Chapter 15

1. (a) During simple harmonic motion, the speed is (momentarily) zero when the object is at a "turning point" (that is, when $x = +x_m$ or $x = -x_m$). Consider that it starts at $x = +x_m$ and we are told that $t = 0.25$ second elapses until the object reaches $x = -x_m$. To execute a full cycle of the motion (which takes a period T to complete), the object which started at $x = +x_m$, must return to $x = +x_m$ (which, by symmetry, will occur 0.25 second *after* it was at $x = -x_m$). Thus, $T = 2t = 0.50$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T = 2.0$ Hz.

(c) The 36 cm distance between $x = +x_m$ and $x = -x_m$ is $2x_m$. Thus, $x_m = 36/2 = 18$ cm.

2. (a) The acceleration amplitude is related to the maximum force by Newton's second law: $F_{\max} = ma_m$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). The frequency is the reciprocal of the period: $f = 1/T = 1/0.20 = 5.0$ Hz, so the angular frequency is $\omega = 10\pi$ (understood to be valid to two significant figures). Therefore,

$$F_{\max} = m\omega^2 x_m = (0.12 \text{ kg})(10\pi \text{ rad/s})^2 (0.085 \text{ m}) = 10 \text{ N}.$$

(b) Using Eq. 15-12, we obtain

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = m\omega^2 = (0.12\text{kg})(10\pi \text{ rad/s})^2 = 1.2\times10^2 \text{ N/m}.$$

3. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \omega^2 x_m = (2\pi f)^2 x_m = \left(2\pi(6.60 \text{ Hz})\right)^2 (0.0220 \text{ m}) = 37.8 \text{ m/s}^2.$$

4. (a) Since the problem gives the frequency $f = 3.00$ Hz, we have $\omega = 2\pi f = 6\pi$ rad/s (understood to be valid to three significant figures). Each spring is considered to support one fourth of the mass m_{car} so that Eq. 15-12 leads to

$$\omega = \sqrt{\frac{k}{m_{\text{car}}/4}} \Rightarrow k = \frac{1}{4}(1450\text{kg})(6\pi \text{ rad/s})^2 = 1.29\times10^5 \text{ N/m}.$$

(b) If the new mass being supported by the four springs is m_{total} = [1450 + 5(73)] kg = 1815 kg, then Eq. 15-12 leads to

$$\omega_{new} = \sqrt{\frac{k}{m_{total}/4}} \quad \Rightarrow \; f_{new} = \frac{1}{2\pi}\sqrt{\frac{1.29\times10^5 \text{ N/m}}{(1815/4) \text{ kg}}} = 2.68\text{Hz}.$$

5. (a) The amplitude is half the range of the displacement, or x_m = 1.0 mm.

(b) The maximum speed v_m is related to the amplitude x_m by $v_m = \omega x_m$, where ω is the angular frequency. Since $\omega = 2\pi f$, where f is the frequency,

$$v_m = 2\pi f x_m = 2\pi\left(120 \text{ Hz}\right)\left(1.0\times10^{-3} \text{ m}\right) = 0.75 \text{ m/s}.$$

(c) The maximum acceleration is

$$a_m = \omega^2 x_m = \left(2\pi f\right)^2 x_m = \left(2\pi\left(120 \text{ Hz}\right)\right)^2 \left(1.0\times10^{-3} \text{ m}\right) = 5.7\times10^2 \text{ m/s}^2.$$

6. (a) The angular frequency ω is given by $\omega = 2\pi f = 2\pi/T$, where f is the frequency and T is the period. The relationship $f = 1/T$ was used to obtain the last form. Thus

$$\omega = 2\pi/(1.00 \times 10^{-5} \text{ s}) = 6.28 \times 10^5 \text{ rad/s}.$$

(b) The maximum speed v_m and maximum displacement x_m are related by $v_m = \omega x_m$, so

$$x_m = \frac{v_m}{\omega} = \frac{1.00\times10^3 \text{ m/s}}{6.28\times10^5 \text{ rad/s}} = 1.59\times10^{-3} \text{ m}.$$

7. The magnitude of the maximum acceleration is given by $a_m = \omega^2 x_m$, where ω is the angular frequency and x_m is the amplitude.

(a) The angular frequency for which the maximum acceleration is g is given by $\omega = \sqrt{g/x_m}$, and the corresponding frequency is given by

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{g}{x_m}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2}{1.0\times10^{-6}\text{m}}} = 498 \text{ Hz}.$$

(b) For frequencies greater than 498 Hz, the acceleration exceeds g for some part of the motion.

8. We note (from the graph in the text) that x_m = 6.00 cm. Also the value at t = 0 is x_o = −2.00 cm. Then Eq. 15-3 leads to

$$\phi = \cos^{-1}(-2.00/6.00) = +1.91 \text{ rad or } -4.37 \text{ rad.}$$

The other "root" (+4.37 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

9. (a) Making sure our calculator is in radians mode, we find

$$x = 6.0\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = 3.0 \text{ m.}$$

(b) Differentiating with respect to time and evaluating at $t = 2.0$ s, we find

$$v = \frac{dx}{dt} = -3\pi(6.0)\sin\left(3\pi(2.0) + \frac{\pi}{3}\right) = -49 \text{ m/s.}$$

(c) Differentiating again, we obtain

$$a = \frac{dv}{dt} = -(3\pi)^2(6.0)\cos\left(3\pi(2.0) + \frac{\pi}{3}\right) = -2.7\times10^2 \text{ m/s}^2.$$

(d) In the second paragraph after Eq. 15-3, the textbook defines the phase of the motion. In this case (with $t = 2.0$ s) the phase is $3\pi(2.0) + \pi/3 \approx 20$ rad.

(e) Comparing with Eq. 15-3, we see that $\omega = 3\pi$ rad/s. Therefore, $f = \omega/2\pi = 1.5$ Hz.

(f) The period is the reciprocal of the frequency: $T = 1/f \approx 0.67$ s.

10. (a) The problem describes the time taken to execute one cycle of the motion. The period is $T = 0.75$ s.

(b) Frequency is simply the reciprocal of the period: $f = 1/T \approx 1.3$ Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second.

(c) Since 2π radians are equivalent to a cycle, the angular frequency ω (in radians-per-second) is related to frequency f by $\omega = 2\pi f$ so that $\omega \approx 8.4$ rad/s.

11. When displaced from equilibrium, the net force exerted by the springs is $-2kx$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/dt^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -2kx.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{2k}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}} = \frac{1}{2\pi}\sqrt{\frac{2(7580 \text{ N/m})}{0.245 \text{ kg}}} = 39.6 \text{ Hz}.$$

12. We note (from the graph) that $v_m = \omega x_m = 5.00$ cm/s. Also the value at $t = 0$ is $v_o =$ 4.00 cm/s. Then Eq. 15-6 leads to

$$\phi = \sin^{-1}(-4.00/5.00) = -0.927 \text{ rad or } +5.36 \text{ rad}.$$

The other "root" (+4.07 rad) can be rejected on the grounds that it would lead to a positive slope at $t = 0$.

13. (a) The motion repeats every 0.500 s so the period must be $T = 0.500$ s.

(b) The frequency is the reciprocal of the period: $f = 1/T = 1/(0.500 \text{ s}) = 2.00$ Hz.

(c) The angular frequency ω is $\omega = 2\pi f = 2\pi(2.00 \text{ Hz}) = 12.6$ rad/s.

(d) The angular frequency is related to the spring constant k and the mass m by $\omega = \sqrt{k/m}$. We solve for k and obtain

$$k = m\omega^2 = (0.500 \text{ kg})(12.6 \text{ rad/s})^2 = 79.0 \text{ N/m}.$$

(e) Let x_m be the amplitude. The maximum speed is

$$v_m = \omega x_m = (12.6 \text{ rad/s})(0.350 \text{ m}) = 4.40 \text{ m/s}.$$

(f) The maximum force is exerted when the displacement is a maximum and its magnitude is given by $F_m = kx_m = (79.0 \text{ N/m})(0.350 \text{ m}) = 27.6$ N.

14. Equation 15-12 gives the angular velocity:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{100 \text{ N/m}}{2.00 \text{ kg}}} = 7.07 \text{ rad/s}.$$

Energy methods (discussed in Section 15-4) provide one method of solution. Here, we use trigonometric techniques based on Eq. 15-3 and Eq. 15-6.

(a) Dividing Eq. 15-6 by Eq. 15-3, we obtain

$$\frac{v}{x} = -\omega \tan(\omega t + \phi)$$

so that the phase ($\omega t + \phi$) is found from

$$\omega t + \phi = \tan^{-1}\left(\frac{-v}{\omega x}\right) = \tan^{-1}\left(\frac{-3.415 \text{ m/s}}{(7.07 \text{ rad/s})(0.129 \text{ m})}\right).$$

With the calculator in radians mode, this gives the phase equal to –1.31 rad. Plugging this back into Eq. 15-3 leads to $0.129 \text{ m} = x_m \cos(-1.31) \;\Rightarrow\; x_m = 0.500 \text{ m}$.

(b) Since $\omega t + \phi = -1.31$ rad at $t = 1.00$ s, we can use the above value of ω to solve for the phase constant ϕ. We obtain $\phi = -8.38$ rad (though this, as well as the previous result, can have 2π or 4π (and so on) added to it without changing the physics of the situation). With this value of ϕ, we find $x_o = x_m \cos \phi = -0.251$ m.

(c) And we obtain $v_o = -x_m\omega \sin\phi = 3.06$ m/s.

15. (a) Let

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi t}{T}\right)$$

be the coordinate as a function of time for particle 1 and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right)$$

be the coordinate as a function of time for particle 2. Here T is the period. Note that since the range of the motion is A, the amplitudes are both $A/2$. The arguments of the cosine functions are in radians. Particle 1 is at one end of its path ($x_1 = A/2$) when $t = 0$. Particle 2 is at $A/2$ when $2\pi t/T + \pi/6 = 0$ or $t = -T/12$. That is, particle 1 lags particle 2 by one-twelfth a period. We want the coordinates of the particles 0.50 s later; that is, at $t = 0.50$ s,

$$x_1 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50 \text{ s}}{1.5 \text{ s}}\right) = -0.25A$$

and

$$x_2 = \frac{A}{2}\cos\left(\frac{2\pi \times 0.50 \text{ s}}{1.5 \text{ s}} + \frac{\pi}{6}\right) = -0.43A.$$

Their separation at that time is $x_1 - x_2 = -0.25A + 0.43A = 0.18A$.

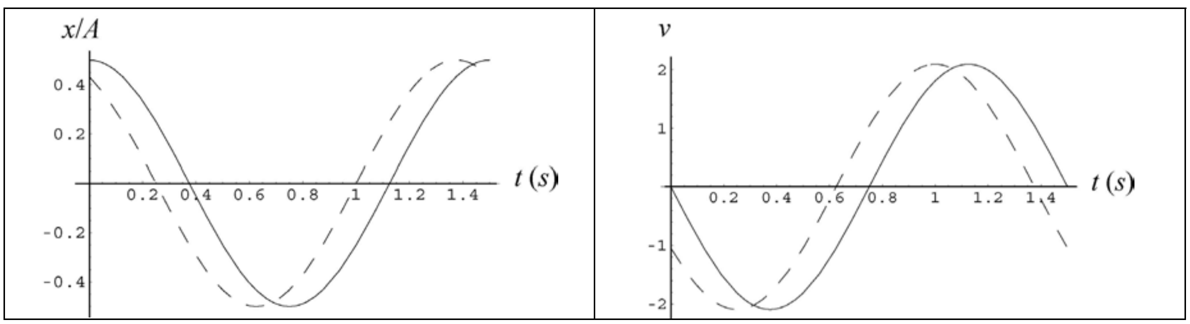
(b) The velocities of the particles are given by

$$v_1 = \frac{dx_1}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T}\right)$$

and

$$v_2 = \frac{dx_2}{dt} = \frac{\pi A}{T}\sin\left(\frac{2\pi t}{T} + \frac{\pi}{6}\right).$$

We evaluate these expressions for t = 0.50 s and find they are both negative-valued, indicating that the particles are moving in the same direction. The plots of x and v as a function of time for particle 1 (so1id) and particle 2 (dashed line) are given below.



16. They pass each other at time t, at $x_1 = x_2 = \frac{1}{2}x_m$ where

$$x_1 = x_m\cos(\omega t + \phi_1) \quad \text{and} \quad x_2 = x_m\cos(\omega t + \phi_2).$$

From this, we conclude that $\cos(\omega t + \phi_1) = \cos(\omega t + \phi_2) = \frac{1}{2}$, and therefore that the phases (the arguments of the cosines) are either both equal to $\pi/3$ or one is $\pi/3$ while the other is $-\pi/3$. Also at this instant, we have $v_1 = -v_2 \neq 0$ where

$$v_1 = -x_m\omega\sin(\omega t + \phi_1) \quad \text{and} \quad v_2 = -x_m\omega\sin(\omega t + \phi_2).$$

This leads to $\sin(\omega t + \phi_1) = -\sin(\omega t + \phi_2)$. This leads us to conclude that the phases have opposite sign. Thus, one phase is $\pi/3$ and the other phase is $-\pi/3$; the wt term cancels if we take the phase difference, which is seen to be $\pi/3 - (-\pi/3) = 2\pi/3$.

17. (a) Equation 15-8 leads to

$$a = -\omega^2 x \;\Rightarrow\; \omega = \sqrt{\frac{-a}{x}} = \sqrt{\frac{123\ \text{m/s}^2}{0.100\ \text{m}}} = 35.07\ \text{rad/s}\,.$$

Therefore, $f = \omega/2\pi = 5.58$ Hz.

(b) Equation 15-12 provides a relation between ω (found in the previous part) and the mass:

$$\omega=\sqrt{\frac{k}{m}} \quad\Rightarrow\quad m=\frac{400\ \text{N/m}}{(35.07\ \text{rad/s})^2}=0.325\text{kg}.$$

(c) By energy conservation, $\frac{1}{2}kx_m^2$ (the energy of the system at a turning point) is equal to the sum of kinetic and potential energies at the time t described in the problem.

$$\frac{1}{2}kx_m^2=\frac{1}{2}mv^2+\frac{1}{2}kx^2 \Rightarrow x_m=\frac{m}{k}v^2+x^2.$$

Consequently, $x_m=\sqrt{(0.325\ \text{kg}/400\ \text{N/m})(13.6\ \text{m/s})^2+(0.100\ \text{m})^2}=0.400\ \text{m}.$

18. From highest level to lowest level is twice the amplitude x_m of the motion. The period is related to the angular frequency by Eq. 15-5. Thus, $x_m=\frac{1}{2}d$ and $\omega = 0.503$ rad/h. The phase constant ϕ in Eq. 15-3 is zero since we start our clock when $x_o = x_m$ (at the highest point). We solve for t when x is one-fourth of the total distance from highest to lowest level, or (which is the same) half the distance from highest level to middle level (where we locate the origin of coordinates). Thus, we seek t when the ocean surface is at $x=\frac{1}{2}x_m=\frac{1}{4}d$. With $x=x_m\cos(\omega t+\phi)$, we obtain

$$\frac{1}{4}d=\left(\frac{1}{2}d\right)\cos\left(0.503t+0\right) \quad\Rightarrow\quad \frac{1}{2}=\cos(0.503t)$$

which has t = 2.08 h as the smallest positive root. The calculator is in radians mode during this calculation.

19. Both parts of this problem deal with the critical case when the maximum acceleration becomes equal to that of free fall. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency; this is the expression we set equal to $g = 9.8\ \text{m/s}^2$.

(a) Using Eq. 15-5 and T = 1.0 s, we have

$$\left(\frac{2\pi}{T}\right)^2 x_m=g \Rightarrow x_m=\frac{gT^2}{4\pi^2}=0.25\ \text{m}.$$

(b) Since $\omega = 2\pi f$, and x_m = 0.050 m is given, we find

$$\left(2\pi f\right)^2 x_m=g \quad\Rightarrow\quad f=\frac{1}{2\pi}\sqrt{\frac{g}{x_m}}=2.2\ \text{Hz}.$$

20. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where $\omega = 1.20$ rad/s in this problem. Evaluating this at $t = 0$ and using the values from the graphs shown in the problem, we find

$$\phi = \tan^{-1}(-v_o/x_o\omega) = \tan^{-1}(+4.00/(2 \times 1.20)) = 1.03 \text{ rad (or } -5.25 \text{ rad).}$$

One can check that the other "root" (4.17 rad) is unacceptable since it would give the wrong signs for the individual values of v_o and x_o.

21. Let the spring constants be k_1 and k_2. When displaced from equilibrium, the magnitude of the net force exerted by the springs is $|k_1x + k_2\,x|$ acting in a direction so as to return the block to its equilibrium position ($x = 0$). Since the acceleration $a = d^2x/d^2$, Newton's second law yields

$$m\frac{d^2x}{dt^2} = -k_1x - k_2x.$$

Substituting $x = x_m \cos(\omega t + \phi)$ and simplifying, we find

$$\omega^2 = \frac{k_1 + k_2}{m}$$

where ω is in radians per unit time. Since there are 2π radians in a cycle, and frequency f measures cycles per second, we obtain

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k_1 + k_2}{m}}.$$

The single springs each acting alone would produce simple harmonic motions of frequency

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k_1}{m}} = 30 \text{ Hz}, \qquad f_2 = \frac{1}{2\pi}\sqrt{\frac{k_2}{m}} = 45 \text{ Hz},$$

respectively. Comparing these expressions, it is clear that

$$f = \sqrt{f_1^2 + f_2^2} = \sqrt{(30 \text{ Hz})^2 + (45 \text{ Hz})^2} = 54 \text{ Hz}.$$

22. The statement that "the spring does not affect the collision" justifies the use of elastic collision formulas in section 10-5. We are told the period of SHM so that we can find the mass of block 2:

$$T = 2\pi\sqrt{\frac{m_2}{k}} \quad \Rightarrow \quad m_2 = \frac{kT^2}{4\pi^2} = 0.600 \text{ kg}.$$

At this point, the rebound speed of block 1 can be found from Eq. 10-30:

$$|v_{1f}| = \left|\frac{0.200\text{ kg} - 0.600\text{ kg}}{0.200\text{ kg} + 0.600\text{ kg}}\right|(8.00\text{ m/s}) = 4.00\text{ m/s}\ .$$

This becomes the initial speed v_0 of the projectile motion of block 1. A variety of choices for the positive axis directions are possible, and we choose left as the $+x$ direction and down as the $+y$ direction, in this instance. With the "launch" angle being zero, Eq. 4-21 and Eq. 4-22 (with $-g$ replaced with $+g$) lead to

$$x - x_0 = v_0 t = v_0\sqrt{\frac{2h}{g}} = (4.00\text{ m/s})\sqrt{\frac{2(4.90\text{ m})}{9.8\text{ m/s}^2}}\ .$$

Since $x - x_0 = d$, we arrive at $d = 4.00$ m.

23. The maximum force that can be exerted by the surface must be less than $\mu_s F_N$ or else the block will not follow the surface in its motion. Here, μ_s is the coefficient of static friction and F_N is the normal force exerted by the surface on the block. Since the block does not accelerate vertically, we know that $F_N = mg$, where m is the mass of the block. If the block follows the table and moves in simple harmonic motion, the magnitude of the maximum force exerted on it is given by

$$F = ma_m = m\omega^2 x_m = m(2\pi f)^2 x_m,$$

where a_m is the magnitude of the maximum acceleration, ω is the angular frequency, and f is the frequency. The relationship $\omega = 2\pi f$ was used to obtain the last form. We substitute $F = m(2\pi f)^2 x_m$ and $F_N = mg$ into $F < \mu_s F_N$ to obtain $m(2\pi f)^2 x_m < \mu_s mg$. The largest amplitude for which the block does not slip is

$$x_m = \frac{\mu_s g}{(2\pi f)^2} = \frac{(0.50)(9.8\text{ m/s}^2)}{(2\pi \times 2.0\text{ Hz})^2} = 0.031\text{ m}.$$

A larger amplitude requires a larger force at the end points of the motion. The surface cannot supply the larger force and the block slips.

24. We wish to find the effective spring constant for the combination of springs shown in the figure. We do this by finding the magnitude F of the force exerted on the mass when the total elongation of the springs is Δx. Then $k_{\text{eff}} = F/\Delta x$. Suppose the left-hand spring is elongated by Δx_ℓ and the right-hand spring is elongated by Δx_r. The left-hand spring exerts a force of magnitude $k\Delta x_\ell$ on the right-hand spring and the right-hand spring exerts a force of magnitude $k\Delta x_r$ on the left-hand spring. By Newton's third law these must be equal, so $\Delta x_\ell = \Delta x_r$. The two elongations must be the same, and the total elongation is twice the elongation of either spring: $\Delta x = 2\Delta x_\ell$. The left-hand spring exerts a force on

the block and its magnitude is $F = k\Delta x_\ell$. Thus $k_{\text{eff}} = k\Delta x_\ell / 2\Delta x_r = k/2$. The block behaves as if it were subject to the force of a single spring, with spring constant $k/2$. To find the frequency of its motion, replace k_{eff} in $f = (1/2\pi)\sqrt{k_{\text{eff}}/m}$ with $k/2$ to obtain

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}}.$$

With m = 0.245 kg and k = 6430 N/m, the frequency is f = 18.2 Hz.

25. (a) We interpret the problem as asking for the equilibrium position; that is, the block is gently lowered until forces balance (as opposed to being suddenly released and allowed to oscillate). If the amount the spring is stretched is x, then we examine force-components along the incline surface and find

$$kx = mg\sin\theta \;\Rightarrow\; x = \frac{mg\sin\theta}{k} = \frac{(14.0\text{ N})\sin 40.0°}{120\text{ N/m}} = 0.0750\text{ m}$$

at equilibrium. The calculator is in degrees mode in the above calculation. The distance from the top of the incline is therefore (0.450 + 0.75) m = 0.525 m.

(b) Just as with a vertical spring, the effect of gravity (or one of its components) is simply to shift the equilibrium position; it does not change the characteristics (such as the period) of simple harmonic motion. Thus, Eq. 15-13 applies, and we obtain

$$T = 2\pi\sqrt{\frac{14.0\text{ N}/9.80\text{ m/s}^2}{120\text{ N/m}}} = 0.686\text{ s}.$$

26. To be on the verge of slipping means that the force exerted on the smaller block (at the point of maximum acceleration) is $f_{\max} = \mu_s mg$. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where $\omega = \sqrt{k/(m+M)}$ is the angular frequency (from Eq. 15-12). Therefore, using Newton's second law, we have

$$ma_m = \mu_s mg \Rightarrow \frac{k}{m+M}x_m = \mu_s g$$

which leads to

$$x_m = \frac{\mu_s g(m+M)}{k} = \frac{(0.40)(9.8\text{ m/s}^2)(1.8\text{ kg} + 10\text{ kg})}{200\text{ N/m}} = 0.23\text{ m} = 23\text{ cm}.$$

27. The total energy is given by $E = \frac{1}{2}kx_m^2$, where k is the spring constant and x_m is the amplitude. We use the answer from part (b) to do part (a), so it is best to look at the solution for part (b) first.

(a) The fraction of the energy that is kinetic is

$$\frac{K}{E}=\frac{E-U}{E}=1-\frac{U}{E}=1-\frac{1}{4}=\frac{3}{4}=0.75$$

where the result from part (b) has been used.

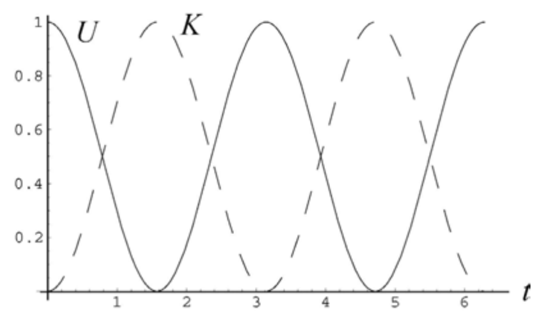
(b) When $x=\frac{1}{2}x_m$ the potential energy is $U=\frac{1}{2}kx^2=\frac{1}{8}kx_m^2$. The ratio is

$$\frac{U}{E}=\frac{kx_m^2/8}{kx_m^2/2}=\frac{1}{4}=0.25.$$

(c) Since $E=\frac{1}{2}kx_m^2$ and $U=\frac{1}{2}kx^2$, $U/E = x^2/x_m^2$. We solve x^2/x_m^2 = 1/2 for x. We should get $x=x_m/\sqrt{2}$.

The figure to the right depicts the potential energy (solid line) and kinetic energy (dashed line) as a function of time, assuming $x(0)=x_m$. The two curves intersect when $K=U=E/2$, or equivalently,

$$\cos^2\omega t=\sin^2\omega t=1/2.$$



28. The total mechanical energy is equal to the (maximum) kinetic energy as it passes through the equilibrium position (x = 0):

$$\tfrac{1}{2}mv^2=\tfrac{1}{2}(2.0\text{ kg})(0.85\text{ m/s})^2=0.72\text{ J}.$$

Looking at the graph in the problem, we see that $U(x = 10) = 0.5$ J. Since the potential function has the form $U(x)=bx^2$, the constant is $b=5.0\times10^{-3}\,\text{J/cm}^2$. Thus, $U(x) = 0.72$ J when x = 12 cm.

(a) Thus, the mass does turn back before reaching x = 15 cm.

(b) It turns back at x = 12 cm.

29. When the block is at the end of its path and is momentarily stopped, its displacement is equal to the amplitude and all the energy is potential in nature. If the spring potential energy is taken to be zero when the block is at its equilibrium position, then

$$E=\frac{1}{2}kx_m^2=\frac{1}{2}\left(1.3\times10^2\text{ N/m}\right)\left(0.024\text{ m}\right)^2=3.7\times10^{-2}\text{ J}.$$

30. (a) The energy at the turning point is all potential energy: $E = \frac{1}{2}kx_m^2$ where E = 1.00 J and x_m = 0.100 m. Thus,

$$k = \frac{2E}{x_m^2} = 200 \text{ N / m}.$$

(b) The energy as the block passes through the equilibrium position (with speed v_m = 1.20 m/s) is purely kinetic:

$$E = \frac{1}{2}mv_m^2 \Rightarrow m = \frac{2E}{v_m^2} = 1.39 \text{ kg}.$$

(c) Equation 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.91 \text{ Hz}.$$

31. (a) Equation 15-12 (divided by 2π) yields

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{1000 \text{ N / m}}{5.00 \text{ kg}}} = 2.25 \text{ Hz}.$$

(b) With x_0 = 0.500 m, we have $U_0 = \frac{1}{2}kx_0^2 = 125 \text{ J}$.

(c) With v_0 = 10.0 m/s, the initial kinetic energy is $K_0 = \frac{1}{2}mv_0^2 = 250 \text{ J}$.

(d) Since the total energy $E = K_0 + U_0$ = 375 J is conserved, then consideration of the energy at the turning point leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = 0.866 \text{ m}.$$

32. We infer from the graph (since mechanical energy is conserved) that the *total* energy in the system is 6.0 J; we also note that the amplitude is apparently x_m = 12 cm = 0.12 m. Therefore we can set the maximum *potential* energy equal to 6.0 J and solve for the spring constant k:

$$\tfrac{1}{2}k\,x_m^2 = 6.0 \text{ J} \quad \Rightarrow \quad k = 8.3 \times 10^2 \text{ N/m}.$$

33. The problem consists of two distinct parts: the completely inelastic collision (which is assumed to occur instantaneously, the bullet embedding itself in the block before the block moves through significant distance) followed by simple harmonic motion (of mass $m + M$ attached to a spring of spring constant k).

(a) Momentum conservation readily yields $v' = mv/(m + M)$. With m = 9.5 g, M = 5.4 kg, and v = 630 m/s, we obtain $v' = 1.1$ m/s.

(b) Since v' occurs at the equilibrium position, then $v' = v_m$ for the simple harmonic motion. The relation $v_m = \omega x_m$ can be used to solve for x_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter:

$$\frac{1}{2}(m+M)v'^2 = \frac{1}{2}kx_m^2 \quad \Rightarrow \quad \frac{1}{2}(m+M)\frac{m^2v^2}{(m+M)^2} = \frac{1}{2}kx_m^2$$

which simplifies to

$$x_m = \frac{mv}{\sqrt{k(m+M)}} = \frac{(9.5\times10^{-3}\text{kg})(630\text{ m/s})}{\sqrt{(6000\text{ N/m})(9.5\times10^{-3}\text{kg}+5.4\text{kg})}} = 3.3\times10^{-2}\text{ m}.$$

34. We note that the spring constant is

$$k = 4\pi^2 m_1/T^2 = 1.97\times10^5\text{ N/m}.$$

It is important to determine where in its simple harmonic motion (which "phase" of its motion) block 2 is when the impact occurs. Since $\omega = 2\pi/T$ and the given value of t (when the collision takes place) is one-fourth of T, then $\omega t = \pi/2$ and the location then of block 2 is $x = x_m\cos(\omega t + \phi)$ where $\phi = \pi/2$ which gives $x = x_m\cos(\pi/2 + \pi/2) = -x_m$. This means block 2 is at a turning point in its motion (and thus has zero speed right before the impact occurs); this means, too, that the spring is stretched an amount of 1 cm = 0.01 m at this moment. To calculate its after-collision speed (which will be the same as that of block 1 right after the impact, since they stick together in the process) we use momentum conservation and obtain v = (4.0 kg)(6.0 m/s)/(6.0 kg) = 4.0 m/s. Thus, at the end of the impact itself (while block 1 is still at the same position as before the impact) the system (consisting now of a total mass M = 6.0 kg) has kinetic energy

$$K = \tfrac{1}{2}(6.0\text{ kg})(4.0\text{ m/s})^2 = 48\text{ J}$$

and potential energy

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}(1.97\times10^5\text{ N/m})(0.010\text{ m})^2 \approx 10\text{ J},$$

meaning the total mechanical energy in the system at this stage is approximately $E = K + U$ = 58 J. When the system reaches its new turning point (at the new amplitude X) then this amount must equal its (maximum) potential energy there: $E = \tfrac{1}{2}(1.97\times10^5\text{ N/m})X^2$. Therefore, we find

$$X = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(58\text{ J})}{1.97\times10^5\text{ N/m}}} = 0.024\text{ m}.$$

35. The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency and $x_m = 0.0020$ m is the amplitude. Thus, $a_m = 8000$ m/s^2 leads to $\omega = 2000$ rad/s. Using Newton's second law with $m = 0.010$ kg, we have

$$F = ma = m\left(-a_m\cos(\omega t+\phi)\right) = -(80\text{ N})\cos\left(2000t - \frac{\pi}{3}\right)$$

where t is understood to be in seconds.

(a) Equation 15-5 gives $T = 2\pi/\omega = 3.1 \times 10^{-3}$ s.

(b) The relation $v_m = \omega x_m$ can be used to solve for v_m, or we can pursue the alternate (though related) approach of energy conservation. Here we choose the latter. By Eq. 15-12, the spring constant is $k = \omega^2 m = 40000$ N/m. Then, energy conservation leads to

$$\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 \quad \Rightarrow \quad v_m = x_m\sqrt{\frac{k}{m}} = 4.0\text{ m/s}.$$

(c) The total energy is $\frac{1}{2}kx_m^2 = \frac{1}{2}mv_m^2 = 0.080\text{ J}$.

(d) At the maximum displacement, the force acting on the particle is

$$F = kx = (4.0\times10^4\text{ N/m})(2.0\times10^{-3}\text{m}) = 80\text{ N}.$$

(e) At half of the maximum displacement, $x = 1.0$ mm, and the force is

$$F = kx = (4.0\times10^4\text{ N/m})(1.0\times10^{-3}\text{m}) = 40\text{ N}.$$

36. We note that the ratio of Eq. 15-6 and Eq. 15-3 is $v/x = -\omega\tan(\omega t + \phi)$ where ω is given by Eq. 15-12. Since the kinetic energy is $\frac{1}{2}mv^2$ and the potential energy is $\frac{1}{2}kx^2$ (which may be conveniently written as $\frac{1}{2}m\omega^2x^2$) then the ratio of kinetic to potential energy is simply

$$(v/x)^2/\omega^2 = \tan^2(\omega t + \phi),$$

which at $t = 0$ is $\tan^2\phi$. Since $\phi = \pi/6$ in this problem, then the ratio of kinetic to potential energy at $t = 0$ is $\tan^2(\pi/6) = 1/3$.

37. (a) The object oscillates about its equilibrium point, where the downward force of gravity is balanced by the upward force of the spring. If ℓ is the elongation of the spring at equilibrium, then $k\ell = mg$, where k is the spring constant and m is the mass of the object. Thus $k/m = g/\ell$ and

$$f = \omega/2\pi = (1/2\pi)\sqrt{k/m} = (1/2\pi)\sqrt{g/\ell}\,.$$

Now the equilibrium point is halfway between the points where the object is momentarily at rest. One of these points is where the spring is unstretched and the other is the lowest point, 10 cm below. Thus $\ell = 5.0\,\text{cm} = 0.050\,\text{m}$ and

$$f = \frac{1}{2\pi}\sqrt{\frac{9.8\ \text{m/s}^2}{0.050\ \text{m}}} = 2.2\ \text{Hz}.$$

(b) Use conservation of energy. We take the zero of gravitational potential energy to be at the initial position of the object, where the spring is unstretched. Then both the initial potential and kinetic energies are zero. We take the y axis to be positive in the downward direction and let y = 0.080 m. The potential energy when the object is at this point is $U = \frac{1}{2}ky^2 - mgy$. The energy equation becomes

$$0 = \tfrac{1}{2}ky^2 - mgy + \tfrac{1}{2}mv^2\,.$$

We solve for the speed:

$$v = \sqrt{2gy - \frac{k}{m}y^2} = \sqrt{2gy - \frac{g}{\ell}y^2} = \sqrt{2\left(9.8\,\text{m/s}^2\right)\left(0.080\,\text{m}\right) - \left(\frac{9.8\,\text{m/s}^2}{0.050\,\text{m}}\right)\left(0.080\,\text{m}\right)^2}$$
$$= 0.56\,\text{m/s}$$

(c) Let m be the original mass and Δm be the additional mass. The new angular frequency is $\omega' = \sqrt{k/(m+\Delta m)}$. This should be half the original angular frequency, or $\frac{1}{2}\sqrt{k/m}$. We solve $\sqrt{k/(m+\Delta m)} = \frac{1}{2}\sqrt{k/m}$ for m. Square both sides of the equation, then take the reciprocal to obtain $m + \Delta m = 4m$. This gives

$$m = \Delta m/3 = (300\ \text{g})/3 = 100\ \text{g} = 0.100\ \text{kg}.$$

(d) The equilibrium position is determined by the balancing of the gravitational and spring forces: $ky = (m + \Delta m)g$. Thus $y = (m + \Delta m)g/k$. We will need to find the value of the spring constant k. Use $k = m\omega^2 = m(2\pi f)^2$. Then

$$y\frac{(m+\Delta m)g}{m(2\pi f)^2} = \frac{(0.100\ \text{kg} + 0.300\ \text{kg})(9.80\ \text{m/s}^2)}{(0.100\ \text{kg})(2\pi \times 2.24\ \text{Hz})^2} = 0.200\ \text{m}.$$

This is measured from the initial position.

38. From Eq. 15-23 (in absolute value) we find the torsion constant:

$$\kappa = \left|\frac{\tau}{\theta}\right| = \frac{0.20\ \text{N}\cdot\text{m}}{0.85\ \text{rad}} = 0.235\ \ \text{N}\cdot\text{m/rad}\ .$$

With $I = 2mR^2/5$ (the rotational inertia for a solid sphere — from Chapter 11), Eq. 15–23 leads to

$$T = 2\pi\sqrt{\frac{\frac{2}{5}mR^2}{\kappa}} = 2\pi\sqrt{\frac{\frac{2}{5}(95\ \text{kg})(0.15\ \text{m})^2}{0.235\ \ \text{N}\cdot\text{m/rad}}} = 12\ \text{s}.$$

39. (a) We take the angular displacement of the wheel to be $\theta = \theta_m \cos(2\pi t/T)$, where θ_m is the amplitude and T is the period. We differentiate with respect to time to find the angular velocity: $\Omega = -(2\pi/T)\theta_m \sin(2\pi t/T)$. The symbol Ω is used for the angular velocity of the wheel so it is not confused with the angular frequency. The maximum angular velocity is

$$\Omega_m = \frac{2\pi\theta_m}{T} = \frac{(2\pi)(\pi\ \text{rad})}{0.500\ \text{s}} = 39.5\ \text{rad/s}.$$

(b) When $\theta = \pi/2$, then $\theta/\theta_m = 1/2$, $\cos(2\pi t/T) = 1/2$, and

$$\sin(2\pi t/T) = \sqrt{1-\cos^2(2\pi t/T)} = \sqrt{1-(1/2)^2} = \sqrt{3}/2$$

where the trigonometric identity $\cos^2\theta + \sin^2\theta = 1$ is used. Thus,

$$\Omega = -\frac{2\pi}{T}\theta_m \sin\left(\frac{2\pi t}{T}\right) = -\left(\frac{2\pi}{0.500\ \text{s}}\right)(\pi\ \text{rad})\left(\frac{\sqrt{3}}{2}\right) = -34.2\ \text{rad / s}.$$

During another portion of the cycle its angular speed is +34.2 rad/s when its angular displacement is $\pi/2$ rad.

(c) The angular acceleration is

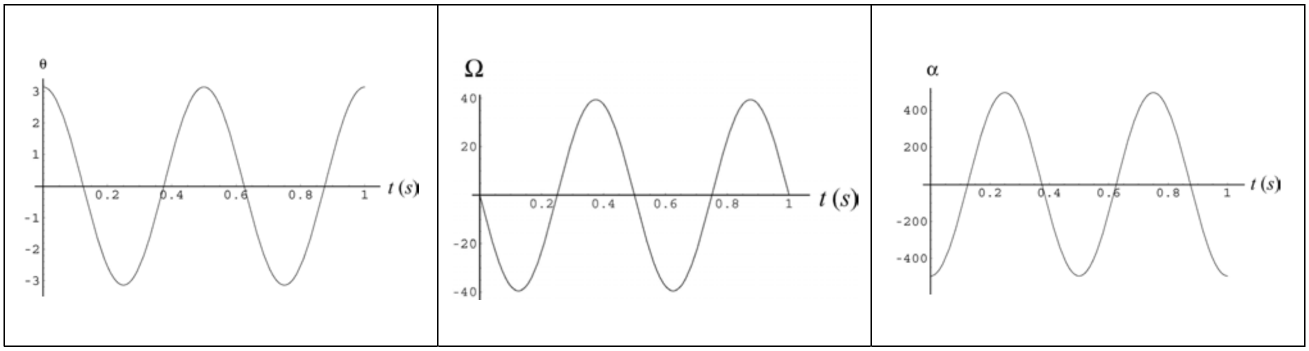
$$\alpha = \frac{d^2\theta}{dt^2} = -\left(\frac{2\pi}{T}\right)^2 \theta_m \cos(2\pi t/T) = -\left(\frac{2\pi}{T}\right)^2 \theta\ .$$

When $\theta = \pi/4$,

$$\alpha = -\left(\frac{2\pi}{0.500\ \text{s}}\right)^2\left(\frac{\pi}{4}\right) = -124\ \text{rad/s}^2,$$

or $|\alpha| = 124\ \text{rad/s}^2$.

The angular displacement, angular velocity, and angular acceleration as a function of time are plotted next.



40. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$, the unknown. For a meter stick of mass m, the rotational inertia about its center of mass is $I_{cm} = mL^2/12$ where $L = 1.0$ m. Thus, for $T = 2.5$ s, we obtain

$$T = 2\pi\sqrt{\frac{mL^2/12 + md^2}{mgd}} = 2\pi\sqrt{\frac{L^2}{12gd} + \frac{d}{g}}.$$

Squaring both sides and solving for d leads to the quadratic formula:

$$d = \frac{g(T/2\pi)^2 \pm \sqrt{d^2(T/2\pi)^4 - L^2/3}}{2}.$$

Choosing the plus sign leads to an impossible value for d ($d = 1.5 > L$). If we choose the minus sign, we obtain a physically meaningful result: $d = 0.056$ m.

41. (a) A uniform disk pivoted at its center has a rotational inertia of $\frac{1}{2}Mr^2$, where M is its mass and r is its radius. The disk of this problem rotates about a point that is displaced from its center by $r + L$, where L is the length of the rod, so, according to the parallel-axis theorem, its rotational inertia is $\frac{1}{2}Mr^2 + \frac{1}{2}M(L+r)^2$. The rod is pivoted at one end and has a rotational inertia of $mL^2/3$, where m is its mass. The total rotational inertia of the disk and rod is

$$\begin{aligned} I &= \frac{1}{2}Mr^2 + M(L+r)^2 + \frac{1}{3}mL^2 \\ &= \frac{1}{2}(0.500\text{ kg})(0.100\text{ m})^2 + (0.500\text{ kg})(0.500\text{ m} + 0.100\text{ m})^2 + \frac{1}{3}(0.270\text{ kg})(0.500\text{ m})^2 \\ &= 0.205\text{ kg}\cdot\text{m}^2. \end{aligned}$$

(b) We put the origin at the pivot. The center of mass of the disk is

$$\ell_d = L + r = 0.500\text{ m} + 0.100\text{ m} = 0.600\text{ m}$$

away and the center of mass of the rod is $\ell_r = L/2 = (0.500\text{ m})/2 = 0.250$ m away, on the same line. The distance from the pivot point to the center of mass of the disk−rod system is

$$d = \frac{M\ell_d + m\ell_r}{M+m} = \frac{(0.500\text{ kg})(0.600\text{ m}) + (0.270\text{ kg})(0.250\text{ m})}{0.500\text{ kg} + 0.270\text{ kg}} = 0.477\text{ m}.$$

(c) The period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{(M+m)gd}} = 2\pi\sqrt{\frac{0.205\text{ kg}\cdot\text{m}^2}{(0.500\text{ kg}+0.270\text{ kg})(9.80\text{ m/s}^2)(0.447\text{ m})}} = 1.50\text{ s}.$$

42. (a) Comparing the given expression to Eq. 15-3 (after changing notation $x \to \theta$), we see that $\omega = 4.43$ rad/s. Since $\omega = \sqrt{g/L}$ then we can solve for the length: $L = 0.499$ m.

(b) Since $v_m = \omega x_m = \omega L\theta_m = (4.43\text{ rad/s})(0.499\text{ m})(0.0800\text{ rad})$ and $m = 0.0600$ kg, then we can find the maximum kinetic energy: $\frac{1}{2}mv_m^{\,2} = 9.40 \times 10^{-4}$ J.

43. (a) Referring to Sample Problem – "Physical pendulum, period and length," we see that the distance between P and C is $h = \frac{2}{3}L - \frac{1}{2}L = \frac{1}{6}L$. The parallel axis theorem (see Eq. 15–30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = \left(\frac{1}{12} + \frac{1}{36}\right)mL^2 = \frac{1}{9}mL^2.$$

Equation 15-29 then gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{L^2/9}{gL/6}} = 2\pi\sqrt{\frac{2L}{3g}}$$

which yields $T = 1.64$ s for $L = 1.00$ m.

(b) We note that this T is identical to that computed in Sample Problem – "Physical pendulum, period and length." As far as the characteristics of the periodic motion are concerned, the center of oscillation provides a pivot that is equivalent to that chosen in the Sample Problem (pivot at the edge of the stick).

44. To use Eq. 15-29 we need to locate the center of mass and we need to compute the rotational inertia about A. The center of mass of the stick shown horizontal in the figure is at A, and the center of mass of the other stick is 0.50 m below A. The two sticks are of equal mass, so the center of mass of the system is $h = \frac{1}{2}(0.50\text{ m}) = 0.25\text{m}$ below A, as shown in the figure. Now, the rotational inertia of the system is the sum of the rotational inertia I_1 of the stick shown horizontal in the figure and the rotational inertia I_2 of the stick shown vertical. Thus, we have

$$I = I_1 + I_2 = \frac{1}{12}ML^2 + \frac{1}{3}ML^2 = \frac{5}{12}ML^2$$

where L = 1.00 m and M is the mass of a meter stick (which cancels in the next step). Now, with $m = 2M$ (the total mass), Eq. 15-29 yields

$$T = 2\pi\sqrt{\frac{\frac{5}{12}ML^2}{2Mgh}} = 2\pi\sqrt{\frac{5L}{6g}}$$

where $h = L/4$ was used. Thus, T = 1.83 s.

45. From Eq. 15-28, we find the length of the pendulum when the period is T = 8.85 s:

$$L = \frac{gT^2}{4\pi^2}.$$

The new length is $L' = L - d$ where d = 0.350 m. The new period is

$$T' = 2\pi\sqrt{\frac{L'}{g}} = 2\pi\sqrt{\frac{L}{g} - \frac{d}{g}} = 2\pi\sqrt{\frac{T^2}{4\pi^2} - \frac{d}{g}}$$

which yields T' = 8.77 s.

46. We require

$$T = 2\pi\sqrt{\frac{L_o}{g}} = 2\pi\sqrt{\frac{I}{mgh}}$$

similar to the approach taken in part (b) of Sample Problem – "Physical pendulum, period and length," but treating in our case a more general possibility for I. Canceling 2π, squaring both sides, and canceling g leads directly to the result; $L_o = I/mh$.

47. We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = d$. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + md^2}{mgd}} = 2\pi\sqrt{\frac{R^2 + 2d^2}{2gd}} = 2\pi\sqrt{\frac{(2.35\text{ cm})^2 + 2(1.75\text{ cm})^2}{2(980\text{ cm/s}^2)(1.75\text{ cm})}} = 0.366\text{ s}.$$

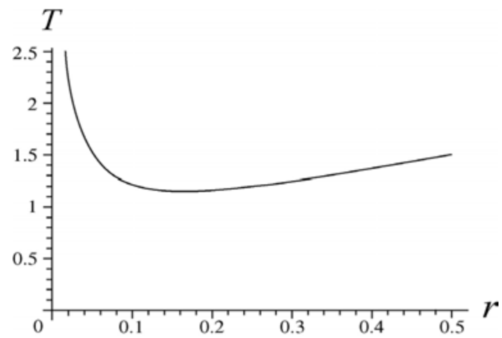
48. (a) For the "physical pendulum" we have

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{I_{com} + mh^2}{mgh}}.$$

If we substitute r for h and use item (i) in Table 10-2, we have

$$T=\frac{2\pi}{\sqrt{g}}\sqrt{\frac{a^2+b^2}{12r}+r}\ .$$

In the figure below, we plot T as a function of r, for $a = 0.35$ m and $b = 0.45$ m.



(b) The minimum of T can be located by setting its derivative to zero, $dT/dr=0$. This yields

$$r=\sqrt{\frac{a^2+b^2}{12}}=\sqrt{\frac{(0.35\text{ m})^2+(0.45\text{ m})^2}{12}}=0.16\text{ m.}$$

(c) The direction from the center does not matter, so the locus of points is a circle around the center, of radius $[(a^2 + b^2)/12]^{1/2}$.

49. Replacing x and v in Eq. 15-3 and Eq. 15-6 with θ and $d\theta/dt$, respectively, we identify 4.44 rad/s as the angular frequency ω. Then we evaluate the expressions at $t = 0$ and divide the second by the first:

$$\left(\frac{d\theta/dt}{\theta}\right)_{\text{at } t=0} = -\,\omega\tan\phi\ .$$

(a) The value of θ at $t = 0$ is 0.0400 rad, and the value of $d\theta/dt$ then is –0.200 rad/s, so we are able to solve for the phase constant:

$$\phi = \tan^{-1}[0.200/(0.0400 \times 4.44)] = 0.845\text{ rad.}$$

(b) Once ϕ is determined we can plug back in to $\theta_o = \theta_m\cos\phi$ to solve for the angular amplitude. We find $\theta_m = 0.0602$ rad.

50. (a) The rotational inertia of a uniform rod with pivot point at its end is $I = mL^2/12 + mL^2 = 1/3ML^2$. Therefore, Eq. 15-29 leads to

$$T = 2\pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg\left(L/2\right)}} \quad \Rightarrow \quad \frac{3gT^2}{8\pi^2}$$

so that L = 0.84 m.

(b) By energy conservation

$$E_{\text{bottom of swing}} = E_{\text{end of swing}} \quad \Rightarrow \quad K_m = U_m$$

where $U = Mg\ell(1-\cos\theta)$ with ℓ being the distance from the axis of rotation to the center of mass. If we use the small-angle approximation ($\cos\theta \approx 1-\frac{1}{2}\theta^2$ with θ in radians (Appendix E)), we obtain

$$U_m = \left(0.5\text{ kg}\right)\left(9.8\text{ m/s}^2\right)\left(\frac{L}{2}\right)\left(\frac{1}{2}\theta_m^2\right)$$

where θ_m = 0.17 rad. Thus, K_m = U_m = 0.031 J. If we calculate (1 – cosθ) straightforwardly (without using the small angle approximation) then we obtain within 0.3% of the same answer.

51. This is similar to the situation treated in Sample Problem — "Physical pendulum, period and length," except that O is no longer at the end of the stick. Referring to the center of mass as C (assumed to be the geometric center of the stick), we see that the distance between O and C is $h = x$. The parallel axis theorem (see Eq. 15-30) leads to

$$I = \frac{1}{12}mL^2 + mh^2 = m\left(\frac{L^2}{12} + x^2\right).$$

Equation 15-29 gives

$$T = 2\pi\sqrt{\frac{I}{mgh}} = 2\pi\sqrt{\frac{\left(\frac{L^2}{12} + x^2\right)}{gx}} = 2\pi\sqrt{\frac{\left(L^2 + 12x^2\right)}{12gx}}.$$

(a) Minimizing T by graphing (or special calculator functions) is straightforward, but the standard calculus method (setting the derivative equal to zero and solving) is somewhat awkward. We pursue the calculus method but choose to work with $12gT^2/2\pi$ instead of T (it should be clear that $12gT^2/2\pi$ is a minimum whenever T is a minimum). The result is

$$\frac{d\left(\frac{12gT^2}{2\pi}\right)}{dx} = 0 = \frac{d\left(\frac{L^2}{x} + 12x\right)}{dx} = -\frac{L^2}{x^2} + 12$$

which yields $x = L/\sqrt{12} = (1.85\text{ m})/\sqrt{12} = 0.53\text{ m}$ as the value of x that should produce the smallest possible value of T.

(b) With L = 1.85 m and x = 0.53 m, we obtain T = 2.1 s from the expression derived in part (a).

52. Consider that the length of the spring as shown in the figure (with one of the block's corners lying directly above the block's center) is some value L (its rest length). If the (constant) distance between the block's center and the point on the wall where the spring attaches is a distance r, then $r\cos\theta = d/\sqrt{2}$, and $r\cos\theta = L$ defines the angle θ measured from a line on the block drawn from the center to the top corner to the line of r (a straight line from the center of the block to the point of attachment of the spring on the wall). In terms of this angle, then, the problem asks us to consider the dynamics that results from increasing θ from its original value θ_o to $\theta_o + 3°$ and then releasing the system and letting it oscillate. If the new (stretched) length of spring is L' (when $\theta = \theta_o + 3°$), then it is a straightforward trigonometric exercise to show that

$$(L')^2 = r^2 + (d/\sqrt{2}\,)^2 - 2r(d/\sqrt{2}\,)\cos(\theta_o + 3°) = L^2 + d^2 - d^2\cos(3°) + \sqrt{2}\,Ld\sin(3°)$$

since θ_o = 45°. The difference between L' (as determined by this expression) and the original spring length L is the amount the spring has been stretched (denoted here as x_m). If one plots x_m versus L over a range that seems reasonable considering the figure shown in the problem (say, from L = 0.03 m to L = 0.10 m) one quickly sees that $x_m \approx 0.00222$ m is an excellent approximation (and is very close to what one would get by approximating x_m as the arc length of the path made by that upper block corner as the block is turned through 3°, even though this latter procedure should in principle overestimate x_m). Using this value of x_m with the given spring constant leads to a potential energy of $U = \frac{1}{2}k\,x_m^{\,2}$ = 0.00296 J. Setting this equal to the kinetic energy the block has as it passes back through the initial position, we have

$$K = 0.00296 \text{ J} = \tfrac{1}{2}\, I\, \omega_m^{\,2}$$

where ω_m is the maximum angular speed of the block (and is not to be confused with the angular frequency ω of the oscillation, though they are related by $\omega_m = \theta_o\omega$ if θ_o is expressed in radians). The rotational inertia of the block is $I = \frac{1}{6}Md^2$ = 0.0018 kg·m^2. Thus, we can solve the above relation for the maximum angular speed of the block:

$$\omega_m = \sqrt{\frac{2K}{I}} = \sqrt{\frac{2(0.00296\text{ J})}{0.0018\text{ kg}\cdot\text{m}^2}} = 1.81 \text{ rad/s}.$$

Therefore the angular frequency of the oscillation is $\omega = \omega_m/\theta_o$ = 34.6 rad/s. Using Eq. 15-5, then, the period is T = 0.18 s.

53. If the torque exerted by the spring on the rod is proportional to the angle of rotation of the rod and if the torque tends to pull the rod toward its equilibrium orientation, then the rod will oscillate in simple harmonic motion. If $\tau = -C\theta$, where τ is the torque, θ is the

angle of rotation, and C is a constant of proportionality, then the angular frequency of oscillation is $\omega = \sqrt{C/I}$ and the period is

$$T = 2\pi/\omega = 2\pi\sqrt{I/C},$$

where I is the rotational inertia of the rod. The plan is to find the torque as a function of θ and identify the constant C in terms of given quantities. This immediately gives the period in terms of given quantities. Let ℓ_0 be the distance from the pivot point to the wall. This is also the equilibrium length of the spring. Suppose the rod turns through the angle θ, with the left end moving away from the wall. This end is now $(L/2)\sin\theta$ further from the wall and has moved a distance $(L/2)(1 - \cos\theta)$ to the right. The length of the spring is now

$$\ell = \sqrt{(L/2)^2(1-\cos\theta)^2 + [\ell_0 + (L/2)\sin\theta]^2}\ .$$

If the angle θ is small we may approximate $\cos\theta$ with 1 and $\sin\theta$ with θ in radians. Then the length of the spring is given by $\ell \approx \ell_0 + L\theta/2$ and its elongation is $\Delta x = L\theta/2$. The force it exerts on the rod has magnitude $F = k\Delta x = kL\theta/2$. Since θ is small we may approximate the torque exerted by the spring on the rod by $\tau = -FL/2$, where the pivot point was taken as the origin. Thus $\tau = -(kL^2/4)\theta$. The constant of proportionality C that relates the torque and angle of rotation is $C = kL^2/4$. The rotational inertia for a rod pivoted at its center is $I = mL^2/12$, where m is its mass. See Table 10-2. Thus the period of oscillation is

$$T = 2\pi\sqrt{\frac{I}{C}} = 2\pi\sqrt{\frac{mL^2/12}{kL^2/4}} = 2\pi\sqrt{\frac{m}{3k}}.$$

With $m = 0.600$ kg and $k = 1850$ N/m, we obtain $T = 0.0653$ s.

54. We note that the initial angle is $\theta_o = 7° = 0.122$ rad (though it turns out this value will cancel in later calculations). If we approximate the initial stretch of the spring as the arc-length that the corresponding point on the plate has moved through ($x = r\,\theta_o$ where r = 0.025 m) then the initial potential energy is approximately $\frac{1}{2}kx^2 =$ 0.0093 J. This should equal to the kinetic energy of the plate ($\frac{1}{2}I\,\omega_m{}^2$ where this ω_m is the maximum angular speed of the plate, not the angular frequency ω). Noting that the maximum angular speed of the plate is $\omega_m = \omega\theta_o$ where $\omega = 2\pi/T$ with T = 20 ms = 0.02 s as determined from the graph, then we can find the rotational inertial from $\frac{1}{2}\,I\omega_m{}^2$ = 0.0093 J. Thus, $I = 1.3\times10^{-5}\ \text{kg}\cdot\text{m}^2$.

55. (a) The period of the pendulum is given by $T = 2\pi\sqrt{I/mgd}$, where I is its rotational inertia, m = 22.1 g is its mass, and d is the distance from the center of mass to the pivot point. The rotational inertia of a rod pivoted at its center is $mL^2/12$ with L = 2.20 m. According to the parallel-axis theorem, its rotational inertia when it is pivoted a distance d from the center is $I = mL^2/12 + md^2$. Thus,

$$T = 2\pi\sqrt{\frac{m(L^2/12+d^2)}{mgd}} = 2\pi\sqrt{\frac{L^2+12d^2}{12gd}}.$$

Minimizing T with respect to d, $dT/d(d) = 0$, we obtain $d = L/\sqrt{12}$. Therefore, the minimum period T is

$$T_{\min} = 2\pi\sqrt{\frac{L^2+12(L/\sqrt{12})^2}{12g(L/\sqrt{12})}} = 2\pi\sqrt{\frac{2L}{\sqrt{12}g}} = 2\pi\sqrt{\frac{2(2.20\text{ m})}{\sqrt{12}(9.80\text{ m/s}^2)}} = 2.26\text{ s}.$$

(b) If d is chosen to minimize the period, then as L is increased the period will increase as well.

(c) The period does not depend on the mass of the pendulum, so T does not change when m increases.

56. The table of moments of inertia in Chapter 11, plus the parallel axis theorem found in that chapter, leads to

$$I_P = \tfrac{1}{2}MR^2 + Mh^2 = \tfrac{1}{2}(2.5\text{ kg})(0.21\text{ m})^2 + (2.5\text{ kg})(0.97\text{ m})^2 = 2.41\text{ kg}\cdot\text{m}^2$$

where P is the hinge pin shown in the figure (the point of support for the physical pendulum), which is a distance $h = 0.21$ m + 0.76 m away from the center of the disk.

(a) Without the torsion spring connected, the period is

$$T = 2\pi\sqrt{\frac{I_P}{Mgh}} = 2.00\text{ s}.$$

(b) Now we have two "restoring torques" acting in tandem to pull the pendulum back to the vertical position when it is displaced. The magnitude of the torque-sum is $(Mgh + \kappa)\theta = I_P\,\alpha$, where the small-angle approximation ($\sin\theta \approx \theta$ in radians) and Newton's second law (for rotational dynamics) have been used. Making the appropriate adjustment to the period formula, we have

$$T' = 2\pi\sqrt{\frac{I_P}{Mgh+\kappa}}.$$

The problem statement requires $T = T' + 0.50$ s. Thus, $T' = (2.00 - 0.50)\text{s} = 1.50$ s. Consequently,

$$\kappa = \frac{4\pi^2}{T'^2}I_P - Mgh = 18.5\ \text{N}\cdot\text{m/rad}.$$

57. Since the energy is proportional to the amplitude squared (see Eq. 15-21), we find the fractional change (assumed small) is

$$\frac{E'-E}{E} \approx \frac{dE}{E} = \frac{dx_m^2}{x_m^2} = \frac{2x_m dx_m}{x_m^2} = 2\frac{dx_m}{x_m}.$$

Thus, if we approximate the fractional change in x_m as dx_m/x_m, then the above calculation shows that multiplying this by 2 should give the fractional energy change. Therefore, if x_m decreases by 3%, then E must decrease by 6.0%.

58. Referring to the numbers in Sample Problem – "Damped harmonic oscillator, time to decay, energy," we have m = 0.25 kg, b = 0.070 kg/s, and T = 0.34 s. Thus, when t = 20T, the damping factor becomes

$$e^{-bt/2m} = e^{-(0.070)(20)(0.34)/2(0.25)} = 0.39.$$

59. (a) We want to solve $e^{-bt/2m}$ = 1/3 for t. We take the natural logarithm of both sides to obtain $-bt/2m$ = ln(1/3). Therefore, $t = -(2m/b)\ln(1/3) = (2m/b)\ln 3$. Thus,

$$t = \frac{2(1.50\text{ kg})}{0.230\text{ kg/s}}\ln 3 = 14.3\text{ s}.$$

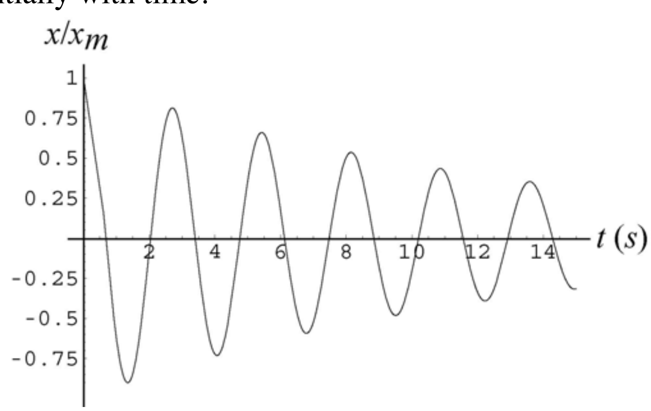
(b) The angular frequency is

$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}} = \sqrt{\frac{8.00\text{ N/m}}{1.50\text{ kg}} - \frac{(0.230\text{ kg/s})^2}{4(1.50\text{ kg})^2}} = 2.31\text{ rad/s}.$$

The period is $T = 2\pi/\omega'$ = (2π)/(2.31 rad/s) = 2.72 s and the number of oscillations is

$$t/T = (14.3\text{ s})/(2.72\text{ s}) = 5.27.$$

The displacement $x(t)$ as a function of time is shown below. The amplitude, $x_m e^{-bt/2m}$, decreases exponentially with time.



60. (a) From Hooke's law, we have

$$k = \frac{(500\text{ kg})(9.8\text{ m/s}^2)}{10\text{cm}} = 4.9\times10^2\text{ N/cm.}$$

(b) The amplitude decreasing by 50% during one period of the motion implies

$$e^{-bT/2m} = \frac{1}{2} \quad \text{where} \quad T = \frac{2\pi}{\omega'}.$$

Since the problem asks us to estimate, we let $\omega' \approx \omega = \sqrt{k/m}$. That is, we let

$$\omega' \approx \sqrt{\frac{49000\text{ N/m}}{500\text{ kg}}} \approx 9.9\text{ rad/s},$$

so that $T \approx 0.63$ s. Taking the (natural) log of both sides of the above equation, and rearranging, we find

$$b = \frac{2m}{T}\ln 2 \approx \frac{2(500\text{ kg})}{0.63\text{ s}}(0.69) = 1.1\times10^3\text{ kg/s.}$$

Note: if one worries about the $\omega' \approx \omega$ approximation, it is quite possible (though messy) to use Eq. 15-43 in its full form and solve for b. The result would be (quoting more figures than are significant)

$$b = \frac{2\ln 2\sqrt{mk}}{\sqrt{(\ln 2)^2 + 4\pi^2}} = 1086\text{ kg/s}$$

which is in good agreement with the value gotten "the easy way" above.

61. (a) We set $\omega = \omega_d$ and find that the given expression reduces to $x_m = F_m/b\omega$ at resonance.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. Thus, at resonance, we have $v_m = \omega F_m/b\omega = F_m/b$.

62. With $\omega = 2\pi/T$ then Eq. 15-28 can be used to calculate the angular frequencies for the given pendulums. For the given range of $2.00 < \omega < 4.00$ (in rad/s), we find only two of the given pendulums have appropriate values of ω: pendulum (d) with length of 0.80 m (for which ω = 3.5 rad/s) and pendulum (e) with length of 1.2 m (for which ω = 2.86 rad/s).

63. With M = 1000 kg and m = 82 kg, we adapt Eq. 15-12 to this situation by writing

$$\omega = \frac{2\pi}{T} = \sqrt{\frac{k}{M+4m}} \ .$$

If d = 4.0 m is the distance traveled (at constant car speed v) between impulses, then we may write $T = v/d$, in which case the above equation may be solved for the spring constant:

$$\frac{2\pi v}{d} = \sqrt{\frac{k}{M+4m}} \ \Rightarrow \ k = (M+4m)\left(\frac{2\pi v}{d}\right)^2 .$$

Before the people got out, the equilibrium compression is $x_i = (M + 4m)g/k$, and afterward it is $x_f = Mg/k$. Therefore, with v = 16000/3600 = 4.44 m/s, we find the rise of the car body on its suspension is

$$x_i - x_f = \frac{4mg}{k} = \frac{4mg}{M+4m}\left(\frac{d}{2\pi v}\right)^2 = 0.050 \text{ m}.$$

64. Since $\omega = 2\pi f$ where f = 2.2 Hz, we find that the angular frequency is ω = 13.8 rad/s. Thus, with x = 0.010 m, the acceleration amplitude is $a_m = x_m\,\omega^2$ = 1.91 m/s^2. We set up a ratio:

$$a_m = \left(\frac{a_m}{g}\right)g = \left(\frac{1.91}{9.8}\right)g = 0.19g.$$

65. (a) The problem gives the frequency f = 440 Hz, where the SI unit abbreviation Hz stands for Hertz, which means a cycle-per-second. The angular frequency ω is similar to frequency except that ω is in radians-per-second. Recalling that 2π radians are equivalent to a cycle, we have $\omega = 2\pi f \approx 2.8\times10^3$ rad/s.

(b) In the discussion immediately after Eq. 15-6, the book introduces the velocity amplitude $v_m = \omega x_m$. With x_m = 0.00075 m and the above value for ω, this expression yields v_m = 2.1 m/s.

(c) In the discussion immediately after Eq. 15-7, the book introduces the acceleration amplitude $a_m = \omega^2 x_m$, which (if the more precise value ω = 2765 rad/s is used) yields a_m = 5.7 km/s.

66. (a) First consider a single spring with spring constant k and unstretched length L. One end is attached to a wall and the other is attached to an object. If it is elongated by Δx the magnitude of the force it exerts on the object is $F = k\,\Delta x$. Now consider it to be two springs, with spring constants k_1 and k_2, arranged so spring 1 is attached to the object. If spring 1 is elongated by Δx_1 then the magnitude of the force exerted on the object is $F = k_1\,\Delta x_1$. This must be the same as the force of the single spring, so $k\,\Delta x = k_1\,\Delta x_1$. We must determine the relationship between Δx and Δx_1. The springs are uniform so equal unstretched lengths are elongated by the same amount and the elongation of any portion of the spring is proportional to its unstretched length. This means spring 1 is elongated by

$\Delta x_1 = CL_1$ and spring 2 is elongated by $\Delta x_2 = CL_2$, where C is a constant of proportionality. The total elongation is

$$\Delta x = \Delta x_1 + \Delta x_2 = C(L_1 + L_2) = CL_2(n + 1),$$

where $L_1 = nL_2$ was used to obtain the last form. Since $L_2 = L_1/n$, this can also be written $\Delta x = CL_1(n + 1)/n$. We substitute $\Delta x_1 = CL_1$ and $\Delta x = CL_1(n + 1)/n$ into $k\ \Delta x = k_1\ \Delta x_1$ and solve for k_1. With $k = 8600$ N/m and $n = L_1/L_2 = 0.70$, we obtain

$$k_1 = \left(\frac{n+1}{n}\right)k = \left(\frac{0.70+1.0}{0.70}\right)(8600 \text{ N/m}) = 20886 \text{ N/m} \approx 2.1\times10^4 \text{N/m}.$$

(b) Now suppose the object is placed at the other end of the composite spring, so spring 2 exerts a force on it. Now $k\ \Delta x = k_2\ \Delta x_2$. We use $\Delta x_2 = CL_2$ and $\Delta x = CL_2(n + 1)$, then solve for k_2. The result is $k_2 = k(n + 1)$.

$$k_2 = (n+1)k = (0.70+1.0)(8600 \text{ N/m}) = 14620 \text{ N/m} \approx 1.5\times10^4 \text{N/m}$$

(c) To find the frequency when spring 1 is attached to mass m, we replace k in $(1/2\pi)\sqrt{k/m}$ with $k(n + 1)/n$. With $f = (1/2\pi)\sqrt{k/m}$, we obtain, for $f = 200$ Hz and $n =$ 0.70,

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{nm}} = \sqrt{\frac{n+1}{n}}f = \sqrt{\frac{0.70+1.0}{0.70}}(200 \text{ Hz}) = 3.1\times10^2 \text{ Hz}.$$

(d) To find the frequency when spring 2 is attached to the mass, we replace k with $k(n + 1)$ to obtain

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{(n+1)k}{m}} = \sqrt{n+1}f = \sqrt{0.70+1.0}(200 \text{ Hz}) = 2.6\times10^2 \text{Hz}.$$

67. The magnitude of the downhill component of the gravitational force acting on each ore car is

$$w_x = (10000 \text{ kg})(9.8 \text{ m/s}^2)\sin\theta$$

where $\theta = 30°$ (and it is important to have the calculator in degrees mode during this problem). We are told that a downhill pull of $3\omega_x$ causes the cable to stretch $x = 0.15$ m. Since the cable is expected to obey Hooke's law, its spring constant is

$$k = \frac{3w_x}{x} = 9.8\times10^5 \text{ N/m}.$$

(a) Noting that the oscillating mass is that of *two* of the cars, we apply Eq. 15-12 (divided by 2π).

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{9.8\times10^5 \text{ N/m}}{20000 \text{ kg}}} = 1.1 \text{ Hz}.$$

(b) The difference between the equilibrium positions of the end of the cable when supporting two as opposed to three cars is

$$\Delta x = \frac{3w_x - 2w_x}{k} = 0.050 \text{ m}.$$

68. (a) Hooke's law readily yields (0.300 kg)(9.8 m/s^2)/(0.0200 m) = 147 N/m.

(b) With m = 2.00 kg, the period is

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.733 \text{ s}.$$

69. We use $v_m = \omega x_m = 2\pi f x_m$, where the frequency is 180/(60 s) = 3.0 Hz and the amplitude is half the stroke, or x_m = 0.38 m. Thus,

$$v_m = 2\pi(3.0 \text{ Hz})(0.38 \text{ m}) = 7.2 \text{ m/s}.$$

70. (a) The rotational inertia of a hoop is $I = mR^2$, and the energy of the system becomes

$$E = \frac{1}{2}I\omega^2 + \frac{1}{2}kx^2$$

and θ is in radians. We note that $r\omega = v$ (where $v = dx/dt$). Thus, the energy becomes

$$E = \frac{1}{2}\left(\frac{mR^2}{r^2}\right)v^2 + \frac{1}{2}kx^2$$

which looks like the energy of the simple harmonic oscillator discussed in Section15-4 if we identify the mass m in that section with the term mR^2/r^2 appearing in this problem. Making this identification, Eq. 15-12 yields

$$\omega = \sqrt{\frac{k}{mR^2/r^2}} = \frac{r}{R}\sqrt{\frac{k}{m}}.$$

(b) If $r = R$ the result of part (a) reduces to $\omega = \sqrt{k/m}$.

(c) And if $r = 0$ then $\omega = 0$ (the spring exerts no restoring torque on the wheel so that it is not brought back toward its equilibrium position).

71. Since $T = 0.500$ s, we note that $\omega = 2\pi/T = 4\pi$ rad/s. We work with SI units, so m = 0.0500 kg and $v_m = 0.150$ m/s.

(a) Since $\omega = \sqrt{k/m}$, the spring constant is

$$k = \omega^2 m = \left(4\pi \text{ rad/s}\right)^2 \left(0.0500 \text{ kg}\right) = 7.90 \text{ N/m}.$$

(b) We use the relation $v_m = x_m\omega$ and obtain

$$x_m = \frac{v_m}{\omega} = \frac{0.150}{4\pi} = 0.0119 \text{ m}.$$

(c) The frequency is $f = \omega/2\pi = 2.00$ Hz (which is equivalent to $f = 1/T$).

72. (a) We use Eq. 15-29 and the parallel-axis theorem $I = I_{cm} + mh^2$ where $h = R = 0.126$ m. For a solid disk of mass m, the rotational inertia about its center of mass is $I_{cm} = mR^2/2$. Therefore,

$$T = 2\pi\sqrt{\frac{mR^2/2 + mR^2}{mgR}} = 2\pi\sqrt{\frac{3R}{2g}} = 0.873\text{s}.$$

(b) We seek a value of $r \neq R$ such that

$$2\pi\sqrt{\frac{R^2 + 2r^2}{2gr}} = 2\pi\sqrt{\frac{3R}{2g}}$$

and are led to the quadratic formula:

$$r = \frac{3R \pm \sqrt{(3R)^2 - 8R^2}}{4} = R \quad \text{or} \quad \frac{R}{2}.$$

Thus, our result is $r = 0.126/2 = 0.0630$ m.

73. (a) The spring stretches until the magnitude of its upward force on the block equals the magnitude of the downward force of gravity: $ky = mg$, where $y = 0.096$ m is the elongation of the spring at equilibrium, k is the spring constant, and $m = 1.3$ kg is the mass of the block. Thus

$$k = mg/y = (1.3 \text{ kg})(9.8 \text{ m/s}^2)/(0.096 \text{ m}) = 1.33 \times 10^2 \text{ N/m}.$$

(b) The period is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1.3\text{ kg}}{133\text{ N/m}}} = 0.62\text{ s}.$$

(c) The frequency is $f = 1/T = 1/0.62$ s = 1.6 Hz.

(d) The block oscillates in simple harmonic motion about the equilibrium point determined by the forces of the spring and gravity. It is started from rest 5.0 cm below the equilibrium point so the amplitude is 5.0 cm.

(e) The block has maximum speed as it passes the equilibrium point. At the initial position, the block is not moving but it has potential energy,

$$U_i = -mgy_i + \frac{1}{2}ky_i^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.146\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.146\text{ m})^2 = -0.44\text{ J}.$$

When the block is at the equilibrium point, the elongation of the spring is y = 9.6 cm and the potential energy is

$$U_f = -mgy + \frac{1}{2}ky^2 = -(1.3\text{ kg})(9.8\text{ m/s}^2)(0.096\text{ m}) + \frac{1}{2}(133\text{ N/m})(0.096\text{ m})^2 = -0.61\text{ J}.$$

We write the equation for conservation of energy as $U_i = U_f + \frac{1}{2}mv^2$ and solve for v:

$$v = \sqrt{\frac{2(U_i - U_f)}{m}} = \sqrt{\frac{2(-0.44\text{ J} + 0.61\text{ J})}{1.3\text{kg}}} = 0.51\text{ m/s}.$$

74. The distance from the relaxed position of the bottom end of the spring to its equilibrium position when the body is attached is given by Hooke's law:

$$\Delta x = F/k = (0.20\text{ kg})(9.8\text{ m/s}^2)/(19\text{ N/m}) = 0.103\text{ m}.$$

(a) The body, once released, will not only fall through the Δx distance but continue through the equilibrium position to a "turning point" equally far on the other side. Thus, the total descent of the body is $2\Delta x = 0.21$ m.

(b) Since $f = \omega/2\pi$, Eq. 15-12 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 1.6\text{ Hz}.$$

(c) The maximum distance from the equilibrium position gives the amplitude: $x_m = \Delta x$ = 0.10 m.

75. (a) Assume the bullet becomes embedded and moves with the block before the block moves a significant distance. Then the momentum of the bullet–block system is conserved during the collision. Let m be the mass of the bullet, M be the mass of the block, v_0 be the initial speed of the bullet, and v be the final speed of the block and bullet. Conservation of momentum yields $mv_0 = (m + M)v$, so

$$v = \frac{mv_0}{m+M} = \frac{(0.050 \text{ kg})(150 \text{ m/s})}{0.050 \text{ kg} + 4.0 \text{ kg}} = 1.85 \text{ m/s}.$$

When the block is in its initial position the spring and gravitational forces balance, so the spring is elongated by Mg/k. After the collision, however, the block oscillates with simple harmonic motion about the point where the spring and gravitational forces balance with the bullet embedded. At this point the spring is elongated a distance $\ell = (M+m)g/k$, somewhat different from the initial elongation. Mechanical energy is conserved during the oscillation. At the initial position, just after the bullet is embedded, the kinetic energy is $\frac{1}{2}(M+m)v^2$ and the elastic potential energy is $\frac{1}{2}k(Mg/k)^2$. We take the gravitational potential energy to be zero at this point. When the block and bullet reach the highest point in their motion the kinetic energy is zero. The block is then a distance y_m above the position where the spring and gravitational forces balance. Note that y_m is the amplitude of the motion. The spring is compressed by $y_m - \ell$, so the elastic potential energy is $\frac{1}{2}k(y_m - \ell)^2$. The gravitational potential energy is $(M + m)gy_m$. Conservation of mechanical energy yields

$$\frac{1}{2}(M+m)v^2 + \frac{1}{2}k\left(\frac{Mg}{k}\right)^2 = \frac{1}{2}k\left(y_m - \ell\right)^2 + (M+m)gy_m.$$

We substitute $\ell = (M+m)g/k$. Algebraic manipulation leads to

$$\begin{aligned} y_m &= \sqrt{\frac{(m+M)v^2}{k} - \frac{mg^2}{k^2}(2M+m)} \\ &= \sqrt{\frac{(0.050 \text{ kg} + 4.0 \text{ kg})(1.85 \text{ m/s})^2}{500 \text{ N/m}} - \frac{(0.050 \text{ kg})\left(9.8 \text{ m/s}^2\right)^2}{(500 \text{ N/m})^2}\left[2(4.0 \text{ kg}) + 0.050 \text{ kg}\right]} \\ &= 0.166 \text{ m}. \end{aligned}$$

(b) The original energy of the bullet is $E_0 = \frac{1}{2}mv_0^2 = \frac{1}{2}(0.050 \text{ kg})(150 \text{ m/s})^2 = 563 \text{ J}$. The kinetic energy of the bullet–block system just after the collision is

$$E = \frac{1}{2}(m+M)v^2 = \frac{1}{2}(0.050 \text{ kg} + 4.0 \text{ kg})(1.85 \text{ m/s})^2 = 6.94 \text{ J}.$$

Since the block does not move significantly during the collision, the elastic and gravitational potential energies do not change. Thus, E is the energy that is transferred. The ratio is

$$E/E_0 = (6.94 \text{ J})/(563 \text{ J}) = 0.0123 \text{ or } 1.23\%.$$

76. (a) We note that

$$\omega = \sqrt{k/m} = \sqrt{1500/0.055} = 165.1 \text{ rad/s}.$$

We consider the most direct path in each part of this problem. That is, we consider in part (a) the motion directly from $x_1 = +0.800x_m$ at time t_1 to $x_2 = +0.600x_m$ at time t_2 (as opposed to, say, the block moving from $x_1 = +0.800x_m$ through $x = +0.600x_m$, through $x = 0$, reaching $x = -x_m$ and after returning back through $x = 0$ then getting to $x_2 = +0.600x_m$). Equation 15-3 leads to

$$\omega t_1 + \phi = \cos^{-1}(0.800) = 0.6435 \text{ rad}$$

$$\omega t_2 + \phi = \cos^{-1}(0.600) = 0.9272 \text{ rad} .$$

Subtracting the first of these equations from the second leads to

$$\omega(t_2 - t_1) = 0.9272 - 0.6435 = 0.2838 \text{ rad} .$$

Using the value for ω computed earlier, we find $t_2 - t_1 = 1.72 \times 10^{-3}$ s.

(b) Let t_3 be when the block reaches $x = -0.800x_m$ in the direct sense discussed above. Then the reasoning used in part (a) leads here to

$$\omega(t_3 - t_1) = (\ 2.4981 - 0.6435) \text{ rad} = 1.8546 \text{ rad}$$

and thus to $t_3 - t_1 = 11.2 \times 10^{-3}$ s.

77. (a) From the graph, we find $x_m = 7.0$ cm = 0.070 m, and $T = 40$ ms = 0.040 s. Thus, the angular frequency is $\omega = 2\pi/T = 157$ rad/s. Using $m = 0.020$ kg, the maximum kinetic energy is then $\frac{1}{2}mv^2 = \frac{1}{2}m\,\omega^2 x_m^{\ 2} = 1.2$ J.

(b) Using Eq. 15-5, we have $f = \omega/2\pi = 50$ oscillations per second. Of course, Eq. 15-2 can also be used for this.

78. (a) From the graph we see that $x_m = 7.0$ cm = 0.070 m and $T = 40$ ms = 0.040 s. The maximum speed is $x_m\omega = x_m 2\pi/T = 11$ m/s.

(b) The maximum acceleration is $x_m\omega^2 = x_m(2\pi/T)^2 = 1.7 \times 10^3$ m/s^2.

79. Setting 15 mJ (0.015 J) equal to the maximum kinetic energy leads to $v_{\max} = 0.387$ m/s. Then one can use either an "exact" approach using $v_{\max} = \sqrt{2gL(1-\cos\theta_{\max})}$ or the "SHM" approach where

$$v_{max} = L\omega_{max} = L\omega\theta_{max} = L\sqrt{g/L}\ \theta_{max}$$

to find L. Both approaches lead to L = 1.53 m.

80. Its total mechanical energy is equal to its maximum potential energy $\frac{1}{2}kx_m{}^2$, and its potential energy at t = 0 is $\frac{1}{2}kx_o{}^2$ where $x_o = x_m\cos(\pi/5)$ in this problem. The ratio is therefore $\cos^2(\pi/5) = 0.655 = 65.5\%$.

81. (a) From the graph, it is clear that x_m = 0.30 m.

(b) With $F = -kx$, we see k is the (negative) slope of the graph — which is 75/0.30 = 250 N/m. Plugging this into Eq. 15-13 yields

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.28 \text{ s.}$$

(c) As discussed in Section 15-2, the maximum acceleration is

$$a_m = \omega^2 x_m = \frac{k}{m} x_m = 1.5\times10^2 \text{ m/s}^2.$$

Alternatively, we could arrive at this result using $a_m = (2\pi/T)^2\, x_m$.

(d) Also in Section 15-2 is $v_m = \omega x_m$ so that the maximum kinetic energy is

$$K_m = \frac{1}{2}mv_m^2 = \frac{1}{2}m\omega^2 x_m^2 = \frac{1}{2}kx_m^2$$

which yields 11.3 ≈ 11 J. We note that the above manipulation reproduces the notion of energy conservation for this system (maximum kinetic energy being equal to the maximum potential energy).

82. Since the centripetal acceleration is horizontal and Earth's gravitational $\vec{g}$ is downward, we can define the magnitude of an "effective" gravitational acceleration using the Pythagorean theorem:

$$g_{eff} = \sqrt{g^2 + (v^2/R)^2}.$$

Then, since frequency is the reciprocal of the period, Eq. 15-28 leads to

$$f = \frac{1}{2\pi}\sqrt{\frac{g_{eff}}{L}} = \frac{1}{2\pi}\sqrt{\frac{\sqrt{g^2 + v^4/R^2}}{L}}.$$

With v = 70 m/s, R = 50 m, and L = 0.20 m, we have $f \approx 3.5\ \text{s}^{-1} = 3.5$ Hz.

83. (a) Hooke's law readily yields

$$k = (15\ \text{kg})(9.8\ \text{m/s}^2)/(0.12\ \text{m}) = 1225\ \text{N/m}.$$

Rounding to three significant figures, the spring constant is therefore 1.23 kN/m.

(b) We are told f = 2.00 Hz = 2.00 cycles/sec. Since a cycle is equivalent to 2π radians, we have ω = 2π(2.00) = 4π rad/s (understood to be valid to three significant figures). Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow m = \frac{1225\ \text{N/m}}{(4\pi\ \text{rad/s})^2} = 7.76\ \text{kg}.$$

Consequently, the weight of the package is mg = 76.0 N.

84. (a) Comparing with Eq. 15-3, we see ω = 10 rad/s in this problem. Thus, $f = \omega/2\pi$ = 1.6 Hz.

(b) Since $v_m = \omega x_m$ and x_m = 10 cm (see Eq. 15-3), then v_m = (10 rad/s)(10 cm) = 100 cm/s or 1.0 m/s.

(c) The maximum occurs at t = 0.

(d) Since $a_m = \omega^2 x_m$, then $v_m = (10\ \text{rad/s})^2(10\ \text{cm}) = 1000\ \text{cm/s}^2$ or 10 m/s^2.

(e) The acceleration extremes occur at the displacement extremes: $x = \pm x_m$ or x = ±10 cm.

(f) Using Eq. 15-12, we find

$$\omega = \sqrt{\frac{k}{m}} \Rightarrow k = (0.10\ \text{kg})(10\ \text{rad/s})^2 = 10\ \text{N/m}.$$

Thus, Hooke's law gives $F = -kx = -10x$ in SI units.

85. Using Δm = 2.0 kg, T_1 = 2.0 s and T_2 = 3.0 s, we write

$$T_1 = 2\pi\sqrt{\frac{m}{k}} \quad \text{and} \quad T_2 = 2\pi\sqrt{\frac{m+\Delta m}{k}}.$$

Dividing one relation by the other, we obtain

$$\frac{T_2}{T_1} = \sqrt{\frac{m + \Delta m}{m}}$$

which (after squaring both sides) simplifies to $m = \dfrac{\Delta m}{(T_2/T_1)^2 - 1} = 1.6\text{kg}.$

86. (a) The textbook notes (in the discussion immediately after Eq. 15-7) that the acceleration amplitude is $a_m = \omega^2 x_m$, where ω is the angular frequency ($\omega = 2\pi f$ since there are 2π radians in one cycle). Therefore, in this circumstance, we obtain

$$a_m = \left(2\pi(1000\text{ Hz})\right)^2 (0.00040\text{ m}) = 1.6 \times 10^4\text{ m/s}^2.$$

(b) Similarly, in the discussion after Eq. 15-6, we find $v_m = \omega x_m$ so that

$$v_m = \left(2\pi(1000\text{ Hz})\right)(0.00040\text{ m}) = 2.5\text{ m/s}.$$

(c) From Eq. 15-8, we have (in absolute value)

$$|a| = \left(2\pi(1000\text{ Hz})\right)^2 (0.00020\text{ m}) = 7.9 \times 10^3\text{ m/s}^2.$$

(d) This can be approached with the energy methods of Section 15-4, but here we will use trigonometric relations along with Eq. 15-3 and Eq. 15-6. Thus, allowing for both roots stemming from the square root,

$$\sin(\omega t + \phi) = \pm\sqrt{1 - \cos^2(\omega t + \phi)} \quad \Rightarrow \quad -\frac{v}{\omega x_m} = \pm\sqrt{1 - \frac{x^2}{x_m^2}}.$$

Taking absolute values and simplifying, we obtain

$$|v| = 2\pi f\sqrt{x_m^2 - x^2} = 2\pi(1000)\sqrt{0.00040^2 - 0.00020^2} = 2.2\text{ m/s}.$$

87. (a) The rotational inertia is $I = \frac{1}{2}MR^2 = \frac{1}{2}(3.00\text{ kg})(0.700\text{ m})^2 = 0.735\text{ kg}\cdot\text{m}^2$.

(b) Using Eq. 15-22 (in absolute value), we find

$$\kappa = \frac{\tau}{\theta} = \frac{0.0600\text{ N}\cdot\text{m}}{2.5\text{ rad}} = 0.0240\text{ N}\cdot\text{m/rad}.$$

(c) Using Eq. 15-5, Eq. 15-23 leads to

$$\omega = \sqrt{\frac{\kappa}{I}} = \sqrt{\frac{0.024\text{N}\cdot\text{m/rad}}{0.735\text{kg}\cdot\text{m}^2}} = 0.181\text{ rad/s}.$$

88. (a) The Hooke's law force (of magnitude (100)(0.30) = 30 N) is directed upward and the weight (20 N) is downward. Thus, the net force is 10 N upward.

(b) The equilibrium position is where the upward Hooke's law force balances the weight, which corresponds to the spring being stretched (from unstretched length) by 20 N/100 N/m = 0.20 m. Thus, relative to the equilibrium position, the block (at the instant described in part (a)) is at what one might call *the bottom turning point* (since $v = 0$) at $x = -x_m$ where the amplitude is $x_m = 0.30 - 0.20 = 0.10$ m.

(c) Using Eq. 15-13 with $m = W/g \approx 2.0$ kg, we have

$$T = 2\pi\sqrt{\frac{m}{k}} = 0.90 \text{ s}.$$

(d) The maximum kinetic energy is equal to the maximum potential energy $\frac{1}{2}kx_m^2$. Thus,

$$K_m = U_m = \frac{1}{2}(100 \text{ N/m})(0.10 \text{ m})^2 = 0.50 \text{ J}.$$

89. (a) We require $U = \frac{1}{2}E$ at some value of x. Using Eq. 15-21, this becomes

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kx_m^2\right) \Rightarrow x = \frac{x_m}{\sqrt{2}}.$$

We compare the given expression x as a function of t with Eq. 15-3 and find $x_m = 5.0$ m. Thus, the value of x we seek is $x = 5.0/\sqrt{2} \approx 3.5 \text{ m}$.

(b) We solve the given expression (with $x = 5.0/\sqrt{2}$), making sure our calculator is in radians mode:

$$t = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = 1.54 \text{ s}.$$

Since we are asked for the interval $t_{eq} - t$ where t_{eq} specifies the instant the particle passes through the equilibrium position, then we set $x = 0$ and find

$$t_{eq} = \frac{\pi}{4} + \frac{3}{\pi}\cos^{-1}(0) = 2.29 \text{ s}.$$

Consequently, the time interval is $t_{eq} - t = 0.75$ s.

90. Since the particle has zero speed (momentarily) at $x \neq 0$, then it must be at its turning point; thus, $x_o = x_m = 0.37$ cm. It is straightforward to infer from this that the phase

constant ϕ in Eq. 15-2 is zero. Also, $f = 0.25$ Hz is given, so we have $\omega = 2\pi f = \pi/2$ rad/s. The variable t is understood to take values in seconds.

(a) The period is $T = 1/f = 4.0$ s.

(b) As noted above, $\omega = \pi/2$ rad/s.

(c) The amplitude, as observed above, is 0.37 cm.

(d) Equation 15-3 becomes $x = (0.37 \text{ cm}) \cos(\pi t/2)$.

(e) The derivative of x is $v = -(0.37 \text{ cm/s})(\pi/2) \sin(\pi t/2) \approx (-0.58 \text{ cm/s}) \sin(\pi t/2)$.

(f) From the previous part, we conclude $v_m = 0.58$ cm/s.

(g) The acceleration-amplitude is $a_m = \omega^2 x_m = 0.91 \text{ cm/s}^2$.

(h) Making sure our calculator is in radians mode, we find $x = (0.37) \cos(\pi(3.0)/2) = 0$. It is important to avoid rounding off the value of π in order to get precisely zero, here.

(i) With our calculator still in radians mode, we obtain $v = -(0.58 \text{ cm/s})\sin(\pi(3.0)/2) = 0.58$ cm/s.

91. (a) The frequency for small-amplitude oscillations is $f = (1/2\pi)\sqrt{g/L}$, where L is the length of the pendulum. This gives

$$f = (1/2\pi)\sqrt{(9.80 \text{ m/s}^2)/(2.0 \text{ m})} = 0.35 \text{ Hz}.$$

(b) The forces acting on the pendulum are the tension force $\vec{T}$ of the rod and the force of gravity $m\vec{g}$. Newton's second law yields $\vec{T} + m\vec{g} = m\vec{a}$, where m is the mass and $\vec{a}$ is the acceleration of the pendulum. Let $\vec{a} = \vec{a}_e + \vec{a}'$, where $\vec{a}_e$ is the acceleration of the elevator and $\vec{a}'$ is the acceleration of the pendulum relative to the elevator. Newton's second law can then be written $m(\vec{g} - \vec{a}_e) + \vec{T} = m\vec{a}'$. Relative to the elevator the motion is exactly the same as it would be in an inertial frame where the acceleration due to gravity is $\vec{g} - \vec{a}_e$. Since $\vec{g}$ and $\vec{a}_e$ are along the same line and in opposite directions, we can find the frequency for small-amplitude oscillations by replacing g with $g + a_e$ in the expression $f = (1/2\pi)\sqrt{g/L}$. Thus

$$f = \frac{1}{2\pi}\sqrt{\frac{g + a_e}{L}} = \frac{1}{2\pi}\sqrt{\frac{9.8 \text{ m/s}^2 + 2.0 \text{ m/s}^2}{2.0 \text{ m}}} = 0.39 \text{ Hz}.$$

(c) Now the acceleration due to gravity and the acceleration of the elevator are in the same direction and have the same magnitude. That is, $\vec{g} - \vec{a}_e = 0$. To find the frequency for small-amplitude oscillations, replace g with zero in $f = (1/2\pi)\sqrt{g/L}$. The result is zero. The pendulum does not oscillate.

92. The period formula, Eq. 15-29, requires knowing the distance h from the axis of rotation and the center of mass of the system. We also need the rotational inertia I about the axis of rotation. From the figure, we see $h = L + R$ where $R = 0.15$ m. Using the parallel-axis theorem, we find

$$I = \frac{1}{2}MR^2 + M\left(L+R\right)^2 ,$$

where $M = 1.0$ kg. Thus, Eq. 15-29, with $T = 2.0$ s, leads to

$$2.0 = 2\pi\sqrt{\frac{\frac{1}{2}MR^2 + M\left(L+R\right)^2}{Mg\left(L+R\right)}}$$

which leads to $L = 0.8315$ m.

93. (a) Hooke's law provides the spring constant:

$$k = (4.00 \text{ kg})(9.8 \text{ m/s}^2)/(0.160 \text{ m}) = 245 \text{ N/m}.$$

(b) The attached mass is $m = 0.500$ kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.500}{245}} = 0.284 \text{ s}.$$

94. We note (from the graph) that $a_m = \omega^2 x_m = 4.00$ cm/s^2. Also, the value at $t = 0$ is $a_o = 1.00$ cm/s^2. Then Eq. 15-7 leads to

$$\phi = \cos^{-1}(-1.00/4.00) = +1.82 \text{ rad or } -4.46 \text{ rad}.$$

The other "root" (+4.46 rad) can be rejected on the grounds that it would lead to a negative slope at $t = 0$.

95. The time for one cycle is $T = (50 \text{ s})/20 = 2.5$ s. Thus, from Eq. 15-23, we find

$$I = \kappa\left(\frac{T}{2\pi}\right)^2 = (0.50)\left(\frac{2.5}{2\pi}\right)^2 = 0.079 \text{ kg}\cdot\text{m}^2.$$

96. The angular frequency of the simple harmonic oscillation is given by Eq. 15-13:

$$\omega = \sqrt{\frac{k}{m}}\ .$$

Thus, for two different masses m_1 and m_2, with the same spring constant k, the ratio of the frequencies would be

$$\frac{\omega_1}{\omega_2} = \frac{\sqrt{k/m_1}}{\sqrt{k/m_2}} = \sqrt{\frac{m_2}{m_1}}\ .$$

In our case, with $m_1 = m$ and $m_2 = 2.5m$, the ratio is $\frac{\omega_1}{\omega_2} = \sqrt{\frac{m_2}{m_1}} = \sqrt{2.5} = 1.58$.

97. (a) The graphs suggest that T = 0.40 s and κ = 4/0.2 = 0.02 N·m/rad. With these values, Eq. 15-23 can be used to determine the rotational inertia:

$$I = \kappa T^2/4\pi^2 = 8.11 \times 10^{-5}\ \text{kg}\cdot\text{m}^2.$$

(b) We note (from the graph) that θ_{max} = 0.20 rad. Setting the maximum kinetic energy ($\frac{1}{2}I\omega_{max}{}^2$) equal to the maximum potential energy (see the hint in the problem) leads to $\omega_{max} = \theta_{max}\sqrt{\kappa/I}$ = 3.14 rad/s.

98. (a) Hooke's law provides the spring constant: k = (20 N)/(0.20 m) = 1.0×10^2 N/m.

(b) The attached mass is m = (5.0 N)/(9.8 m/s^2) = 0.51 kg. Consequently, Eq. 15-13 leads to

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.51\ \text{kg}}{100\ \text{N/m}}} = 0.45\ \text{s}.$$

99. For simple harmonic motion, Eq. 15-24 must reduce to

$$\tau = -L\left(F_g \sin\theta\right) \to -L\left(F_g\theta\right)$$

where θ is in radians. We take the percent difference (in absolute value)

$$\left|\frac{\left(-LF_g\sin\theta\right) - \left(-LF_g\theta\right)}{-LF_g\sin\theta}\right| = \left|1 - \frac{\theta}{\sin\theta}\right|$$

and set this equal to 0.010 (corresponding to 1.0%). In order to solve for θ (since this is not possible "in closed form"), several approaches are available. Some calculators have built-in numerical routines to facilitate this, and most math software packages have this capability. Alternatively, we could expand $\sin\theta \approx \theta - \theta^3/6$ (valid for small θ) and thereby find an approximate solution (which, in turn, might provide a seed value for a numerical search). Here we show the latter approach:

$$\left|1-\frac{\theta}{\theta-\theta^3/6}\right| \approx 0.010 \Rightarrow \frac{1}{1-\theta^2/6} \approx 1.010$$

which leads to $\theta \approx \sqrt{6(0.01/1.01)} = 0.24 \text{ rad} = 14.0°$. A more accurate value (found numerically) for θ that results in a 1.0% deviation is 13.986°.

100. (a) The potential energy at the turning point is equal (in the absence of friction) to the total kinetic energy (translational plus rotational) as it passes through the equilibrium position:

$$\frac{1}{2}kx_m^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}I_{\text{cm}}^2\omega^2 = \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{2}\left(\frac{1}{2}MR^2\right)\left(\frac{v_{\text{cm}}}{R}\right)^2$$
$$= \frac{1}{2}Mv_{\text{cm}}^2 + \frac{1}{4}Mv_{\text{cm}}^2 = \frac{3}{4}Mv_{\text{cm}}^2$$

which leads to $Mv_{\text{cm}}^2 = 2kx_m^2/3$ = 0.125 J. The translational kinetic energy is therefore $\frac{1}{2}Mv_{\text{cm}}^2 = kx_m^2/3 = 0.0625\text{ J}$.

(b) And the rotational kinetic energy is $\frac{1}{4}Mv_{\text{cm}}^2 = kx_m^2/6 = 0.03125\text{J} \approx 3.13\times10^{-2}\text{ J}$.

(c) In this part, we use v_{cm} to denote the speed at any instant (and not just the maximum speed as we had done in the previous parts). Since the energy is constant, then

$$\frac{dE}{dt} = \frac{d}{dt}\left(\frac{3}{4}Mv_{\text{cm}}^2\right) + \frac{d}{dt}\left(\frac{1}{2}kx^2\right) = \frac{3}{2}Mv_{\text{cm}}a_{\text{cm}} + kxv_{\text{cm}} = 0$$

which leads to

$$a_{\text{cm}} = -\left(\frac{2k}{3M}\right)x.$$

Comparing with Eq. 15-8, we see that $\omega = \sqrt{2k/3M}$ for this system. Since $\omega = 2\pi/T$, we obtain the desired result: $T = 2\pi\sqrt{3M/2k}$.

101. We note that for a horizontal spring, the relaxed position is the equilibrium position (in a regular simple harmonic motion setting); thus, we infer that the given v = 5.2 m/s at $x = 0$ is the maximum value v_m (which equals ωx_m where $\omega = \sqrt{k/m} = 20\text{ rad/s}$).

(a) Since $\omega = 2\pi f$, we find f = 3.2 Hz.

(b) We have v_m = 5.2 m/s = (20 rad/s)x_m, which leads to x_m = 0.26 m.

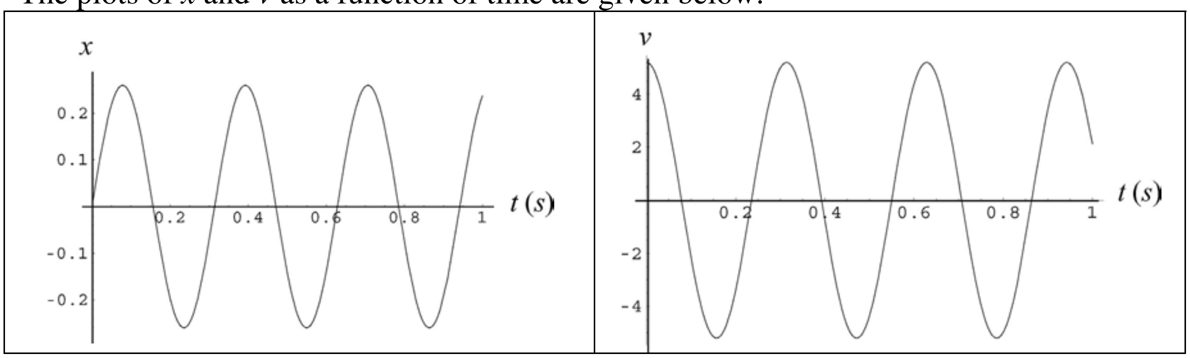
(c) With meters, seconds, and radians understood,

$$x = (0.26\text{ m})\cos(20t + \phi)$$
$$v = -(5.2\text{ m/s})\sin(20t + \phi).$$

The requirement that $x = 0$ at $t = 0$ implies (from the first equation above) that either $\phi = +\pi/2$ or $\phi = -\pi/2$. Only one of these choices meets the further requirement that $v > 0$ when $t = 0$; that choice is $\phi = -\pi/2$. Therefore,

$$x = (0.26\text{ m})\cos\left(20t - \frac{\pi}{2}\right) = (0.26\text{ m})\sin\left(20t\right).$$

The plots of x and v as a function of time are given below:



102. (a) Equation 15-21 leads to

$$E = \frac{1}{2}kx_m^2 \Rightarrow x_m = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(4.0\text{ J})}{200\text{ N/m}}} = 0.20\text{ m}.$$

(b) Since $T = 2\pi\sqrt{m/k} = 2\pi\sqrt{0.80\text{ kg}/200\text{ N/m}} \approx 0.4\text{ s}$, then the block completes 10/0.4 = 25 cycles during the specified interval.

(c) The maximum kinetic energy is the total energy, 4.0 J.

(d) This can be approached more than one way; we choose to use energy conservation:

$$E = K + U \Rightarrow 4.0 = \frac{1}{2}mv^2 + \frac{1}{2}kx^2.$$

Therefore, when $x = 0.15$ m, we find $v = 2.1$ m/s.

103. (a) By Eq. 15-13, the mass of the block is

$$m_b = \frac{kT_0^{\,2}}{4\pi^2} = 2.43 \text{ kg}.$$

Therefore, with $m_p = 0.50$ kg, the new period is

$$T = 2\pi\sqrt{\frac{m_p + m_b}{k}} = 0.44 \text{ s}.$$

(b) The speed before the collision (since it is at its maximum, passing through equilibrium) is $v_0 = x_m\omega_0$ where $\omega_0 = 2\pi/T_0$; thus, $v_0 = 3.14$ m/s. Using momentum conservation (along the horizontal direction) we find the speed after the collision:

$$V = v_0 \frac{m_b}{m_p + m_b} = 2.61 \text{ m/s}.$$

The equilibrium position has not changed, so (for the new system of greater mass) this represents the maximum speed value for the subsequent harmonic motion: $V = x'_m\omega$ where $\omega = 2\pi/T = 14.3$ rad/s. Therefore, $x'_m = 0.18$ m.

104. (a) We are told that when $t = 4T$, with $T = 2\pi/\omega' \approx 2\pi\sqrt{m/k}$ (neglecting the second term in Eq. 15-43),

$$e^{-bt/2m} = \frac{3}{4}.$$

Thus,

$$T \approx 2\pi\sqrt{(2.00\text{kg})/(10.0\text{ N/m})} = 2.81\text{ s}$$

and we find

$$\frac{b(4T)}{2m} = \ln\left(\frac{4}{3}\right) = 0.288 \quad \Rightarrow \quad b = \frac{2(2.00\text{ kg})(0.288)}{4(2.81\text{ s})} = 0.102 \text{ kg/s}.$$

(b) Initially, the energy is $E_o = \frac{1}{2}kx_{mo}^2 = \frac{1}{2}(10.0)(0.250)^2 = 0.313\text{ J}$. At $t = 4T$,

$$E = \tfrac{1}{2}k(\tfrac{3}{4}x_{mo})^2 = 0.176\text{ J}.$$

Therefore, $E_o - E = 0.137$ J.

105. (a) From Eq. 16-12, $T = 2\pi\sqrt{m/k} = 0.45\text{ s}$.

(b) For a vertical spring, the distance between the unstretched length and the equilibrium length (with a mass m attached) is mg/k, where in this problem $mg = 10$ N and $k = 200$ N/m (so that the distance is 0.05 m). During simple harmonic motion, the convention is to establish $x = 0$ at the equilibrium length (the middle level for the oscillation) and to write the total energy without any gravity term; that is, $E = K + U$, where $U = kx^2/2$. Thus, as the block passes through the unstretched position, the energy is $E = 2.0 + \frac{1}{2}k(0.05)^2 = 2.25\text{ J}$. At its topmost and bottommost points of oscillation, the energy (using this convention) is all elastic potential: $\frac{1}{2}kx_m^2$. Therefore, by energy conservation,

$$2.25 = \frac{1}{2}kx_m^2 \Rightarrow x_m = \pm 0.15\text{ m}.$$

This gives the amplitude of oscillation as 0.15 m, but how far are these points from the *unstretched* position? We add (or subtract) the 0.05 m value found above and obtain 0.10 m for the top-most position and 0.20 m for the bottom-most position.

(c) As noted in part (b), $x_m = \pm 0.15$ m.

(d) The maximum kinetic energy equals the maximum potential energy (found in part (b)) and is equal to 2.25 J.

106. (a) The graph makes it clear that the period is $T = 0.20$ s.

(b) The period of the simple harmonic oscillator is given by Eq. 15-13: $T = 2\pi\sqrt{\frac{m}{k}}$.

Thus, using the result from part (a) with $k = 200$ N/m, we obtain $m = 0.203 \approx 0.20$ kg.

(c) The graph indicates that the speed is (momentarily) zero at $t = 0$, which implies that the block is at $x_0 = \pm x_m$. From the graph we also note that the slope of the velocity curve (hence, the acceleration) is positive at $t = 0$, which implies (from $ma = -kx$) that the value of x is negative. Therefore, with $x_m = 0.20$ m, we obtain $x_0 = -0.20$ m.

(d) We note from the graph that $v = 0$ at $t = 0.10$ s, which implied $a = \pm a_m = \pm\omega^2 x_m$. Since acceleration is the instantaneous slope of the velocity graph, then (looking again at the graph) we choose the negative sign. Recalling $\omega^2 = k/m$ we obtain $a = -197 \approx -2.0 \times 10^2$ m/s^2.

(e) The graph shows $v_m = 6.28$ m/s, so $K_m = \frac{1}{2}mv_m^2 = 4.0$ J.

107. The mass is $m = \frac{0.108\text{ kg}}{6.02 \times 10^{23}} = 1.8 \times 10^{-25}$ kg. Using Eq. 15-12 and the fact that $f = \omega/2\pi$, we have

$$1\times 10^{13}\text{ Hz} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \Rightarrow k = \left(2\pi \times 10^{13}\right)^2\left(1.8\times 10^{-25}\right) \approx 7\times 10^2\text{ N/m}.$$