxed{3000\text{ N/m}}$$

Answers will vary based on the assumptions made.

6. The addition of the support will force the bridge to have an oscillation node at the center of the span. This makes the new fundamental frequency equal to the first overtone of the original fundamental frequency. If the wave speed in the bridge material doesn't change, then the resonant frequency will double, to $\boxed{6.0\text{ Hz}}$. Since earthquakes don't do significant shaking at that frequency, the modifications would be effective at keeping the bridge from having large oscillations during an earthquake.