

# Circular Motion and Gravitation

## Circular Motion and Gravitation, Practice A

*Givens*

*Solutions*

1.  $a_c = 3.0 \text{ m/s}^2$   
 $r = 2.1 \text{ m}$

$$v_t = \sqrt{a_c r} = \sqrt{(3.0 \text{ m/s}^2)(2.1 \text{ m})} = \boxed{2.5 \text{ m/s}}$$

2.  $a_c = 250 \text{ m/s}^2$   
 $r = 0.50 \text{ m}$

$$v_t = \sqrt{a_c r} = \sqrt{(250 \text{ m/s}^2)(0.50 \text{ m})} = \boxed{11 \text{ m/s}}$$

3.  $r = 1.5 \text{ m}$   
 $v_t = 1.5 \text{ m/s}$

$$a_c = \frac{v_t^2}{r} = \frac{(1.5 \text{ m/s})^2}{1.5 \text{ m}} = \boxed{1.5 \text{ m/s}^2}$$

4.  $a_c = 15.4 \text{ m/s}^2$   
 $v_t = 30.0 \text{ m/s}$

$$r = \frac{v_t^2}{a_c} = \frac{(30.0 \text{ m/s})^2}{15.4 \text{ m/s}^2} = \boxed{58.4 \text{ m}}$$

## Circular Motion and Gravitation, Practice B

1.  $r = 2.10 \text{ m}$   
 $v_t = 2.50 \text{ m/s}$   
 $F_c = 88.0 \text{ N}$

$$m = F_c \frac{r}{v_t^2} = (88.0 \text{ N}) \frac{(2.10 \text{ m})}{(2.50 \text{ m/s})^2} = \boxed{29.6 \text{ kg}}$$

2.  $v_t = 13.2 \text{ m/s}$   
 $F_c = 377 \text{ N}$   
 $m = 86.5 \text{ kg}$

$$r = \frac{mv_t^2}{F_c} = \frac{(86.5 \text{ kg})(13.2 \text{ m/s})^2}{377 \text{ N}} = \boxed{40.0 \text{ m}}$$

3.  $r = 1.50 \text{ m}$   
 $v_t = 1.80 \text{ m/s}$   
 $m = 18.5 \text{ kg}$

$$F_c = \frac{mv_t^2}{r} = \frac{(18.5 \text{ kg})(1.80 \text{ m/s})^2}{1.50 \text{ m}} = \boxed{40.0 \text{ N}}$$

4.  $m = 905 \text{ kg}$   
 $r = \frac{3.25 \text{ km}}{2\pi}$   
 $F_c = 2140 \text{ N}$

$$v_t = \sqrt{\frac{r F_c}{m}} = \sqrt{\left(\frac{3.25 \times 10^3 \text{ m}}{2\pi}\right)\left(\frac{2140 \text{ N}}{905 \text{ kg}}\right)} = \boxed{35.0 \text{ m/s}}$$

## Circular Motion and Gravitation, Section 1 Review

Givens

Solutions

2.  $r = 12 \text{ m}$   
 $a_c = 17 \text{ m/s}^2$

$$v_t = \sqrt{a_c r} = \sqrt{(17 \text{ m/s}^2)(12 \text{ m})} = \boxed{14 \text{ m/s}}$$

5.  $m = 90.0 \text{ kg}$   
 $r = 11.5 \text{ m}$   
 $v_t = 13.2 \text{ m/s}$

$$F_c = \frac{mv_t^2}{r} = \frac{(90.0 \text{ kg})(13.2 \text{ m/s})^2}{11.5 \text{ m}} = \boxed{1360 \text{ N}}$$

## Circular Motion and Gravitation, Practice C

1.  $m_1 = m_2 = 0.800 \text{ kg}$   
 $F_g = 8.92 \times 10^{-11} \text{ N}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{G m_1 m_2}{F_g}} = \sqrt{\frac{(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(0.800 \text{ kg})(0.800 \text{ kg})}{8.92 \times 10^{-11} \text{ N}}}$$

$$r = \boxed{0.692 \text{ m}}$$

2.  $m_1 = 6.4 \times 10^{23} \text{ kg}$   
 $m_2 = 9.6 \times 10^{15} \text{ kg}$   
 $F_g = 4.6 \times 10^{15} \text{ N}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$r = \sqrt{\frac{G m_1 m_2}{F_g}} = \sqrt{\frac{(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(6.4 \times 10^{23} \text{ kg})(9.6 \times 10^{15} \text{ kg})}{4.6 \times 10^{15} \text{ N}}}$$

$$r = \boxed{9.4 \times 10^6 \text{ m} = 9.4 \times 10^3 \text{ km}}$$

3.  $m_1 = 66.5 \text{ kg}$   
 $m_2 = 5.97 \times 10^{24} \text{ kg}$   
 $r = 6.38 \times 10^6 \text{ m}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

a.  $F_g = G \frac{m_1 m_2}{r^2} = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(66.5 \text{ kg})(5.97 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2} = \boxed{651 \text{ N}}$

$m_2 = 6.42 \times 10^{23} \text{ kg}$   
 $r = 3.40 \times 10^6 \text{ m}$

b.  $F_g = G \frac{m_1 m_2}{r^2} = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(66.5 \text{ kg})(6.42 \times 10^{23} \text{ kg})}{(3.40 \times 10^6 \text{ m})^2} = \boxed{246 \text{ N}}$

$m_2 = 1.25 \times 10^{22} \text{ kg}$   
 $r = 1.20 \times 10^6 \text{ m}$

c.  $F_g = G \frac{m_1 m_2}{r^2} = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(66.5 \text{ kg})(1.25 \times 10^{22} \text{ kg})}{(1.20 \times 10^6 \text{ m})^2} = \boxed{38.5 \text{ N}}$

## Circular Motion and Gravitation, Section 2 Review

### Givens

3.  $m_E = 5.97 \times 10^{24}$  kg  
 $r_E = 6.38 \times 10^6$  m  
 $m = 65.0$  kg  
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$m_E = 5.97 \times 10^{24}$  kg  
 $r = 7.38 \times 10^6$  m  
 $m = 65.0$  kg

$m_S = 5.68 \times 10^{26}$  kg  
 $r_S = 6.03 \times 10^7$  m  
 $m = 65.0$  kg

$m_S = 5.68 \times 10^{26}$  kg  
 $r = 6.13 \times 10^7$  m  
 $m = 65.0$  kg

5.  $r_E = 6.38 \times 10^6$  m  
 $g = 9.81$  m/s<sup>2</sup>  
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

### Solutions

a.  $F_g = G \frac{mm_E}{r_E^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(65.0 \text{ kg})(5.97 \times 10^{24} \text{ kg})}{(6.38 \times 10^6 \text{ m})^2} = \boxed{636 \text{ N}}$

b.  $F_g = G \frac{mm_E}{r^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(65.0 \text{ kg})(5.97 \times 10^{24} \text{ kg})}{(7.38 \times 10^6 \text{ m})^2} = \boxed{475 \text{ N}}$

c.  $F_g = \frac{G mm_S}{r_S^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(65.0 \text{ kg})(5.68 \times 10^{26} \text{ kg})}{(6.03 \times 10^7 \text{ m})^2} = \boxed{678 \text{ N}}$

d.  $F_g = G \frac{mm_S}{r^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(65.0 \text{ kg})(5.68 \times 10^{26} \text{ kg})}{(6.13 \times 10^7 \text{ m})^2} = \boxed{656 \text{ N}}$

$g = G \frac{m_E}{r_E^2}$ , so  $m_E = \frac{gr_E^2}{G}$   
 $m_E = \frac{(9.81 \text{ m/s}^2)(6.38 \times 10^6 \text{ m})^2}{6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2} = \boxed{5.98 \times 10^{24} \text{ kg}}$

## Circular Motion and Gravitation, Practice D

### Givens

$$\begin{aligned}
 1. \quad r &= 3.61 \times 10^5 \text{ m} \\
 m_E &= 5.97 \times 10^{24} \text{ kg} \\
 r_E &= 6.38 \times 10^6 \text{ m} \\
 m_J &= 1.90 \times 10^{27} \text{ kg} \\
 r_J &= 7.15 \times 10^7 \text{ m} \\
 m_m &= 7.35 \times 10^{22} \text{ kg} \\
 r_m &= 1.74 \times 10^6 \text{ m} \\
 G &= 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}
 \end{aligned}$$

### Solutions

Above Earth:

$$r_1 = r + r_E = 3.61 \times 10^5 \text{ m} + 6.38 \times 10^6 \text{ m} = 6.74 \times 10^6 \text{ m}$$

$$v_t = \sqrt{G \frac{m_E}{r_1}}$$

$$v = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \left(\frac{5.97 \times 10^{24} \text{ kg}}{6.74 \times 10^6 \text{ m}}\right)} = 7.69 \times 10^3 \text{ m/s}$$

$$T = 2\pi \sqrt{\frac{r_1^3}{Gm_E}}$$

$$T = 2\pi \sqrt{\frac{(6.74 \times 10^6 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) (5.97 \times 10^{24} \text{ kg})}} = 5.51 \times 10^3 \text{ s}$$

Above Jupiter:

$$r_2 = r + r_J = 3.61 \times 10^5 \text{ m} + 7.15 \times 10^7 \text{ m} = 7.19 \times 10^7 \text{ m}$$

$$v_t = \sqrt{G \frac{m_J}{r_2}}$$

$$v_t = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \left(\frac{1.90 \times 10^{27} \text{ kg}}{7.19 \times 10^7 \text{ m}}\right)} = 4.20 \times 10^4 \text{ m/s}$$

$$T = 2\pi \sqrt{\frac{r_2^3}{Gm_J}}$$

$$T = 2\pi \sqrt{\frac{(7.19 \times 10^7 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) (1.90 \times 10^{27} \text{ kg})}} = 1.08 \times 10^4 \text{ s}$$

Above Earth's moon:

$$r_3 = r + r_m = 3.61 \times 10^5 \text{ m} + 1.74 \times 10^6 \text{ m} = 2.10 \times 10^6 \text{ m}$$

$$v_t = \sqrt{G \frac{m_m}{r_3}}$$

$$v_t = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \left(\frac{7.35 \times 10^{22} \text{ kg}}{2.10 \times 10^6 \text{ m}}\right)} = 1.53 \times 10^3 \text{ m/s}$$

$$T = 2\pi \sqrt{\frac{r_3^3}{Gm_m}}$$

$$T = 2\pi \sqrt{\frac{(2.10 \times 10^6 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) (7.35 \times 10^{22} \text{ kg})}} = 8.63 \times 10^3 \text{ s}$$

**Givens**

2.  $T = 125 \text{ min}$   
 $r_E = 6.38 \times 10^6 \text{ m}$   
 $m_E = 5.97 \times 10^{24} \text{ kg}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

**Solutions**

$$T^2 = \frac{4\pi^2 r^3}{Gm_E}$$

$$r^3 = \frac{T^2 Gm_E}{4\pi^2}$$

$$r = \sqrt[3]{\frac{[(125 \text{ min})(60 \text{ s/min})]^2 (6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})}{4\pi^2}}$$

$$r = 8.28 \times 10^6 \text{ m}$$

height above Earth =  $r - r_E = 8.28 \times 10^6 \text{ m} - 6.38 \times 10^6 \text{ m} = \boxed{1.90 \times 10^6 \text{ m}}$

**Circular Motion and Gravitation, Section 3 Review**

5.  $r = 3.84 \times 10^8 \text{ m}$   
 $m_E = 5.97 \times 10^{24} \text{ kg}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$v_t = \sqrt{G \frac{m_E}{r}}$$

$$v_t = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \left(\frac{5.97 \times 10^{24} \text{ kg}}{3.84 \times 10^8 \text{ m}}\right)} = \boxed{1.02 \times 10^3 \text{ m/s}}$$

$$T = 2\pi \sqrt{\frac{r^3}{Gm_E}}$$

$$T = 2\pi \sqrt{\frac{(3.84 \times 10^8 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) (5.97 \times 10^{24} \text{ kg})}} = \boxed{2.37 \times 10^6 \text{ s}}$$

**Circular Motion and Gravitation, Practice E**

1.  $F = 3.0 \text{ N}$   
 $d = 0.25 \text{ m}$   
 $\theta = 90.0^\circ$

$$\tau = Fd(\sin \theta) = (3.0 \text{ N})(0.25 \text{ m})(\sin 90.0^\circ) = \boxed{0.75 \text{ N}\cdot\text{m}}$$

2.  $m = 3.0 \text{ kg}$   
 $d = 2.0 \text{ m}$   
 $\theta_1 = 5.0^\circ$   
 $g = 9.81 \text{ m/s}^2$   
 $\theta_2 = 15.0^\circ$

a.  $\tau = Fd(\sin \theta_1) = mgd(\sin \theta_1)$   
 $\tau = (3.0 \text{ kg})(9.81 \text{ m/s}^2)(2.0 \text{ m})(\sin 5.0^\circ) = \boxed{5.1 \text{ N}\cdot\text{m}}$

b.  $\tau = mgd(\sin \theta_2) = (3.0 \text{ kg})(9.81 \text{ m/s}^2)(2.0 \text{ m})(\sin 15.0^\circ) = \boxed{15 \text{ N}\cdot\text{m}}$

3.  $\tau = 40.0 \text{ N}\cdot\text{m}$   
 $d = 30.0 \text{ cm}$

For a given torque, the minimum force must be applied perpendicular to the lever arm, or  $\sin \theta = 1$ . Therefore,

$$F = \frac{\tau}{d} = \frac{40.0 \text{ N}\cdot\text{m}}{0.300 \text{ m}} = \boxed{133 \text{ N}}$$

## Circular Motion and Gravitation, Section 4 Review

### Givens

5.  $eff = 0.73$   
 $d_{in} = 18.0 \text{ m}$   
 $d_{out} = 3.0 \text{ m}$   
 $m = 58 \text{ kg}$   
 $g = 9.81 \text{ m/s}^2$

### Solutions

$$eff = \frac{W_{out}}{W_{in}}$$

$$eff = \frac{F_{out}d_{out}}{F_{in}d_{in}} \quad \text{where } F_{out} = mg$$

$$F_{in} = \frac{mgd_{out}}{effd_{in}} = \frac{(58 \text{ kg})(9.81 \text{ m/s}^2)(3.0 \text{ m})}{(0.73)(18.0 \text{ m})} = \boxed{1.3 \times 10^2 \text{ N}}$$

6.  $F_g = 950 \text{ N}$   
 $F_{applied} = 350 \text{ N}$

$$MA = \frac{F_{out}}{F_{in}} = \frac{F_g}{F_{applied}} = \frac{950 \text{ N}}{350 \text{ N}} = \boxed{2.7}$$

8.  $F_{30} = 30.0 \text{ N}$   
 $\theta_{30} = 45^\circ$   
 $d_{30} = 0 \text{ m}$   
 $F_{25} = 25.0 \text{ N}$   
 $\theta_{25} = 59^\circ$   
 $d_{25} = 2.0 \text{ m}$   
 $F_{10} = 10.0 \text{ N}$   
 $\theta_{10} = 23^\circ$   
 $d_{10} = 4.0 \text{ m}$

$$\tau_{30} = F_{30}d_{30}(\sin \theta_{30}) = (30.0 \text{ N})(0 \text{ m})(\sin 45^\circ) = \boxed{0 \text{ N}\cdot\text{m}}$$

$$\tau_{25} = F_{25}d_{25}(\sin \theta_{25}) = (25.0 \text{ N})(2.0 \text{ m})(\sin 59^\circ) = \boxed{43 \text{ N}\cdot\text{m}}$$

$$\tau_{10} = F_{10}d_{10}(\sin \theta_{10}) = (-10.0 \text{ N})(4.0 \text{ m})(\sin 23^\circ) = \boxed{-16 \text{ N}\cdot\text{m}}$$

The bar will rotate counterclockwise because  $\tau_{net}$  is positive  
 $(43 \text{ N}\cdot\text{m} - 16 \text{ N}\cdot\text{m} = +27 \text{ N}\cdot\text{m})$ .

## Circular Motion and Gravitation, Chapter Review

8.  $a_c = 145 \text{ m/s}^2$   
 $r = 0.34 \text{ m}$

$$v_t = \sqrt{ra_c} = \sqrt{(0.34 \text{ m})(145 \text{ m/s}^2)} = \boxed{7.0 \text{ m/s}}$$

9.  $a_c = 28 \text{ m/s}^2$   
 $r = 27 \text{ cm}$

$$v_t = \sqrt{ra_c} = \sqrt{(27 \times 10^{-2} \text{ m})(28 \text{ m/s}^2)} = \boxed{2.7 \text{ m/s}}$$

10.  $v_t = 20.0 \text{ m/s}$   
 $F_n = 2.06 \times 10^4 \text{ N}$   
 $r_1 = 10.0 \text{ m}$   
 $g = 9.81 \text{ m/s}^2$

a.  $F_{net} = F_n - F_g = F_n - mg$

$$F_{net} = F_c = \frac{mv_t^2}{r_1}$$

$$F_n - mg = \frac{mv_t^2}{r_1}$$

$$mv_t^2 = r_1(F_n - mg)$$

$$m(v_t^2 + r_1g) = r_1F_n$$

$$m = \frac{r_1F_n}{v_t^2 + r_1g} = \frac{(10.0 \text{ m})(2.06 \times 10^4 \text{ N})}{(20.0 \text{ m/s})^2 + (10.0 \text{ m})(9.81 \text{ m/s}^2)}$$

$$m = \frac{2.06 \times 10^5 \text{ N}\cdot\text{m}}{4.00 \times 10^2 \text{ m}^2/\text{s}^2 + 98.1 \text{ m}^2/\text{s}^2} = \frac{2.06 \times 10^5 \text{ N}\cdot\text{m}}{498 \text{ m}^2/\text{s}^2}$$

$$m = \boxed{414 \text{ kg}}$$

## Givens

$$r_2 = 15.0 \text{ m}$$

## Solutions

$$\mathbf{b.} \quad F_c = mg$$

$$\frac{mv_t^2}{r_2} = mg$$

$$v_t = \sqrt{gr_2} = \sqrt{(9.81 \text{ m/s}^2)(15.0 \text{ m})} = \boxed{12.1 \text{ m/s}}$$

$$\mathbf{11.} \quad r = 10.0 \text{ m}$$

$$v_t = 8.0 \text{ m/s}$$

$$F_{\text{rope, max}} = 1.0 \times 10^3 \text{ N}$$

$$g = 9.81 \text{ m/s}^2$$

$$F_{\text{total}} = F_c + F_g = \frac{mv_t^2}{r} + mg$$

$$F_{\text{total}} \leq F_{\text{rope, max}}$$

$$F_{\text{rope, max}} = m_{\text{max}} \left( \frac{v_t^2}{r} + g \right)$$

$$m_{\text{max}} = \frac{F_{\text{rope, max}}}{\frac{v_t^2}{r} + g} = \frac{1.0 \times 10^3 \text{ N}}{\left[ \frac{(8.0 \text{ m/s})^2}{10.0 \text{ m}} \right] + 9.81 \text{ m/s}^2}$$

$$m_{\text{max}} = \frac{1.0 \times 10^3 \text{ N}}{6.4 \text{ m/s}^2 + 9.81 \text{ m/s}^2} = \frac{1.0 \times 10^3 \text{ N}}{16.2 \text{ m/s}^2}$$

$$m_{\text{max}} = \boxed{62 \text{ kg}}$$

$$\mathbf{18.} \quad F_g = 3.20 \times 10^{-8} \text{ N}$$

$$m_1 = 50.0 \text{ kg}$$

$$m_2 = 60.0 \text{ kg}$$

$$G = 6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \sqrt{\frac{(6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(50.0 \text{ kg})(60.0 \text{ kg})}{3.20 \times 10^{-8} \text{ N}}}$$

$$r = \boxed{2.50 \text{ m}}$$

$$\mathbf{19.} \quad m_1 = 9.11 \times 10^{-31} \text{ kg}$$

$$m_2 = 1.67 \times 10^{-27} \text{ kg}$$

$$F_g = 1.0 \times 10^{-47} \text{ N}$$

$$G = 6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$$

$$r = \sqrt{\frac{Gm_1m_2}{F_g}} = \frac{(6.673 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{1.0 \times 10^{-47} \text{ N}}$$

$$r = \boxed{1.0 \times 10^{-10} \text{ m} = 0.10 \text{ nm}}$$

$$\mathbf{27.} \quad r = 1.44 \times 10^8 \text{ m}$$

$$r_E = 6.38 \times 10^6 \text{ m}$$

$$m_E = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

$$r_I = r + r_E = 1.44 \times 10^8 \text{ m} + 6.38 \times 10^6 \text{ m} = 1.50 \times 10^8 \text{ m}$$

$$v_t = \sqrt{G \frac{m_E}{r_I}}$$

$$v_t = \sqrt{\left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \left( \frac{5.97 \times 10^{24} \text{ kg}}{1.50 \times 10^8 \text{ m}} \right)} = \boxed{1630 \text{ m/s}}$$

$$T = 2\pi \sqrt{\frac{r_I^3}{Gm_E}}$$

$$T = 2\pi \sqrt{\frac{(1.50 \times 10^8 \text{ m})^3}{\left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) (5.97 \times 10^{24} \text{ kg})}} = \boxed{5.78 \times 10^5 \text{ s}}$$

## Givens

**28.**  $T = 24.0 \text{ h}$

$$r_E = 6.38 \times 10^6 \text{ m}$$

$$m_E = 5.97 \times 10^{24} \text{ kg}$$

$$G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

## Solutions

$$T = 2\pi \sqrt{\frac{r_1^3}{Gm_E}}$$

$$T^2 = 4\pi^2 \frac{r_1^3}{Gm_E}$$

$$r_1 = \sqrt[3]{\frac{T^2 G m_E}{4\pi^2}}$$

$$r_1 = \sqrt[3]{\frac{(24.0 \text{ h}) \left(3600 \frac{\text{s}}{\text{h}}\right)^2 \left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) (5.97 \times 10^{24} \text{ kg})}{4\pi^2}}$$

$$r_1 = 4.22 \times 10^7 \text{ m (from Earth's center)}$$

$$r = r_1 - r_E = 4.22 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = \boxed{3.58 \times 10^7 \text{ m}}$$

**29.**  $r = 2.0 \times 10^8 \text{ m}$

$$T = 5.0 \times 10^4 \text{ s}$$

$$G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$$

$$T = 2\pi \sqrt{\frac{r^3}{Gm}}$$

$$T^2 = 4\pi^2 \frac{r^3}{Gm}$$

$$m = \frac{4\pi^2 r^3}{T^2 G} = \frac{4\pi^2 (2.0 \times 10^8 \text{ m})^3}{(5.0 \times 10^4 \text{ s})^2 \left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)}$$

$$m = \boxed{1.9 \times 10^{27} \text{ kg}}$$

**37.**  $m = 54 \text{ kg}$

$$r = 0.050 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\theta = 90^\circ$$

$$\tau = Fd(\sin \theta) = mgr(\sin \theta)$$

$$\tau = (54 \text{ kg})(9.81 \text{ m/s}^2)(0.050 \text{ m})(\sin 90^\circ) = \boxed{26 \text{ N}\cdot\text{m}}$$

**38.**  $\theta = 90.0^\circ - 8.0^\circ = 82.0^\circ$

$$m = 1130 \text{ kg}$$

$$d = 3.05 \text{ m} - 1.12 \text{ m} - 0.40 \text{ m} = 1.53 \text{ m}$$

$$g = 9.81 \text{ m/s}^2$$

$$\tau_{\text{net}} = \tau_g + \tau_{\text{jack}} = 0$$

$$mgd(\sin \theta) + \tau_{\text{jack}} = 0$$

$$\tau_{\text{jack}} = -mgd(\sin \theta) = -(1130 \text{ kg})(9.81 \text{ m/s}^2)(1.53 \text{ m})(\sin 82.0^\circ)$$

$$\text{magnitude of } \tau_{\text{jack}} = \boxed{1.68 \times 10^4 \text{ N}\cdot\text{m}}$$

**39.**  $m = 2.00 \times 10^3 \text{ kg}$

$$r = 20.0 \text{ m}$$

$$\mu_k = 0.70$$

$$g = 9.81 \text{ m/s}^2$$

$$F_k = \mu_k F_n = \mu_k mg$$

$$F_c = m \frac{v_t^2}{r}$$

$$F_k = F_c$$

$$\mu_k mg = m \frac{v_t^2}{r}$$

$$v_t = \sqrt{r\mu_k g} = \sqrt{(20.0 \text{ m})(0.70)(9.81 \text{ m/s}^2)}$$

$$v_t = \boxed{12 \text{ m/s}}$$



## Givens

**40.**  $m_m = 7.35 \times 10^{22}$  kg  
 $m_s = 1.99 \times 10^{30}$  kg  
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$r = 3.84 \times 10^8$  m  
 $m_E = 5.97 \times 10^{24}$  kg  
 $m_m = 7.35 \times 10^{22}$  kg

$m_E = 5.97 \times 10^{24}$  kg  
 $m_s = 1.99 \times 10^{30}$  kg  
 $r = 1.50 \times 10^{11}$  m

## Solutions

$r = 1.50 \times 10^{11}$  m  $-$   $0.00384 \times 10^{11}$  m  $=$   $1.50 \times 10^{11}$  m

**a.**  $F_g = G \frac{m_m m_s}{r^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(7.35 \times 10^{22} \text{ kg})(1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = \boxed{4.34 \times 10^{20} \text{ N}}$

**b.**  $F_g = G \frac{m_E m_m}{r^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(5.97 \times 10^{24} \text{ kg})(7.35 \times 10^{22} \text{ kg})}{(3.84 \times 10^8 \text{ m})^2} = \boxed{1.99 \times 10^{20} \text{ N}}$

**c.**  $F_g = G \frac{m_E m_s}{r^2}$   
 $F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \frac{(5.97 \times 10^{24} \text{ kg})(1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} = \boxed{3.52 \times 10^{22} \text{ N}}$

**41.**  $m = 75$  kg  
 $r = 0.075$  m  
 $d = 0.25$  m  
 $g = 9.81 \text{ m/s}^2$

For a force perpendicular to  $d$ ,  $\tau = Fd$ .

$F = \frac{\tau}{d} = \frac{mgr}{d} = \frac{(75 \text{ kg})(9.81 \text{ m/s}^2)(0.075 \text{ m})}{0.25 \text{ m}} = \boxed{2.2 \times 10^2 \text{ N}}$

**42.**  $\tau = 58 \text{ N}\cdot\text{m}$   
 $d = 0.35$  m  
 $\theta = 56^\circ$

$\tau = Fd(\sin \theta)$

$F = \frac{\tau}{d(\sin \theta)} = \frac{58 \text{ N}\cdot\text{m}}{(0.35 \text{ m})(\sin 56^\circ)} = \boxed{2.0 \times 10^2 \text{ N}}$

**43.**  $d = 1.4$  m  
 $F = 1600$  N  
 $\theta = 53.5^\circ$

$\tau = Fd(\sin \theta) = (1600 \text{ N})(1.4 \text{ m})(\sin 53.5^\circ) = \boxed{1800 \text{ N}\cdot\text{m}}$

**44.**  $L_h = 2.7$  m  
 $L_m = 4.5$  m  
 $m_h = 60.0$  kg  
 $m_m = 100.0$  kg  
 $\theta_h = 20.0^\circ$  from 6:00  
 $\theta_m = 60.0^\circ$  from 6:00  
 $g = 9.81 \text{ m/s}^2$

Consider the total mass of each hand to be at the midpoint of that hand.

$\tau_{\text{net}} = -m_h g \left( \frac{L_h}{2} \right) (\sin \theta_h) - m_m g \left( \frac{L_m}{2} \right) (\sin \theta_m)$

$\tau_{\text{net}} = -(60.0 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{2.7 \text{ m}}{2} \right) (\sin 20.0^\circ) - (100.0 \text{ kg})(9.81 \text{ m/s}^2) \left( \frac{4.5 \text{ m}}{2} \right) (\sin 60.0^\circ)$

$\tau_{\text{net}} = -2.7 \times 10^2 \text{ N}\cdot\text{m} - 1.9 \times 10^3 \text{ N}\cdot\text{m} = \boxed{-2.2 \times 10^3 \text{ N}\cdot\text{m}}$

**45.**  $\text{eff} = 0.64$   
 $m = 78$  kg  
 $d_{\text{out}} = 4.0$  m  
 $d_{\text{in}} = 24$  m  
 $g = 9.81 \text{ m/s}^2$

$\text{eff} = \frac{W_{\text{out}}}{W_{\text{in}}} = \frac{F_{\text{out}} d_{\text{out}}}{F_{\text{in}} d_{\text{in}}}$

$F_{\text{in}} = \frac{F_{\text{out}} d_{\text{out}}}{d_{\text{in}}(\text{eff})} = \frac{mg d_{\text{out}}}{d_{\text{in}}(\text{eff})} = \frac{(78 \text{ kg})(9.81 \text{ m/s}^2)(4.0 \text{ m})}{(24 \text{ m})(0.64)}$

$F_{\text{in}} = \boxed{2.0 \times 10^2 \text{ N}}$

## Givens

46.  $d = 2.0 \text{ m}$   
 $\theta = 15^\circ$   
 $\mu_k = 0.160$

## Solutions

$$W_{out} = F_g d (\sin \theta)$$

$$W_{in} = (F_f + F_{g,x})d = [\mu_k F_g (\cos \theta) + F_g (\sin \theta)]d$$

$$W_{in} = F_g d [\mu_k (\cos \theta) + (\sin \theta)]$$

$$eff = \frac{W_{out}}{W_{in}} = \frac{F_g d (\sin \theta)}{F_g d [\mu_k (\cos \theta) + (\sin \theta)]} = \frac{\sin \theta}{\mu_k (\cos \theta) + (\sin \theta)}$$

$$eff = \frac{\sin 15^\circ}{(0.160)(\cos 15^\circ) + (\sin 15^\circ)} = \frac{0.26}{0.15 + 0.26}$$

$$eff = \frac{0.26}{0.41} = 0.63 = \boxed{63\%}$$

47.  $d_{out} = 3.0 \text{ m}$   
 $F_{in} = 2200 \text{ N}$   
 $d_{in} = 14 \text{ m}$   
 $m = 750 \text{ kg}$   
 $g = 9.81 \text{ m/s}^2$

$$eff = \frac{W_{out}}{W_{in}} = \frac{F_{out} d_{out}}{F_{in} d_{in}} = \frac{mg d_{out}}{F_{in} d_{in}}$$

$$eff = \frac{(750 \text{ kg})(9.81 \text{ m/s}^2)(3.0 \text{ m})}{(2200 \text{ N})(14 \text{ m})} = 0.72 = \boxed{72\%}$$

48.  $eff = 0.875$   
 $F_{in} = 648 \text{ N}$   
 $m = 150 \text{ kg}$   
 $d_{out} = 2.46 \text{ m}$   
 $g = 9.81 \text{ m/s}^2$

$$eff = \frac{W_{out}}{W_{in}} = \frac{F_{out} d_{out}}{F_{in} d_{in}} = \frac{mg d_{out}}{F_{in} d_{in}}$$

$$d_{in} = \frac{mg d_{out}}{F_{in} (eff)} = \frac{(150 \text{ kg})(9.81 \text{ m/s}^2)(2.46 \text{ m})}{(648 \text{ N})(0.875)} = \boxed{6.4 \text{ m}}$$

49.  $r_{Io} = 1.82 \times 10^6 \text{ m}$   
 $d = 4.22 \times 10^8 \text{ m}$   
 $r_j = 7.15 \times 10^7 \text{ m}$   
 $m_j = 1.90 \times 10^{27} \text{ kg}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

a.  $r = r_{Io} + d + r_j$

$$r = 1.82 \times 10^6 \text{ m} + 4.22 \times 10^8 \text{ m} + 7.15 \times 10^7 \text{ m}$$

$$r = 4.95 \times 10^8 \text{ m}$$

$$T = 2\pi \sqrt{\frac{r^3}{Gm_j}} = 2\pi \sqrt{\frac{(4.95 \times 10^8 \text{ m})^3}{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(1.90 \times 10^{27} \text{ kg})}}$$

$$T = (1.94 \times 10^5 \text{ s})(1 \text{ h}/3600 \text{ s})(1 \text{ day}/24 \text{ h}) = \boxed{2.25 \text{ days}}$$

- b.  $r = 4.95 \times 10^8 \text{ m}$  (see part a.)

$$v_t = \sqrt{G \frac{m_j}{r}}$$

$$v_t = \sqrt{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right) \left(\frac{1.90 \times 10^{27} \text{ kg}}{4.95 \times 10^8 \text{ m}}\right)} = \boxed{1.60 \times 10^4 \text{ m/s}}$$

## Givens

**50.**  $F_g = 13\,500\text{ N}$   
 $r = 2.00 \times 10^2\text{ m}$   
 $v_t = 50.0\text{ km/h}$   
 $g = 9.81\text{ m/s}^2$

## Solutions

**a.**  $a_c = \frac{v_t^2}{r} = \frac{(50.0 \times 10^3\text{ m/h})^2 (1\text{ h}/3600\text{ s})^2}{2.00 \times 10^2\text{ m}} = \boxed{0.965\text{ m/s}^2}$

**b.**  $F_c = ma_c = \left(\frac{F_g}{g}\right)a_c = \left(\frac{13\,500\text{ N}}{9.81\text{ m/s}^2}\right)(0.965\text{ m/s}^2) = \boxed{1.33 \times 10^3\text{ N}}$

**c.**  $F_c = F_k = \mu_k F_n = \mu_k F_g$

$$\mu_k = \frac{F_c}{F_g} = \frac{1330\text{ N}}{13\,500\text{ N}} = \boxed{0.0985}$$

**51.**  $d = 15.0\text{ m}$   
 $\theta_1 = 90.0^\circ - 20.0^\circ = 70.0^\circ$   
 $F_{g,\max} = 450\text{ N}$

**a.**  $\tau_{\max} = F_{g,\max}d(\sin \theta_1) = (450\text{ N})(15.0\text{ m})(\sin 70.0^\circ) = \boxed{6.3 \times 10^3\text{ N}\cdot\text{m}}$

$\theta_2 = 90.0^\circ - 40.0^\circ = 50.0^\circ$

**b.**  $F_g = \frac{\tau_{\max}}{d(\sin \theta_2)} = \frac{6.3 \times 10^3\text{ N}\cdot\text{m}}{(15.0\text{ m})(\sin 50.0^\circ)} = \boxed{5.5 \times 10^2\text{ N}}$

**52.**  $m_1 = 5.00\text{ kg}$   
 $m_2 = 1.99 \times 10^{30}\text{ kg}$   
 $F_g = 1370\text{ N}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$F_g = G \frac{m_1 m_2}{r^2}$$

$$r = \sqrt{\frac{G m_1 m_2}{F_g}}$$

$$r = \sqrt{\frac{\left(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)(5.00\text{ kg})(1.99 \times 10^{30}\text{ kg})}{1370\text{ N}}} = \boxed{6.96 \times 10^8\text{ m}}$$

**53.**  $v_t = 55.0\text{ km/h}$   
 $r = 40.0\text{ m}$   
 $m = 1350\text{ kg}$   
 $\mu_k = 0.500$   
 $g = 9.81\text{ m/s}^2$

$F_f = \mu_k F_n = \mu_k mg = (0.500)(1350\text{ kg})(9.81\text{ m/s}^2) = \boxed{6620\text{ N}}$

$F_c = \frac{mv_t^2}{r} = \frac{(1350\text{ kg})[(55.0 \times 10^3\text{ m/h})(1\text{ h}/3600\text{ s})]^2}{40.0\text{ m}} = \boxed{7880\text{ N}}$

Because  $F_c > F_f$ , the frictional force is not large enough to maintain the circular motion.

**54.**  $\theta = 60.0^\circ$   
 $d = 0.35\text{ m}$   
 $\tau = 2.0\text{ N}\cdot\text{m}$

$F = \frac{\tau}{d \sin \theta} = \frac{2.0\text{ N}\cdot\text{m}}{(0.35\text{ m})(\sin 60.0^\circ)} = \boxed{6.6\text{ N}}$

$\tau_{\max}$  is produced when  $\theta = 90.0^\circ$

$\tau_{\max} = Fd \sin \theta = (6.6\text{ N})(0.35\text{ m})(\sin 90.0^\circ) = \boxed{2.3\text{ N}\cdot\text{m}}$

# Circular Motion and Gravitation, Standardized Test Prep

## Givens

## Solutions

2.  $v_t = 15 \text{ m/s}$   
 $r = 25 \text{ m}$

$$a_c = \frac{v_t^2}{r} = \frac{(15 \text{ m/s})^2}{25 \text{ m}} = \boxed{9.0 \text{ m/s}^2}$$

4.  $m_E = 5.97 \times 10^{24} \text{ kg}$   
 $m_s = 1.99 \times 10^{30} \text{ kg}$   
 $r = 1.50 \times 10^{11} \text{ m}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$F_g = G \frac{m_E m_s}{r^2}$$

$$F_g = \left( 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2} \right) \left( \frac{(5.97 \times 10^{24} \text{ kg})(1.99 \times 10^{30} \text{ kg})}{(1.50 \times 10^{11} \text{ m})^2} \right)$$

$$F_g = \boxed{3.52 \times 10^{22} \text{ N}}$$

9.  $F_1 = 6.0 \text{ N}$   
 $F_2 = 6.0 \text{ N}$   
 $F_3 = 6.0 \text{ N}$   
 $\theta_1 = 90.0^\circ$   
 $\theta_2 = 90.0^\circ - 60.0^\circ = 30.0^\circ$   
 $\theta_3 = 0.0^\circ$   
 $d = 1.0 \text{ m}$

$$\tau_{net} = \tau_1 + \tau_2 + \tau_3$$

$$\tau_{net} = F_1 d \sin \theta_1 + F_2 d \sin \theta_2 + F_3 d \sin \theta_3$$

$$\tau_{net} = (6.0 \text{ N})(1.0 \text{ m})(\sin 90.0^\circ) + (6.0 \text{ N})(1.0 \text{ m})(\sin 30.0^\circ) + (6.0 \text{ N})(1.0 \text{ m})(\sin 0.0^\circ)$$

$$\tau_{net} = 6.0 \text{ N}\cdot\text{m} + 3.0 \text{ N}\cdot\text{m} + 0.0 \text{ N}\cdot\text{m} = \boxed{9.0 \text{ N}\cdot\text{m}}$$

10.  $F_{in} = 75 \text{ N}$   
 $F_{out} = 225 \text{ N}$

$$MA = \frac{F_{out}}{F_{in}} = \frac{225 \text{ N}}{75 \text{ N}} = \boxed{3}$$

11.  $eff = 87.5\% = 0.875$   
 $F_{out} = 1320 \text{ N}$   
 $d_{out} = 1.50 \text{ m}$

$$eff = \frac{W_{out}}{W_{in}} = \frac{F_{out} d_{out}}{W_{in}}$$

$$W_{in} = \frac{F_{out} d_{out}}{eff} = \frac{(1320 \text{ N})(1.50 \text{ m})}{0.875} = \boxed{2260 \text{ J}}$$

17.  $m_s = 1.99 \times 10^{30} \text{ kg}$   
 $r = 2.28 \times 10^{11} \text{ m}$   
 $G = 6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$

$$T = 2\pi \sqrt{\frac{r^3}{Gm_s}}$$

$$T = \left[ 2\pi \sqrt{\frac{(2.28 \times 10^{11} \text{ m})^3}{(6.673 \times 10^{-11} \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2})(1.99 \times 10^{30} \text{ kg})}} \right] \left( \frac{1 \text{ h}}{3600 \text{ s}} \right) \left( \frac{1 \text{ day}}{24 \text{ h}} \right)$$

$$T = \boxed{687 \text{ days}}$$