

Momentum and Collisions

Momentum and Collisions, Practice A

Givens

1. $m = 146 \text{ kg}$
 $v = 17 \text{ m/s south}$

Solutions

$$p = mv = (146 \text{ kg})(17 \text{ m/s south})$$

$$p = \boxed{2.5 \times 10^3 \text{ kg}\cdot\text{m/s to the south}}$$

2. $m_1 = 21 \text{ kg}$
 $m_2 = 5.9 \text{ kg}$
 $v = 4.5 \text{ m/s to the northwest}$

$$\mathbf{a. p_{tot}} = m_{tot}\mathbf{v} = (m_1 + m_2)\mathbf{v} = (21 \text{ kg} + 5.9 \text{ kg})(4.5 \text{ m/s})$$

$$\mathbf{p_{tot}} = (27 \text{ kg})(4.5 \text{ m/s}) = \boxed{1.2 \times 10^2 \text{ kg}\cdot\text{m/s to the northwest}}$$

$$\mathbf{b. p_1} = m_1\mathbf{v} = (21 \text{ kg})(4.5 \text{ m/s}) = \boxed{94 \text{ kg}\cdot\text{m/s to the northwest}}$$

$$\mathbf{c. p_2} = m_2\mathbf{v} = (5.9 \text{ kg})(4.5 \text{ m/s}) = \boxed{27 \text{ kg}\cdot\text{m/s to the northwest}}$$

3. $m = 1210 \text{ kg}$
 $p = 5.6 \times 10^4 \text{ kg}\cdot\text{m/s to the east}$

$$v = \frac{p}{m} = \frac{5.6 \times 10^4 \text{ kg}\cdot\text{m/s}}{1210 \text{ kg}} = \boxed{46 \text{ m/s to the east}}$$

Momentum and Collisions, Practice B

1. $m = 0.50 \text{ kg}$
 $v_i = 15 \text{ m/s to the right}$
 $\Delta t = 0.020 \text{ s}$
 $v_f = 0 \text{ m/s}$

$$\mathbf{F} = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.50 \text{ kg})(0 \text{ m/s}) - (0.50 \text{ kg})(15 \text{ m/s})}{0.020 \text{ s}} \text{ to the right}$$

$$\mathbf{F} = -3.8 \times 10^2 \text{ N to the right}$$

$$\mathbf{F} = \boxed{3.8 \times 10^2 \text{ N to the left}}$$

2. $m = 82 \text{ kg}$
 $\Delta y = -3.0 \text{ m}$
 $\Delta t = 0.55 \text{ s}$
 $v_i = 0 \text{ m/s}$
 $a = -9.81 \text{ m/s}^2$

$$v_f = \pm\sqrt{2a\Delta y} = \pm\sqrt{(2)(-9.81 \text{ m/s}^2)(-3.0 \text{ m})} = \pm 7.7 \text{ m/s} = -7.7 \text{ m/s}$$

For the time the man is in the water,

$$v_i = 7.7 \text{ m/s downward} = -7.7 \text{ m/s} \quad v_f = 0 \text{ m/s}$$

$$\mathbf{F} = \frac{mv_f - mv_i}{\Delta t} = \frac{(82 \text{ kg})(0 \text{ m/s}) - (82 \text{ kg})(-7.7 \text{ m/s})}{0.55 \text{ s}} = 1.1 \times 10^3 \text{ N}$$

$$\mathbf{F} = \boxed{1.1 \times 10^3 \text{ N upward}}$$

3. $m = 0.40 \text{ kg}$
 $v_i = 18 \text{ m/s to the north}$
 $= +18 \text{ m/s}$
 $v_f = 22 \text{ m/s to the south}$
 $= -22 \text{ m/s}$

$$\Delta p = mv_f - mv_i = (0.40 \text{ kg})(-22 \text{ m/s}) - (0.40 \text{ kg})(18 \text{ m/s})$$

$$\Delta p = -8.8 \text{ kg}\cdot\text{m/s} - 7.2 \text{ kg}\cdot\text{m/s} = -16.0 \text{ kg}\cdot\text{m/s}$$

$$\Delta p = \boxed{16 \text{ kg}\cdot\text{m/s to the south}}$$

Givens

4. $m = 0.50 \text{ kg}$
 $F_1 = 3.00 \text{ N}$ to the right
 $\Delta t_1 = 1.50 \text{ s}$
 $v_{i,1} = 0 \text{ m/s}$
 $F_2 = 4.00 \text{ N}$ to the left
 $= -4.00 \text{ N}$
 $\Delta t_2 = 3.00 \text{ s}$
 $v_{i,2} = 9.0 \text{ m/s}$ to the right

Solutions

a. $v_{f,1} = \frac{F_1 \Delta t_1 + m v_{i,1}}{m} = \frac{(3.00 \text{ N})(1.50 \text{ s}) + (0.50 \text{ kg})(0 \text{ m/s})}{0.50 \text{ kg}}$
 $v_{f,1} = 9.0 \text{ m/s} = \boxed{9.0 \text{ m/s to the right}}$

b. $v_{f,2} = \frac{F_2 \Delta t_2 + m v_{i,2}}{m} = \frac{(-4.00 \text{ N})(3.00 \text{ s}) + (0.50 \text{ kg})(9.0 \text{ m/s})}{0.50 \text{ kg}}$
 $v_{f,2} = \frac{-12.0 \text{ kg}\cdot\text{m/s} + 4.5 \text{ kg}\cdot\text{m/s}}{0.50 \text{ kg}} = \frac{-7.5 \text{ kg}\cdot\text{m/s}}{0.50 \text{ kg}} = -15 \text{ m/s}$
 $v_{f,2} = \boxed{15 \text{ m/s to the left}}$

Momentum and Collisions, Practice C

1. $m = 2240 \text{ kg}$
 $v_i = 20.0 \text{ m/s}$ to the west,
 $v_i = -20.0 \text{ m/s}$
 $v_f = 0$
 $F = 8410 \text{ N}$ to the east,
 $F = +8410 \text{ N}$

a. $\Delta t = \frac{\Delta p}{F} = \frac{m v_f - m v_i}{F}$
 $\Delta t = \frac{(2240 \text{ kg})(0) - (2240 \text{ kg})(-20.0 \text{ m/s})}{(8410 \text{ N})} = \frac{44\,800 \text{ kg}\cdot\text{m/s}}{8410 \text{ kg}\cdot\text{m/s}^2}$
 $\Delta t = \boxed{5.33 \text{ s}}$

b. $\Delta x = \frac{1}{2}(v_i + v_f)\Delta t$
 $\Delta x = \frac{1}{2}(-20.0 \text{ m/s} - 0)(5.33 \text{ s})$
 $\Delta x = \boxed{-53.3 \text{ m or } 53.3 \text{ m to the west}}$

2. $m = 2500 \text{ kg}$
 $v_i = 20.0 \text{ m/s}$ to the north
 $= +20.0 \text{ m/s}$
 $F = 6250 \text{ N}$ to the south
 $= -6250 \text{ N}$
 $\Delta t = 2.50 \text{ s}$

a. $v_f = \frac{F \Delta t + m v_i}{m} = \frac{(-6250 \text{ N})(2.50 \text{ s}) + (2500 \text{ kg})(20.0 \text{ m/s})}{2500 \text{ kg}}$
 $v_f = \frac{(-1.56 \times 10^4 \text{ kg}\cdot\text{m/s}) + (5.0 \times 10^4 \text{ kg}\cdot\text{m/s})}{2500 \text{ kg}} = \frac{3.4 \times 10^4 \text{ kg}\cdot\text{m/s}}{2500 \text{ kg}}$
 $v_f = 14 \text{ m/s} = \boxed{14 \text{ m/s to the north}}$

b. $\Delta x = \frac{1}{2}(v_i + v_f)(\Delta t) = \frac{1}{2}(20.0 \text{ m/s} + 14 \text{ m/s})(2.50 \text{ s})$
 $\Delta x = \frac{1}{2}(34 \text{ m/s})(2.50 \text{ s}) = \boxed{42 \text{ m to the north}}$

$v_f = 0 \text{ m/s}$

c. $\Delta t = \frac{m v_f - m v_i}{F} = \frac{(2500 \text{ kg})(0 \text{ m/s}) - (2500 \text{ kg})(20.0 \text{ m/s})}{-6250 \text{ N}} = \boxed{8.0 \text{ s}}$

3. $m = 3250 \text{ kg}$
 $v_i = 20.0 \text{ m/s}$ to the west
 $= -20.0 \text{ m/s}$
 $v_f = 0 \text{ m/s}$
 $\Delta t = 5.33 \text{ s}$

a. $F = \frac{m v_f - m v_i}{\Delta t} = \frac{(3250 \text{ kg})(0 \text{ m/s}) - (3250 \text{ kg})(-20.0 \text{ m/s})}{5.33 \text{ s}}$
 $F = 1.22 \times 10^4 \text{ N} = \boxed{1.22 \times 10^4 \text{ N to the east}}$

b. $\Delta x = \frac{1}{2}(v_i + v_f)(\Delta t) = \frac{1}{2}(-20.0 \text{ m/s} + 0 \text{ m/s})(5.33 \text{ s}) = -53.3 \text{ m}$
 $\Delta x = \boxed{53.3 \text{ m to the west}}$

Momentum and Collisions, Section 1 Review

Givens

2. $m_1 = 0.145 \text{ kg}$
 $m_2 = 3.00 \text{ g}$
 $v_2 = 1.50 \times 10^3 \text{ m/s}$

Solutions

- a. $m_1 v_1 = m_2 v_2$

$$v_1 = \frac{m_2 v_2}{m_1} = \frac{(3.00 \times 10^{-3} \text{ kg})(1.50 \times 10^3 \text{ m/s})}{(0.145 \text{ kg})} = \boxed{31.0 \text{ m/s}}$$
- b. $KE_1 = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} (0.145 \text{ kg})(31.0 \text{ m/s})^2 = 69.7 \text{ J}$
 $KE_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} (3.00 \times 10^{-3} \text{ kg})(1.50 \times 10^3 \text{ m/s})^2 = 3380 \text{ J}$
 $\boxed{KE_2 > KE_1}$ The bullet has greater kinetic energy.

3. $m = 0.42 \text{ kg}$
 $\mathbf{v}_i = 12 \text{ m/s downfield}$
 $\mathbf{v}_f = 18 \text{ m/s downfield}$
 $\Delta t = 0.020 \text{ s}$

- a. $\Delta \mathbf{p} = m \mathbf{v}_f - m \mathbf{v}_i = (0.42 \text{ kg})(18 \text{ m/s}) - (0.42 \text{ kg})(12 \text{ m/s})$
 $\Delta \mathbf{p} = 7.6 \text{ kg} \cdot \text{m/s} - 5.0 \text{ kg} \cdot \text{m/s} = \boxed{2.6 \text{ kg} \cdot \text{m/s downfield}}$
- b. $\mathbf{F} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{2.6 \text{ kg} \cdot \text{m/s}}{0.020 \text{ s}} = \boxed{1.3 \times 10^2 \text{ N downfield}}$

Momentum and Collisions, Practice D

1. $m_1 = 63.0 \text{ kg}$
 $m_2 = 10.0 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_{2,f} = 12.0 \text{ m/s}$
 $v_{1,i} = 0 \text{ m/s}$

$$v_{1,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_2 v_{2,f}}{m_1}$$

$$v_{1,f} = \frac{(63.0 \text{ kg})(0 \text{ m/s}) + (10.0 \text{ kg})(0 \text{ m/s}) - (10.0 \text{ kg})(12.0 \text{ m/s})}{63.0 \text{ kg}} = -1.90 \text{ m/s}$$

astronaut speed = $\boxed{1.90 \text{ m/s}}$

2. $m_1 = 85.0 \text{ kg}$
 $m_2 = 135.0 \text{ kg}$
 $\mathbf{v}_{1,i} = 4.30 \text{ m/s to the west}$
 $= -4.30 \text{ m/s}$
 $\mathbf{v}_{2,i} = 0 \text{ m/s}$

$$\mathbf{v}_f = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(85.0 \text{ kg})(-4.30 \text{ m/s}) + (135.0 \text{ kg})(0 \text{ m/s})}{85.0 \text{ kg} + 135.0 \text{ kg}}$$

$$\mathbf{v}_f = \frac{(85.0 \text{ kg})(-4.30 \text{ m/s})}{220.0 \text{ kg}} = -1.66 \text{ m/s} = \boxed{1.66 \text{ m/s to the west}}$$

3. $m_1 = 0.50 \text{ kg}$
 $v_{1,i} = 12.0 \text{ m/s}$
 $v_{2,i} = 0 \text{ m/s}$
 $m_2 = 0.50 \text{ kg}$
 $v_{1,f} = 0 \text{ m/s}$
 $v_{1,f} = 2.4 \text{ m/s}$

a. $v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$
 $m_1 = m_2$
 $v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f} = 12.0 \text{ m/s} + 0 \text{ m/s} - 0 \text{ m/s} = \boxed{12.0 \text{ m/s}}$

b. $v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f} = 12.0 \text{ m/s} + 0 \text{ m/s} - 2.4 \text{ m/s} = \boxed{9.6 \text{ m/s}}$

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4. $m_1 = 2.0 \text{ kg} + m_b$
 $m_2 = 8.0 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_{2,f} = 3.0 \text{ m/s}$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{1,f} = -0.60 \text{ m/s}$

Solutions

$$m_1 v_{1,i} + m_2 v_{2,i} = m_1 v_{1,f} + m_2 v_{2,f}$$

$$(2.0 \text{ kg} + m_b)(0 \text{ m/s}) + (8.0 \text{ kg})(0 \text{ m/s}) = (2.0 \text{ kg} + m_b)(-0.60 \text{ m/s}) + (8.0 \text{ kg})(3.0 \text{ m/s})$$

$$(2.0 \text{ kg} + m_b)(0.60 \text{ m/s}) = (8.0 \text{ kg})(3.0 \text{ m/s})$$

$$m_b = \frac{24 \text{ kg} \cdot \text{m/s} - 1.2 \text{ kg} \cdot \text{m/s}}{0.60 \text{ m/s}} = \frac{23 \text{ kg} \cdot \text{m/s}}{0.60 \text{ m/s}}$$

$$m_b = \boxed{38 \text{ kg}}$$

Momentum and Collisions, Section 2 Review

1. $m_1 = 44 \text{ kg}$ $m_2 = 22 \text{ kg}$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_{1,f} = 3.5 \text{ m/s backward}$
 $= -3.5 \text{ m/s}$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{2,i} = 4.6 \text{ m/s to the right}$
 $= +4.6 \text{ m/s}$

a. $v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$

$$v_{2,f} = \frac{(44 \text{ kg})(0 \text{ m/s}) + (22 \text{ kg})(0 \text{ m/s}) - (44 \text{ kg})(-3.5 \text{ m/s})}{22 \text{ kg}} = \boxed{7.0 \text{ m/s forward}}$$

c. $v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(44 \text{ kg})(0 \text{ m/s}) + (22 \text{ kg})(4.6 \text{ m/s})}{44 \text{ kg} + 22 \text{ kg}}$

$$v_f = \frac{(22 \text{ kg})(4.6 \text{ m/s})}{66 \text{ kg}} = \boxed{1.5 \text{ m/s to the right}}$$

3. $m_1 = 215 \text{ g}$
 $v_{1,i} = 55.0 \text{ m/s}$
 $v_{2,i} = 0 \text{ m/s}$
 $m_2 = 46 \text{ g}$
 $v_{1,f} = 42.0 \text{ m/s}$

$$v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$$

$$v_{2,f} = \frac{(0.215 \text{ kg})(55.0 \text{ m/s}) + (0.046 \text{ kg})(0 \text{ m/s}) - (0.215 \text{ kg})(42.0 \text{ m/s})}{0.046 \text{ kg}}$$

$$v_{2,f} = \frac{11.8 \text{ kg} \cdot \text{m/s} - 9.03 \text{ kg} \cdot \text{m/s}}{0.046 \text{ kg}} = \frac{2.8 \text{ kg} \cdot \text{m/s}}{0.046 \text{ kg}} = \boxed{61 \text{ m/s}}$$

Momentum and Collisions, Practice E

1. $m_1 = 1500 \text{ kg}$
 $v_{1,i} = 15.0 \text{ m/s to the south}$
 $= -15.0 \text{ m/s}$
 $m_2 = 4500 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(1500 \text{ kg})(-15.0 \text{ m/s}) + (4500 \text{ kg})(0 \text{ m/s})}{1500 \text{ kg} + 4500 \text{ kg}}$$

$$v_f = \frac{(1500 \text{ kg})(-15.0 \text{ m/s})}{6.0 \times 10^3 \text{ kg}} = -3.8 \text{ m/s} = \boxed{3.8 \text{ m/s to the south}}$$

2. $m_1 = 9.0 \text{ kg}$
 $m_2 = 18.0 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_{1,i} = 5.5 \text{ m/s}$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(9.0 \text{ kg})(5.5 \text{ m/s}) + (18.0 \text{ kg})(0 \text{ m/s})}{9.0 \text{ kg} + 18.0 \text{ kg}}$$

$$v_f = \frac{(9.0 \text{ kg})(5.5 \text{ m/s})}{27.0 \text{ kg}} = \boxed{1.8 \text{ m/s}}$$

3. $m_1 = 1.50 \times 10^4 \text{ kg}$
 $v_{1,i} = 7.00 \text{ m/s north}$
 $m_2 = m_1 = m$
 $v_{2,i} = 1.50 \text{ m/s north}$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{m(v_{1,i} + v_{2,i})}{2m} = \frac{1}{2}(v_{1,i} + v_{2,i})$$

$$v_f = \frac{1}{2}(7.00 \text{ m/s north} + 1.50 \text{ m/s north})$$

$$v_f = \boxed{4.25 \text{ m/s to the north}}$$

Givens

4. $m_1 = 22 \text{ kg}$
 $m_2 = 9.0 \text{ kg}$
 $\mathbf{v}_{2,i} = 0 \text{ m/s}$
 $\mathbf{v}_f = 3.0 \text{ m/s to the right}$

Solutions

$$\mathbf{v}_{1,i} = \frac{(m_1 + m_2)\mathbf{v}_f - m_2\mathbf{v}_{2,i}}{m_1} = \frac{(22 \text{ kg} + 9.0 \text{ kg})(3.0 \text{ m/s}) - (9.0 \text{ kg})(0 \text{ m/s})}{22 \text{ kg}}$$

$$\mathbf{v}_{1,i} = \frac{(31 \text{ kg})(3.0 \text{ m/s})}{22 \text{ kg}} = \boxed{4.2 \text{ m/s to the right}}$$

5. $m_1 = 47.4 \text{ kg}$
 $v_{1,i} = 4.20 \text{ m/s}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_f = 3.95 \text{ m/s}$

$$\text{a. } m_2 = \frac{m_1 v_f - m_1 v_{1,i}}{v_{2,i} - v_f} = \frac{(47.4 \text{ kg})(3.95 \text{ m/s}) - (47.4 \text{ kg})(4.20 \text{ m/s})}{0 \text{ m/s} - 3.95 \text{ m/s}}$$

$$m_2 = \frac{187 \text{ kg}\cdot\text{m/s} - 199 \text{ kg}\cdot\text{m/s}}{-3.95 \text{ m/s}} = \frac{-12 \text{ kg}\cdot\text{m/s}}{-3.95 \text{ m/s}} = \boxed{3.0 \text{ kg}}$$

$$v_f = 5.00 \text{ m/s}$$

$$\text{b. } v_{1,i} = \frac{(m_1 + m_2)v_f - m_2 v_{2,i}}{m_1} = \frac{(47.4 \text{ kg} + 3.0 \text{ kg})(5.00 \text{ m/s}) - (3.0 \text{ kg})(0 \text{ m/s})}{47.4 \text{ kg}}$$

$$v_{1,i} = \frac{(50.4 \text{ kg})(5.00 \text{ m/s})}{47.4 \text{ kg}} = \boxed{5.32 \text{ m/s}}$$

Momentum and Collisions, Practice F

1. $m_1 = 0.25 \text{ kg}$
 $\mathbf{v}_{1,i} = 12 \text{ m/s to the west}$
 $= -12 \text{ m/s}$
 $m_2 = 6.8 \text{ kg}$
 $\mathbf{v}_{2,i} = 0 \text{ m/s}$

$$\text{a. } \mathbf{v}_f = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(0.25 \text{ kg})(-12 \text{ m/s}) + (6.8 \text{ kg})(0 \text{ m/s})}{0.25 \text{ kg} + 6.8 \text{ kg}}$$

$$\mathbf{v}_f = \frac{(0.25 \text{ kg})(-12 \text{ m/s})}{7.0 \text{ kg}} = -0.43 \text{ m/s} = \boxed{0.43 \text{ m/s to the west}}$$

$$\text{b. } \Delta KE = KE_f - KE_i$$

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}(0.25 \text{ kg})(-12 \text{ m/s})^2 + \frac{1}{2}(6.8 \text{ kg})(0 \text{ m/s})^2$$

$$KE_i = 18 \text{ J} + 0 \text{ J} = 18 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(0.25 \text{ kg} + 6.8 \text{ kg})(-0.43 \text{ m/s})^2$$

$$KE_f = \frac{1}{2}(7.0 \text{ kg})(-0.43 \text{ m/s})^2 = 0.65 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 0.65 \text{ J} - 18 \text{ J} = -17 \text{ J}$$

The kinetic energy decreases by $\boxed{17 \text{ J}}$.

2. $m_1 = 0.40 \text{ kg}$
 $\mathbf{v}_{1,i} = 8.5 \text{ m/s to the south}$
 $= -8.5 \text{ m/s}$
 $m_2 = 0.15 \text{ kg}$
 $\mathbf{v}_{2,i} = 0 \text{ m/s}$

$$\text{a. } \mathbf{v}_f = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(0.40 \text{ kg})(-8.5 \text{ m/s}) + (0.15 \text{ kg})(0 \text{ m/s})}{0.40 \text{ kg} + 0.15 \text{ kg}}$$

$$\mathbf{v}_f = \frac{(0.40 \text{ kg})(-8.5 \text{ m/s})}{0.55 \text{ kg}} = -6.2 \text{ m/s} = \boxed{6.2 \text{ m/s to the south}}$$

$$\text{b. } \Delta KE = KE_f - KE_i$$

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}(0.40 \text{ kg})(-8.5 \text{ m/s})^2 + \frac{1}{2}(0.15 \text{ kg})(0 \text{ m/s})^2$$

$$KE_i = 14 \text{ J} + 0 \text{ J} = 14 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(0.40 \text{ kg} + 0.15 \text{ kg})(-6.2 \text{ m/s})^2$$

$$KE_f = \frac{1}{2}(0.55 \text{ kg})(-6.2 \text{ m/s})^2 = 11 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 11 \text{ J} - 14 \text{ J} = -3 \text{ J}$$

The kinetic energy decreases by $\boxed{3 \text{ J}}$.

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3. $m_1 = 56 \text{ kg}$
 $\mathbf{v}_{1,i} = 4.0 \text{ m/s to the north}$
 $= +4.0 \text{ m/s}$
 $m_2 = 65 \text{ kg}$
 $\mathbf{v}_{2,i} = 12.0 \text{ m/s to the south}$
 $= -12.0 \text{ m/s}$

Solutions

$$\mathbf{a. v}_f = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(56 \text{ kg})(4.0 \text{ m/s}) + (65 \text{ kg})(-12.0 \text{ m/s})}{56 \text{ kg} + 65 \text{ kg}}$$

$$\mathbf{v}_f = \frac{220 \text{ kg}\cdot\text{m/s} - 780 \text{ kg}\cdot\text{m/s}}{121 \text{ kg}} = \frac{-560 \text{ kg}\cdot\text{m/s}}{121 \text{ kg}} = -4.6 \text{ m/s}$$

$$\mathbf{v}_f = \boxed{4.6 \text{ m/s to the south}}$$

$$\mathbf{b. \Delta KE = KE_f - KE_i}$$

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}(56 \text{ kg})(4.0 \text{ m/s})^2 + \frac{1}{2}(65 \text{ kg})(-12.0 \text{ m/s})^2$$

$$KE_i = 450 \text{ J} + 4700 \text{ J} = 5200 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(56 \text{ kg} + 65 \text{ kg})(-4.6 \text{ m/s})^2$$

$$KE_f = \frac{1}{2}(121 \text{ kg})(-4.6 \text{ m/s})^2 = 1300 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 1300 \text{ J} - 5200 \text{ J} = -3900 \text{ J}$$

The kinetic energy decreases by $\boxed{3.9 \times 10^3 \text{ J}}$.

Momentum and Collisions, Practice G

1. $m_1 = 0.015 \text{ kg}$
 $\mathbf{v}_{1,i} = 22.5 \text{ cm/s to the right}$
 $= +22.5 \text{ cm/s}$
 $m_2 = 0.015 \text{ kg}$
 $\mathbf{v}_{2,i} = 18.0 \text{ cm/s to the left}$
 $= -18.0 \text{ cm/s}$
 $\mathbf{v}_{1,f} = 18.0 \text{ cm/s to the left}$
 $= -18.0 \text{ cm/s}$

$$\mathbf{a. v}_{2,f} = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} - m_1 \mathbf{v}_{1,f}}{m_2}$$

$$m_1 = m_2$$

$$\mathbf{v}_{2,f} = \mathbf{v}_{1,i} + \mathbf{v}_{2,i} - \mathbf{v}_{1,f} = 22.5 \text{ cm/s} + (-18.0 \text{ cm/s}) - (-18.0 \text{ cm/s})$$

$$\mathbf{v}_{2,f} = 22.5 \text{ cm/s} = \boxed{22.5 \text{ cm/s to the right}}$$

$$\mathbf{b. KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2}$$

$$KE_i = \frac{1}{2}(0.015 \text{ kg})(0.225 \text{ m/s})^2 + \frac{1}{2}(0.015 \text{ kg})(-0.180 \text{ m/s})^2$$

$$KE_i = 3.8 \times 10^{-4} \text{ J} + 2.4 \times 10^{-4} \text{ J} = \boxed{6.2 \times 10^{-4} \text{ J}}$$

$$KE_f = \frac{1}{2}m_1 v_{1,f}^2 + \frac{1}{2}m_2 v_{2,f}^2$$

$$KE_f = \frac{1}{2}(0.015 \text{ kg})(-0.180 \text{ m/s})^2 + \frac{1}{2}(0.015 \text{ kg})(0.225 \text{ m/s})^2$$

$$KE_f = 2.4 \times 10^{-4} \text{ J} + 3.8 \times 10^{-4} \text{ J} = \boxed{6.2 \times 10^{-4} \text{ J}}$$

2. $m_1 = 16.0 \text{ kg}$
 $\mathbf{v}_{1,i} = 12.5 \text{ m/s to the left,}$
 $v_{1,i} = -12.5 \text{ m/s}$
 $m_2 = 14.0 \text{ kg}$
 $\mathbf{v}_{2,i} = 16.0 \text{ m/s to the right,}$
 $v_{2,i} = 16.0 \text{ m/s}$
 $\mathbf{v}_{2,f} = 14.4 \text{ m/s to the left,}$
 $v_{1,f} = -14.4 \text{ m/s}$

$$\mathbf{a. v}_{1,f} = \frac{(m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} - m_2 \mathbf{v}_{2,f})}{m_1}$$

$$v_{1,f} = \frac{(16.0 \text{ kg})(-12.5 \text{ m/s}) + (14.0 \text{ kg})(16.0 \text{ m/s}) - (14.0 \text{ kg})(-14.4 \text{ m/s})}{16.0 \text{ kg}}$$

$$v_{1,f} = \frac{-200 \text{ kg}\cdot\text{m/s} + 224 \text{ kg}\cdot\text{m/s} + 202 \text{ kg}\cdot\text{m/s}}{16.0 \text{ kg}} = 14.1 \text{ m/s}$$

$$v_{1,f} = \boxed{14.1 \text{ m/s to the right}}$$

Givens

Solutions

b. $KE_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$

$$KE_i = \frac{1}{2}(16.0 \text{ kg})(-12.5 \text{ m/s})^2 + \frac{1}{2}(14.0 \text{ kg})(16.0 \text{ m/s})^2$$

$$KE_i = 1.25 \times 10^3 \text{ J} + 1.79 \times 10^3 \text{ J} = \boxed{3.04 \times 10^3 \text{ J}}$$

$$KE_f = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

$$KE_f = \frac{1}{2}(16.0 \text{ kg})(14.1 \text{ m/s})^2 + \frac{1}{2}(14.0 \text{ kg})(-14.4 \text{ m/s})^2$$

$$KE_f = 1.59 \times 10^3 \text{ J} + 1.45 \times 10^3 \text{ J} = \boxed{3.04 \times 10^3 \text{ J}}$$

3. $m_1 = 4.0 \text{ kg}$

$$\mathbf{v}_{1,i} = 8.0 \text{ m/s to the right}$$

$$m_2 = 4.0 \text{ kg}$$

$$\mathbf{v}_{2,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{1,f} = 0 \text{ m/s}$$

a. $\mathbf{v}_{2,f} = \frac{m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} - m_1\mathbf{v}_{1,f}}{m_2}$

$$m_1 = m_2$$

$$\mathbf{v}_{2,f} = \mathbf{v}_{1,i} + \mathbf{v}_{2,i} - \mathbf{v}_{1,f} = 8.0 \text{ m/s} + 0 \text{ m/s} - 0 \text{ m/s}$$

$$\mathbf{v}_{2,f} = 8.0 \text{ m/s} = \boxed{8.0 \text{ m/s to the right}}$$

b. $KE_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$

$$KE_i = \frac{1}{2}(4.0 \text{ kg})(8.0 \text{ m/s})^2 + \frac{1}{2}(4.0 \text{ kg})(0 \text{ m/s})^2$$

$$KE_i = 130 \text{ J} + 0 \text{ J} = \boxed{1.3 \times 10^2 \text{ J}}$$

$$KE_f = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

$$KE_f = \frac{1}{2}(4.0 \text{ kg})(0 \text{ m/s})^2 + \frac{1}{2}(4.0 \text{ kg})(8.0 \text{ m/s})^2$$

$$KE_f = 0 \text{ J} + 130 \text{ J} = \boxed{1.3 \times 10^2 \text{ J}}$$

4. $m_1 = 25.0 \text{ kg}$

$$\mathbf{v}_{1,i} = 5.00 \text{ m/s to the right}$$

$$m_2 = 35.0 \text{ kg}$$

$$\mathbf{v}_{1,f} = 1.50 \text{ m/s to the right}$$

$$\mathbf{v}_{2,f} = 4.50 \text{ m/s to the right}$$

a. $\mathbf{v}_{2,i} = \frac{m_1\mathbf{v}_{1,f} + m_2\mathbf{v}_{2,f} - m_1\mathbf{v}_{1,i}}{m_2}$

$$\mathbf{v}_{2,i} = \frac{(25.0 \text{ kg})(1.50 \text{ m/s}) + (35.0 \text{ kg})(4.50 \text{ m/s}) - (25.0 \text{ kg})(5.00 \text{ m/s})}{35.0 \text{ kg}}$$

$$\mathbf{v}_{2,i} = \frac{37.5 \text{ kg}\cdot\text{m/s} + 158 \text{ kg}\cdot\text{m/s} - 125 \text{ kg}\cdot\text{m/s}}{35.0 \text{ kg}} = \frac{7.0 \times 10^1 \text{ kg}\cdot\text{m/s}}{35.0 \text{ kg}}$$

$$\mathbf{v}_{2,i} = 2.0 \text{ m/s} = \boxed{2.0 \text{ m/s to the right}}$$

b. $KE_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2$

$$KE_i = \frac{1}{2}(25.0 \text{ kg})(5.00 \text{ m/s})^2 + \frac{1}{2}(35.0 \text{ kg})(2.0 \text{ m/s})^2$$

$$KE_i = 312 \text{ J} + 7.0 \times 10^1 \text{ J} = \boxed{382 \text{ J}}$$

$$KE_f = \frac{1}{2}m_1v_{1,f}^2 + \frac{1}{2}m_2v_{2,f}^2$$

$$KE_f = \frac{1}{2}(25.0 \text{ kg})(1.50 \text{ m/s})^2 + \frac{1}{2}(35.0 \text{ kg})(4.50 \text{ m/s})^2$$

$$KE_f = 28.1 \text{ J} + 354 \text{ J} = \boxed{382 \text{ J}}$$

Momentum and Collisions, Section 3 Review

Givens

2. $m_1 = 95.0 \text{ kg}$
 $\mathbf{v}_{1,i} = 5.0 \text{ m/s}$ to the south,
 $v_{1,i} = -5.0 \text{ m/s}$
 $m_2 = 90.0 \text{ kg}$
 $\mathbf{v}_{2,i} = 3.0 \text{ m/s}$ to the north,
 $v_{2,i} = 3.0 \text{ m/s}$

Solutions

a. $\mathbf{v}_f = \frac{(m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i})}{m_1 + m_2}$

$$v_f = \frac{(95.0 \text{ kg})(-5.0 \text{ m/s}) + (90.0 \text{ kg})(3.0 \text{ m/s})}{95.0 \text{ kg} + 90.0 \text{ kg}}$$

$$v_f = \frac{-480 \text{ kg}\cdot\text{m/s} + 270 \text{ kg}\cdot\text{m/s}}{185 \text{ kg}} = \frac{-210 \text{ kg}\cdot\text{m/s}}{185 \text{ kg}} = -1.1 \text{ m/s}$$

$$v_f = \boxed{1.1 \text{ m/s to the south}}$$

b. $\Delta KE = KE_f - KE_i$

$$KE_i = \frac{1}{2}m_1v_{1,i}^2 + \frac{1}{2}m_2v_{2,i}^2 = \frac{1}{2}(95.0 \text{ kg})(-5.0 \text{ m/s})^2 + \frac{1}{2}(90.0 \text{ kg})(3.0 \text{ m/s})^2$$

$$KE_i = 1200 \text{ J} + 400 \text{ J} = 1600 \text{ J}$$

$$KE_f = \frac{1}{2}m_fv_{1,f}^2 = \frac{1}{2}(m_1 + m_2)v_{1,f}^2 = \frac{1}{2}(95.0 \text{ kg} + 90.0 \text{ kg})(1.1 \text{ m/s})^2$$

$$KE_f = \frac{1}{2}(185 \text{ kg})(1.2 \text{ m}^2/\text{s}^2) = 220 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 220 \text{ J} - 1600 \text{ J} = -1400 \text{ J}$$

The kinetic energy decreases by $1.4 \times 10^3 \text{ J}$.

3. $m_1 = m_2 = 0.40 \text{ kg}$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{2,i} = 3.5 \text{ m/s}$
 $v_{2,f} = 0 \text{ m/s}$

a. $v_{1,f} = \frac{m_1v_{1,i} + m_2v_{2,i} - m_2v_{2,f}}{m_1}$

$$m_1 = m_2$$

$$v_{1,f} = v_{1,i} + v_{2,i} - v_{2,f} = 0 \text{ m/s} + 3.5 \text{ m/s} - 0 \text{ m/s} = \boxed{3.5 \text{ m/s}}$$

b. $KE_{1,i} = \frac{1}{2}m_1v_{1,i}^2 = \frac{1}{2}(0.40 \text{ kg})(0 \text{ m/s})^2 = \boxed{0 \text{ J}}$

c. $KE_{2,f} = \frac{1}{2}m_2v_{2,f}^2 = \frac{1}{2}(0.40 \text{ kg})(0 \text{ m/s})^2 = \boxed{0 \text{ J}}$

Momentum and Collisions, Chapter Review

11. $m = 1.67 \times 10^{-27} \text{ kg}$
 $\mathbf{v} = 5.00 \times 10^6 \text{ m/s}$ straight up
 $m = 15.0 \text{ g}$
 $\mathbf{v} = 325 \text{ m/s}$ to the right
 $m = 75.0 \text{ kg}$
 $\mathbf{v} = 10.0 \text{ m/s}$ southwest
 $m = 5.98 \times 10^{24} \text{ kg}$
 $\mathbf{v} = 2.98 \times 10^4 \text{ m/s}$ forward
- a. $\mathbf{p} = m\mathbf{v} = (1.67 \times 10^{-27} \text{ kg})(5.00 \times 10^6 \text{ m/s}) = \boxed{8.35 \times 10^{-21} \text{ kg}\cdot\text{m/s}}$ upward
- b. $\mathbf{p} = m\mathbf{v} = (15.0 \times 10^{-3} \text{ kg})(325 \text{ m/s}) = \boxed{4.88 \text{ kg}\cdot\text{m/s}}$ to the right
- c. $\mathbf{p} = m\mathbf{v} = (75.0 \text{ kg})(10.0 \text{ m/s}) = \boxed{7.50 \times 10^2 \text{ kg}\cdot\text{m/s}}$ to the southwest
- d. $\mathbf{p} = m\mathbf{v} = (5.98 \times 10^{24} \text{ kg})(2.98 \times 10^4 \text{ m/s}) = \boxed{1.78 \times 10^{29} \text{ kg}\cdot\text{m/s}}$ forward

Givens

12. $m_1 = 2.5 \text{ kg}$

$\mathbf{v}_i = 8.5 \text{ m/s to the left}$
 $= -8.5 \text{ m/s}$

$\mathbf{v}_f = 7.5 \text{ m/s to the right}$
 $= +7.5 \text{ m/s}$

$\Delta t = 0.25 \text{ s}$

13. $m = 0.55 \text{ kg}$

$v_i = 0 \text{ m/s}$

$v_f = 8.0 \text{ m/s}$

$\Delta t = 0.25 \text{ s}$

14. $m = 0.15 \text{ kg}$

$v_i = 26 \text{ m/s}$

$v_f = 0 \text{ m/s}$

$F = -390 \text{ N}$

22. $m_1 = 65.0 \text{ kg}$

$\mathbf{v}_{1,i} = 2.50 \text{ m/s forward}$

$m_2 = 0.150 \text{ kg}$

$\mathbf{v}_{2,i} = 2.50 \text{ m/s forward}$

$\mathbf{v}_{2,f} = 32.0 \text{ m/s forward}$

$m_1 = 60.0 \text{ kg}$

$\mathbf{v}_{1,i} = 0 \text{ m/s}$

$m_2 = 0.150 \text{ kg}$

$\mathbf{v}_{2,i} = 32.0 \text{ m/s forward}$

23. $m_1 = 55 \text{ kg}$

$m_2 = 0.057 \text{ kg}$

$\mathbf{v}_{2,i} = 0 \text{ m/s}$

$\mathbf{v}_{1,i} = 0 \text{ m/s}$

$\mathbf{v}_{2,f} = 36 \text{ m/s to the north}$

Solutions

$$\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{\Delta t} = \frac{(2.5 \text{ kg})(7.5 \text{ m/s}) - (2.5 \text{ kg})(-8.5 \text{ m/s})}{0.25 \text{ s}}$$

$$\mathbf{F} = \frac{19 \text{ kg}\cdot\text{m/s} + 21 \text{ kg}\cdot\text{m/s}}{0.25 \text{ s}} = \frac{4.0 \times 10^1 \text{ kg}\cdot\text{m/s}}{0.25 \text{ s}} = \boxed{160 \text{ N to the right}}$$

$$F = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.55 \text{ kg})(8.0 \text{ m/s}) - (0.55 \text{ kg})(0 \text{ m/s})}{0.25 \text{ s}} = \frac{4.4 \text{ kg}\cdot\text{m/s}}{0.25 \text{ s}}$$

$$F = \boxed{18 \text{ N}}$$

$$\Delta t = \frac{mv_f - mv_i}{F} = \frac{(0.15 \text{ kg})(0 \text{ m/s}) - (0.15 \text{ kg})(26 \text{ m/s})}{-390 \text{ N}}$$

$$\Delta t = \frac{-(0.15 \text{ kg})(26 \text{ m/s})}{-390 \text{ N}} = \boxed{0.010 \text{ s}}$$

$$\Delta x = \frac{1}{2}(v_i + v_f)\Delta t = \frac{1}{2}(26.0 \text{ m/s} + 0 \text{ m/s})(0.010 \text{ s}) = \boxed{0.13 \text{ m}}$$

a. $\mathbf{v}_{1,f} = \frac{m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i} - m_2\mathbf{v}_{2,f}}{m_1}$

$$\mathbf{v}_{1,f} = \frac{(65.0 \text{ kg})(2.50 \text{ m/s}) + (0.150 \text{ kg})(2.50 \text{ m/s}) - (0.150 \text{ kg})(32.0 \text{ m/s})}{65.0 \text{ kg}}$$

$$\mathbf{v}_{1,f} = \frac{162 \text{ kg}\cdot\text{m/s} + 0.375 \text{ kg}\cdot\text{m/s} - 4.80 \text{ kg}\cdot\text{m/s}}{65.0 \text{ kg}} = \frac{162 \text{ kg}\cdot\text{m/s} - 4.42 \text{ kg}\cdot\text{m/s}}{65.0 \text{ kg}}$$

$$\mathbf{v}_{1,f} = \frac{158 \text{ kg}\cdot\text{m/s}}{65.0 \text{ kg}} = \boxed{2.43 \text{ m/s forward}}$$

b. $\mathbf{v}_f = \frac{m_1\mathbf{v}_{1,i} + m_2\mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(60.0 \text{ kg})(0 \text{ m/s}) + (0.150 \text{ kg})(32.0 \text{ m/s})}{60.0 \text{ kg} + 0.150 \text{ kg}}$

$$\mathbf{v}_f = \frac{(0.150 \text{ kg})(32.0 \text{ m/s})}{60.2 \text{ kg}} = \boxed{7.97 \times 10^{-2} \text{ m/s forward}}$$

Because the initial momentum is zero,

$$m_1\mathbf{v}_{1,f} = -m_2\mathbf{v}_{2,f}$$

$$\mathbf{v}_{1,f} = \frac{-m_2\mathbf{v}_{2,f}}{m_1} = \frac{-(0.057 \text{ kg})(36 \text{ m/s})}{55 \text{ kg}} = -0.037 \text{ m/s}$$

$$\mathbf{v}_{1,f} = \boxed{0.037 \text{ m/s to the south}}$$

Givens

28. $m_1 = 4.0 \text{ kg}$
 $m_2 = 3.0 \text{ kg}$
 $v_{1,i} = 5.0 \text{ m/s}$
 $v_{2,i} = -4.0 \text{ m/s}$

Solutions

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(4.0 \text{ kg})(5.0 \text{ m/s}) + (3.0 \text{ kg})(-4.0 \text{ m/s})}{4.0 \text{ kg} + 3.0 \text{ kg}}$$

$$v_f = \frac{2.0 \times 10^1 \text{ kg}\cdot\text{m/s} + (-12 \text{ kg}\cdot\text{m/s})}{7.0 \text{ kg}} = \frac{8 \text{ kg}\cdot\text{m/s}}{7.0 \text{ kg}} = \boxed{1 \text{ m/s}}$$

29. $m_1 = 1.20 \text{ kg}$
 $v_{1,i} = 5.00 \text{ m/s}$
 $m_2 = 0.800 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(1.20 \text{ kg})(5.00 \text{ m/s}) + (0.800 \text{ kg})(0 \text{ m/s})}{1.20 \text{ kg} + 0.800 \text{ kg}}$$

$$v_f = \frac{(1.20 \text{ kg})(5.00 \text{ m/s})}{2.00 \text{ kg}} = \boxed{3.00 \text{ m/s}}$$

30. $m_1 = 2.00 \times 10^4 \text{ kg}$
 $v_{1,i} = 3.00 \text{ m/s}$
 $m_2 = 2m_1$
 $v_{2,i} = 1.20 \text{ m/s}$

a. $v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(2.00 \times 10^4 \text{ kg})(3.00 \text{ m/s}) + (2)(2.00 \times 10^4 \text{ kg})(1.20 \text{ m/s})}{(2.00 \times 10^4 \text{ kg}) + (2)(2.00 \times 10^4 \text{ kg})}$

$$v_f = \frac{6.00 \times 10^4 \text{ kg}\cdot\text{m/s} + 4.80 \times 10^4 \text{ kg}\cdot\text{m/s}}{6.00 \times 10^4 \text{ kg}}$$

$$v_f = \frac{10.80 \times 10^4 \text{ kg}\cdot\text{m/s}}{6.00 \times 10^4 \text{ kg}} = \boxed{1.80 \text{ m/s}}$$

b. $\Delta KE = KE_f - KE_i$

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}(2.00 \times 10^4 \text{ kg})(3.00 \text{ m/s})^2 + \frac{1}{2}(2)(2.00 \times 10^4 \text{ kg})(1.20 \text{ m/s})^2$$

$$KE_i = 9.00 \times 10^4 \text{ J} + 2.88 \times 10^4 \text{ J} = 11.88 \times 10^4 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(2.00 \times 10^4 \text{ kg} + 4.00 \times 10^4 \text{ kg})(1.80 \text{ m/s})^2 = 9.72 \times 10^4 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 9.72 \times 10^4 \text{ J} - 11.88 \times 10^4 \text{ J} = -2.16 \times 10^4 \text{ J}$$

The kinetic energy decreases by $\boxed{2.16 \times 10^4 \text{ J}}$.

31. $m_1 = 88 \text{ kg}$
 $\mathbf{v}_{1,i} = 5.0 \text{ m/s to the east}$
 $= +5.0 \text{ m/s}$
 $m_2 = 97 \text{ kg}$
 $\mathbf{v}_{2,i} = 3.0 \text{ m/s to the west}$
 $= -3.0 \text{ m/s}$

a. $\mathbf{v}_f = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i}}{m_1 + m_2} = \frac{(88 \text{ kg})(5.0 \text{ m/s}) + (97 \text{ kg})(-3.0 \text{ m/s})}{88 \text{ kg} + 97 \text{ kg}}$

$$\mathbf{v}_f = \frac{440 \text{ kg}\cdot\text{m/s} - 290 \text{ kg}\cdot\text{m/s}}{185 \text{ kg}} = \frac{150 \text{ kg}\cdot\text{m/s}}{185 \text{ kg}} = \boxed{0.81 \text{ m/s to the east}}$$

b. $\Delta KE = KE_f - KE_i$

$$KE_i = \frac{1}{2}m_1 v_{1,i}^2 + \frac{1}{2}m_2 v_{2,i}^2 = \frac{1}{2}(88 \text{ kg})(5.0 \text{ m/s})^2 + \frac{1}{2}(97 \text{ kg})(-3.0 \text{ m/s})^2$$

$$KE_i = 1100 \text{ J} + 440 \text{ J} = 1.5 \times 10^3 \text{ J}$$

$$KE_f = \frac{1}{2}(m_1 + m_2)v_f^2 = \frac{1}{2}(88 \text{ kg} + 97 \text{ kg})(0.81 \text{ m/s})^2 = \frac{1}{2}(185 \text{ kg})(0.81 \text{ m/s})^2 = 61 \text{ J}$$

$$\Delta KE = KE_f - KE_i = 61 \text{ J} - 1.5 \times 10^3 \text{ J} = -1.4 \times 10^3 \text{ J}$$

The kinetic energy decreases by $\boxed{1.4 \times 10^3 \text{ J}}$.

Givens

32. $m_1 = 5.0 \text{ g}$

$$\mathbf{v}_{1,i} = 25.0 \text{ cm/s to the right}$$
$$= +25.0 \text{ cm/s}$$

$$m_2 = 15.0 \text{ g}$$

$$\mathbf{v}_{2,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{1,f} = 12.5 \text{ cm/s to the left}$$
$$= -12.5 \text{ cm/s}$$

Solutions

a.
$$\mathbf{v}_{2,f} = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} - m_1 \mathbf{v}_{1,f}}{m_2}$$
$$\mathbf{v}_{2,f} = \frac{(5.0 \text{ g})(25.0 \text{ cm/s}) + (15.0 \text{ g})(0 \text{ m/s}) - (5.0 \text{ g})(-12.5 \text{ cm/s})}{15.0 \text{ g}}$$

$$\mathbf{v}_{2,f} = \frac{120 \text{ g}\cdot\text{cm/s} + 62 \text{ g}\cdot\text{cm/s}}{15.0 \text{ g}} = \frac{180 \text{ g}\cdot\text{cm/s}}{15.0 \text{ g}} = 12 \text{ cm/s}$$

$$\mathbf{v}_{2,f} = \boxed{12 \text{ cm/s to the right}}$$

b.
$$\Delta KE_2 = KE_{2,f} - KE_{2,i} = \frac{1}{2}m_2 v_{2,f}^2 - \frac{1}{2}m_2 v_{2,i}^2$$
$$\Delta KE_2 = \frac{1}{2}(15.0 \times 10^{-3} \text{ kg})(0.12 \text{ m/s})^2 - \frac{1}{2}(15.0 \times 10^{-3} \text{ kg})(0 \text{ m/s})^2$$
$$\Delta KE_2 = \boxed{1.1 \times 10^{-4} \text{ J}}$$

33. $v_{1,i} = 4.0 \text{ m/s}$

$$v_{2,i} = 0 \text{ m/s}$$

$$m_1 = m_2$$

$$v_{1,f} = 0 \text{ m/s}$$

$$v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$$

$$m_1 = m_2$$

$$v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f} = 4.0 \text{ m/s} + 0 \text{ m/s} - 0 \text{ m/s} = \boxed{4.0 \text{ m/s}}$$

34. $m_1 = 25.0 \text{ g}$

$$\mathbf{v}_{1,i} = 20.0 \text{ cm/s to the right}$$

$$m_2 = 10.0 \text{ g}$$

$$\mathbf{v}_{2,i} = 15.0 \text{ cm/s to the right}$$

$$\mathbf{v}_{2,f} = 22.1 \text{ cm/s to the right}$$

$$\mathbf{v}_{1,f} = \frac{m_1 \mathbf{v}_{1,i} + m_2 \mathbf{v}_{2,i} - m_2 \mathbf{v}_{2,f}}{m_1}$$
$$\mathbf{v}_{1,f} = \frac{(25.0 \text{ g})(20.0 \text{ cm/s}) + (10.0 \text{ g})(15.0 \text{ cm/s}) - (10.0 \text{ g})(22.1 \text{ cm/s})}{25.0 \text{ g}}$$

$$\mathbf{v}_{1,f} = \frac{5.00 \times 10^2 \text{ g}\cdot\text{cm/s} + 1.50 \times 10^2 \text{ g}\cdot\text{cm/s} - 2.21 \times 10^2 \text{ g}\cdot\text{cm/s}}{25.0 \text{ g}}$$

$$\mathbf{v}_{1,f} = \frac{429 \text{ g}\cdot\text{cm/s}}{25.0 \text{ g}} = \boxed{17.2 \text{ cm/s to the right}}$$

35. $m = 0.147 \text{ kg}$

$$\mathbf{p} = 6.17 \text{ kg}\cdot\text{m/s toward}$$
$$\text{second base}$$

$$\mathbf{v} = \frac{\mathbf{p}}{m} = \frac{6.17 \text{ kg}\cdot\text{m/s}}{0.147 \text{ kg}} = \boxed{42.0 \text{ m/s toward second base}}$$

36. $KE = 150 \text{ J}$

$$p = 30.0 \text{ kg}\cdot\text{m/s}$$

$$KE = \frac{1}{2}mv^2$$

$$m = \frac{p}{v}$$

$$KE = \frac{1}{2}\left(\frac{p}{v}\right)v^2 = \frac{pv}{2}$$

$$v = \frac{2KE}{p} = \frac{(2)(150 \text{ J})}{30.0 \text{ kg}\cdot\text{m/s}} = \boxed{1.0 \times 10^1 \text{ m/s}}$$

$$m = \frac{p}{v} = \frac{30.0 \text{ kg}\cdot\text{m/s}}{1.0 \times 10^1 \text{ m/s}} = \boxed{3.0 \text{ kg}}$$

Givens

37. $m = 0.10 \text{ kg}$

$\mathbf{v}_i = 15.0 \text{ m/s}$ straight up

$a = -9.81 \text{ m/s}^2$

Solutions

a. At its maximum height, $\mathbf{v} = 0 \text{ m/s}$.

$$\mathbf{p} = m\mathbf{v} = (0.10 \text{ kg})(0 \text{ m/s}) = \boxed{0.0 \text{ kg}\cdot\text{m/s}}$$

b. Halfway to its maximum height (where $v_f = 0 \text{ m/s}$),

$$\Delta y = \left(\frac{1}{2}\right)\left(\frac{v_f^2 - v_i^2}{2a}\right) = \frac{(0 \text{ m/s})^2 - (15.0 \text{ m/s})^2}{(4)(-9.81 \text{ m/s}^2)} = 5.73 \text{ m}$$

Now let \mathbf{v}_f represent the velocity at $\Delta y = 5.73 \text{ m}$.

$$v_f = \pm\sqrt{v_i^2 + 2a\Delta x} = \pm\sqrt{(15.0 \text{ m/s})^2 + (2)(-9.81 \text{ m/s}^2)(5.73 \text{ m})}$$

$$v_f = \pm\sqrt{225 \text{ m}^2/\text{s}^2 - 112 \text{ m}^2/\text{s}^2} = \pm\sqrt{113 \text{ m}^2/\text{s}^2} = \pm 10.6 \text{ m/s}$$

$\mathbf{v}_f = 10.6 \text{ m/s}$ upward

$$\mathbf{p} = m\mathbf{v}_f = (0.10 \text{ kg})(10.6 \text{ m/s}) = \boxed{1.1 \text{ kg}\cdot\text{m/s upward}}$$

38. $m_1 = 3.00 \text{ kg}$

$v_{2,i} = 0 \text{ m/s}$

$v_f = \frac{1}{3}v_{1,i}$, or $v_{1,i} = 3v_f$

$$m_1v_{1,i} + m_2v_{2,i} = m_1v_f + m_2v_f$$

$$m_2(v_{2,i} - v_f) = m_1v_f - m_1v_{1,i}$$

$$m_2 = \frac{m_1v_f - m_1v_{1,i}}{v_{2,i} - v_f}, \text{ where } v_{2,i} = 0 \text{ m/s}$$

$$m_2 = \frac{m_1v_f - m_1(3v_f)}{-v_f} = -(m_1 - 3m_1) = -m_1 + 3m_1$$

$$m_2 = 2m_1 = (2)(3.00 \text{ kg}) = \boxed{6.00 \text{ kg}}$$

39. $m_1 = 5.5 \text{ g}$

$m_2 = 22.6 \text{ g}$

$v_{2,i} = 0 \text{ m/s}$

$\Delta y = -1.5 \text{ m}$

$\Delta x = 2.5 \text{ m}$

$a = -9.81 \text{ m/s}^2$

For an initial downward speed of zero,

$$\Delta y = \frac{1}{2}a\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta y}{a}}$$

$$v_f = \frac{\Delta x}{\Delta t} = \frac{\Delta x}{\sqrt{\frac{2\Delta y}{a}}} = \Delta x \sqrt{\frac{a}{2\Delta y}}$$

$$v_f = (2.5 \text{ m}) \sqrt{\frac{-9.81 \text{ m/s}^2}{(2)(-1.5 \text{ m})}} = 4.5 \text{ m/s}$$

$$v_{1,i} = \frac{(m_1 + m_2)v_f - m_2v_{2,i}}{m_1}$$

$$v_{1,i} = \frac{(5.5 \text{ g} + 22.6 \text{ g})(1 \text{ kg}/10^3 \text{ g})(4.5 \text{ m/s}) - (22.6 \times 10^{-3} \text{ kg})(0 \text{ m/s})}{5.5 \times 10^{-3} \text{ kg}}$$

$$v_{1,i} = \frac{(28.1 \times 10^{-3} \text{ kg})(4.5 \text{ m/s})}{5.5 \times 10^{-3} \text{ kg}} = \boxed{23 \text{ m/s}}$$

Givens

$$40. m_1 = \frac{730 \text{ N}}{9.81 \text{ m/s}^2}$$

$$R = 5.0 \text{ m}$$

$$m_2 = 2.6 \text{ kg}$$

$$\mathbf{v}_{1,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{2,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{2,f} = 5.0 \text{ m/s to the north}$$

Solutions

Because the initial momentum is zero,

$$\mathbf{v}_{1,f} = \frac{-m_2 \mathbf{v}_{2,f}}{m_1} = \frac{-(2.6 \text{ kg})(5.0 \text{ m/s})}{\left(\frac{730 \text{ N}}{9.81 \text{ m/s}^2}\right)} = -0.17 \text{ m/s} = 0.17 \text{ m/s to the south}$$

$$\Delta t = \frac{\Delta x}{v_{1,f}} = \frac{-R}{v_{1,f}} = \frac{-5.0 \text{ m}}{-0.17 \text{ m/s}} = \boxed{29 \text{ s}}$$

$$41. m = 0.025 \text{ kg}$$

$$v_i = 18.0 \text{ m/s}$$

$$\Delta t = 5.0 \times 10^{-4} \text{ s}$$

$$v_f = 10.0 \text{ m/s}$$

$$F = \frac{mv_f - mv_i}{\Delta t} = \frac{(0.025 \text{ kg})(10.0 \text{ m/s}) - (0.025 \text{ kg})(18.0 \text{ m/s})}{5.0 \times 10^{-4} \text{ s}}$$

$$F = \frac{0.25 \text{ kg}\cdot\text{m/s} - 0.45 \text{ kg}\cdot\text{m/s}}{5.0 \times 10^{-4} \text{ s}} = \frac{-0.20 \text{ kg}\cdot\text{m/s}}{5.0 \times 10^{-4} \text{ s}} = -4.0 \times 10^2 \text{ N}$$

$$\text{magnitude of the force} = \boxed{4.0 \times 10^2 \text{ N}}$$

$$42. m_1 = 1550 \text{ kg}$$

$$\mathbf{v}_{1,i} = 10.0 \text{ m/s to the south}$$

$$= -10.0 \text{ m/s}$$

$$m_2 = 2550 \text{ kg}$$

$$\mathbf{v}_f = 5.22 \text{ m/s to the north}$$

$$= +5.22 \text{ m/s}$$

$$\mathbf{v}_{2,i} = \frac{(m_1 + m_2)\mathbf{v}_f - m_1\mathbf{v}_{1,i}}{m_2} = \frac{(1550 \text{ kg} + 2550 \text{ kg})(5.22 \text{ m/s}) - (1550 \text{ kg})(-10.0 \text{ m/s})}{2550 \text{ kg}}$$

$$\mathbf{v}_{2,i} = \frac{(4.10 \times 10^3 \text{ kg})(5.22 \text{ m/s}) - (1550 \text{ kg})(-10.0 \text{ m/s})}{2550 \text{ kg}}$$

$$\mathbf{v}_{2,i} = \frac{2.14 \times 10^4 \text{ kg}\cdot\text{m/s} + 1.55 \times 10^4 \text{ kg}\cdot\text{m/s}}{2550 \text{ kg}} = \frac{3.69 \times 10^4 \text{ kg}\cdot\text{m/s}}{2550 \text{ kg}}$$

$$\mathbf{v}_{2,i} = 14.5 \text{ m/s} = \boxed{14.5 \text{ m/s to the north}}$$

$$43. m_1 = 52.0 \text{ g}$$

$$m_2 = 153 \text{ g}$$

$$v_{1,i} = 0 \text{ m/s}$$

$$v_{2,i} = 0 \text{ m/s}$$

$$v_{1,f} = 2.00 \text{ m/s}$$

$$g = 9.81 \text{ m/s}^2$$

Because the initial momentum is zero,

$$v_{2,f} = \frac{-m_1 v_{1,f}}{m_2} = \frac{-(52.0 \text{ g})(2.00 \text{ m/s})}{153 \text{ g}} = -0.680 \text{ m/s}$$

$$KE_i = PE_f$$

$$\frac{1}{2}mv_{2,f}^2 = mgh$$

$$h = \frac{v_{2,f}^2}{2g} = \frac{(-0.680 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)} = \boxed{2.36 \times 10^{-2} \text{ m} = 2.36 \text{ cm}}$$

$$44. m_1 = 85.0 \text{ kg}$$

$$m_2 = 0.500 \text{ kg}$$

$$\mathbf{v}_{1,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{2,i} = 0 \text{ m/s}$$

$$\mathbf{v}_{2,f} = 20.0 \text{ m/s away from ship}$$

$$\text{ship} = -20.0 \text{ m/s}$$

$$\Delta x = 30.0 \text{ m}$$

Because the initial momentum is zero,

$$\mathbf{v}_{1,f} = \frac{-m_2 \mathbf{v}_{2,f}}{m_1} = \frac{-(0.500 \text{ kg})(-20.0 \text{ m/s})}{85.0 \text{ kg}} = 0.118 \text{ m/s toward the ship}$$

$$\Delta t = \frac{\Delta x}{v_{1,f}} = \frac{30.0 \text{ m}}{0.118 \text{ m/s}} = \boxed{254 \text{ s}}$$

Givens

45. $m_1 = 2250 \text{ kg}$
 $v_{1,i} = 10.0 \text{ m/s}$
 $m_2 = 2750 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $d = 2.50 \text{ m}$
 $\theta = 180^\circ$
 $g = 9.81 \text{ m/s}^2$

Solutions

$$v_f = \frac{m_1 v_{1,i} + m_2 v_{2,i}}{m_1 + m_2} = \frac{(2250 \text{ kg})(10.0 \text{ m/s}) + (2750 \text{ kg})(0 \text{ m/s})}{2250 \text{ kg} + 2750 \text{ kg}}$$

$$v_f = \frac{(2250 \text{ kg})(10.0 \text{ m/s})}{5.00 \times 10^3 \text{ kg}} = 4.50 \text{ m/s}$$

From the work-kinetic energy theorem,

$$W_{net} = \Delta KE = KE_f - KE_i = \frac{1}{2}(m_1 + m_2)(v_f')^2 - \frac{1}{2}(m_1 + m_2)(v_i')^2$$

where

$$v_i' = 4.50 \text{ m/s} \quad v_f' = 0 \text{ m/s}$$

$$W_{net} = W_{friction} = F_k d (\cos \theta) = (m_1 + m_2) g \mu_k d (\cos \theta)$$

$$(m_1 + m_2) g \mu_k d (\cos \theta) = \frac{1}{2}(m_1 + m_2)[(v_f')^2 - (v_i')^2]$$

$$\mu_k = \frac{(v_f')^2 - (v_i')^2}{2 g d (\cos \theta)} = \frac{(0 \text{ m/s})^2 - (4.50 \text{ m/s})^2}{(2)(9.81 \text{ m/s}^2)(2.50 \text{ m})(\cos 180^\circ)} = \frac{-(4.50 \text{ m/s})^2}{-(2)(9.81 \text{ m/s}^2)(2.50 \text{ m})}$$

$$\mu_k = \boxed{0.413}$$

46. $F = 2.5 \text{ N}$ to the right

$$m = 1.5 \text{ kg}$$

$$\Delta t = 0.50 \text{ s}$$

$$v_i = 0 \text{ m/s}$$

$$v_i = 2.0 \text{ m/s to the left}$$

$$= -2.0 \text{ m/s}$$

$$\text{a. } v_f = \frac{F \Delta t + m v_i}{m} = \frac{(2.5 \text{ N})(0.50 \text{ s}) + (1.5 \text{ kg})(0 \text{ m/s})}{1.5 \text{ kg}}$$

$$v_f = 0.83 \text{ m/s} \quad \boxed{0.83 \text{ m/s to the right}}$$

$$\text{b. } v_f = \frac{F \Delta t + m v_i}{m} = \frac{(2.5 \text{ N})(0.50 \text{ s}) + (1.5 \text{ kg})(-2.0 \text{ m/s})}{1.5 \text{ kg}}$$

$$v_f = \frac{1.2 \text{ N} \cdot \text{s} + (-3.0 \text{ kg} \cdot \text{m/s})}{1.5 \text{ kg}} = \frac{-1.8 \text{ kg} \cdot \text{m/s}}{1.5 \text{ kg}} = -1.2 \text{ m/s}$$

$$v_f = \boxed{1.2 \text{ m/s to the left}}$$

47. $m_1 = m_2$

$$v_{1,i} = 22 \text{ cm/s}$$

$$v_{2,i} = -22 \text{ cm/s}$$

Because $m_1 = m_2$, $v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f}$.

Because kinetic energy is conserved and the two masses are equal,

$$\frac{1}{2} v_{1,i}^2 + \frac{1}{2} v_{2,i}^2 = \frac{1}{2} v_{1,f}^2 + \frac{1}{2} v_{2,f}^2$$

$$v_{1,i}^2 + v_{2,i}^2 = v_{1,f}^2 + v_{2,f}^2$$

$$v_{1,i}^2 + v_{2,i}^2 = v_{1,f}^2 + (v_{1,i} + v_{2,i} - v_{1,f})^2$$

$$(22 \text{ cm/s})^2 + (-22 \text{ cm/s})^2 = v_{1,f}^2 + (22 \text{ cm/s} - 22 \text{ cm/s} - v_{1,f})^2$$

$$480 \text{ cm}^2/\text{s}^2 + 480 \text{ cm}^2/\text{s}^2 = 2 v_{1,f}^2$$

$$v_{1,f} = \pm \sqrt{480 \text{ cm}^2/\text{s}^2} = \pm 22 \text{ cm/s}$$

Because m_1 cannot pass through m_2 , it follows that $v_{1,f}$ is opposite $v_{1,i}$.

$$v_{1,f} = \boxed{-22 \text{ cm/s}}$$

$$v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f}$$

$$v_{2,f} = 22 \text{ cm/s} + (-22 \text{ cm/s}) - (-22 \text{ cm/s}) = \boxed{22 \text{ cm/s}}$$

Givens

48. $m_1 = 7.50 \text{ kg}$
 $\Delta y = -3.00 \text{ m}$
 $m_2 = 5.98 \times 10^{24} \text{ kg}$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{2,i} = 0 \text{ m/s}$
 $a = -9.81 \text{ m/s}^2$

49. $m = 55 \text{ kg}$

$\Delta y = -5.0 \text{ m}$
 $\Delta t = 0.30 \text{ s}$
 $\mathbf{v}_i = 0 \text{ m/s}$
 $a = -9.81 \text{ m/s}^2$
 $\mathbf{v}_i = 9.9 \text{ m/s downward}$
 $= -9.9 \text{ m/s}$
 $\mathbf{v}_f = 0 \text{ m/s}$

50. $m_{nuc} = 17.0 \times 10^{-27} \text{ kg}$
 $m_1 = 5.0 \times 10^{-27} \text{ kg}$
 $m_2 = 8.4 \times 10^{-27} \text{ kg}$
 $\mathbf{v}_{nuc,i} = 0 \text{ m/s}$
 $\mathbf{v}_{1,f} = 6.0 \times 10^6 \text{ m/s}$ along the positive y -axis
 $\mathbf{v}_{2,f} = 4.0 \times 10^6 \text{ m/s}$ along the positive x -axis

Solutions

a. $v_{1,f} = \pm\sqrt{2a\Delta y} = \pm\sqrt{(2)(-9.81 \text{ m/s}^2)(-3.00 \text{ m})} = \pm 7.67 \text{ m/s} = -7.67 \text{ m/s}$

Because the initial momentum is zero,

$$m_1 v_{1,f} = -m_2 v_{2,f}$$

$$v_{2,f} = \frac{-m_1 v_{1,f}}{m_2} = \frac{-(7.50 \text{ kg})(-7.67 \text{ m/s})}{5.98 \times 10^{24} \text{ kg}} = \boxed{9.62 \times 10^{-24} \text{ m/s}}$$

a. $v_f = \pm\sqrt{2a\Delta y} = \pm\sqrt{(2)(-9.81 \text{ m/s}^2)(-5.0 \text{ m})} = \pm 9.9 \text{ m/s}$

$$\mathbf{v}_f = -9.9 \text{ m/s} = \boxed{9.9 \text{ m/s downward}}$$

b. $\mathbf{F} = \frac{m\mathbf{v}_f - m\mathbf{v}_i}{\Delta t} = \frac{(55 \text{ kg})(0 \text{ m/s}) - (55 \text{ kg})(-9.9 \text{ m/s})}{0.30 \text{ s}}$

$$\mathbf{F} = 1.8 \times 10^3 \text{ N} = \boxed{1.8 \times 10^3 \text{ N upward}}$$

$$m_3 = m_{nuc} - (m_1 + m_2) = (17.0 \times 10^{-27} \text{ kg}) - [(5.0 \times 10^{-27} \text{ kg}) + (8.4 \times 10^{-27} \text{ kg})]$$

$$m_3 = 3.6 \times 10^{-27} \text{ kg}$$

$$p_1 = m_1 v_{1,f} = (5.0 \times 10^{-27} \text{ kg})(6.0 \times 10^6 \text{ m/s}) = 3.0 \times 10^{-20} \text{ kg}\cdot\text{m/s}$$

$$p_2 = m_2 v_{2,f} = (8.4 \times 10^{-27} \text{ kg})(4.0 \times 10^6 \text{ m/s}) = 3.4 \times 10^{-20} \text{ kg}\cdot\text{m/s}$$

Because the initial momentum is zero, the final momentum must also equal zero.

$$\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_3 = 0 \text{ kg}\cdot\text{m/s}$$

$$\mathbf{p}_3 = -(\mathbf{p}_1 + \mathbf{p}_2)$$

Because \mathbf{p}_1 and \mathbf{p}_2 are along the y -axis and the x -axis, respectively, the magnitude of \mathbf{p}_3 can be found by using the Pythagorean theorem.

$$p_3 = \sqrt{p_1^2 + p_2^2} = \sqrt{(3.0 \times 10^{-20} \text{ kg}\cdot\text{m/s})^2 + (3.4 \times 10^{-20} \text{ kg}\cdot\text{m/s})^2}$$

$$p_3 = \sqrt{(9.0 \times 10^{-40} \text{ kg}^2\cdot\text{m}^2/\text{s}^2) + (1.2 \times 10^{-39} \text{ kg}^2\cdot\text{m}^2/\text{s}^2)}$$

$$p_3 = \sqrt{(21 \times 10^{-40} \text{ kg}^2\cdot\text{m}^2/\text{s}^2)} = 4.6 \times 10^{-20} \text{ kg}\cdot\text{m/s}$$

$$v_{3,f} = \frac{p_3}{m_3} = \frac{4.6 \times 10^{-20} \text{ kg}\cdot\text{m/s}}{3.6 \times 10^{-27} \text{ kg}} = \boxed{1.3 \times 10^7 \text{ m/s}}$$

Because $\mathbf{p}_{1,2}$ is between the positive x -axis and the positive y -axis and because $\mathbf{p}_3 = -\mathbf{p}_{1,2}$, \mathbf{p}_3 must be between the negative x -axis and the negative y -axis.

$$\tan \theta = \frac{p_1}{p_2}$$

$$\theta = \tan^{-1}\left(\frac{p_1}{p_2}\right) = \tan^{-1}\left(\frac{3.0}{3.4}\right) = \boxed{41^\circ \text{ below the negative } x\text{-axis}}$$

Momentum and Collisions, Standardized Test Prep

Givens

Solutions

3. $m = 0.148 \text{ kg}$
 $v = 35 \text{ m/s}$ toward home plate

$$p = mv = (0.148 \text{ kg})(35 \text{ m/s}) = \boxed{5.2 \text{ kg}\cdot\text{m/s}} \text{ toward home plate}$$

4. $m_1 = 1.5 \text{ kg}$
 $v_{1,i} = 3.0 \text{ m/s}$ to the right
 $m_2 = 1.5 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $v_{1,f} = 0.5 \text{ m/s}$ to the right

$$\text{a. } v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$$

$$m_1 = m_2$$

$$v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f}$$

$$v_{2,f} = 3.0 \text{ m/s} + 0 \text{ m/s} - 0.5 \text{ m/s} = \boxed{2.5 \text{ m/s}} \text{ to the right}$$

5. $v_{1,f} = 0 \text{ m/s}$

$$v_{2,f} = v_{1,i} + v_{2,i} - v_{1,f} = 3.0 \text{ m/s} + 0 \text{ m/s} - 0 \text{ m/s} = \boxed{3.0 \text{ m/s}} \text{ to the right}$$

8. $m_1 = m_2$
 $v_{1,i} = 0 \text{ m/s}$
 $v_{2,i} = 5.00 \text{ m/s}$ to the right
 $v_{2,f} = 0 \text{ m/s}$

$$v_{1,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_2 v_{2,f}}{m_1}$$

$$m_1 = m_2$$

$$v_{1,f} = v_{1,i} + v_{2,i} - v_{2,f} = 0 \text{ m/s} + 5.00 \text{ m/s} - 0 \text{ m/s}$$

$$v_{1,f} = \boxed{5.00 \text{ m/s}} \text{ to the right}$$

9. $m_1 = 0.400 \text{ kg}$
 $v_{1,i} = 3.50 \text{ cm/s}$ right,
 $v_{1,i} = 3.50 \text{ cm/s}$
 $m_2 = 0.600 \text{ kg}$
 $v_{2,i} = 0$
 $v_{1,f} = 0.07 \text{ cm/s}$ left,
 $v_{1,f} = -0.70 \text{ cm/s}$

$$v_{2,f} = \frac{m_1 v_{1,i} + m_2 v_{2,i} - m_1 v_{1,f}}{m_2}$$

$$v_{2,f} = \frac{(0.400 \text{ kg})(3.50 \text{ cm/s}) + (0.600 \text{ kg})(0) - (0.400 \text{ kg})(-0.70 \text{ cm/s})}{0.600 \text{ kg}}$$

$$v_{2,f} = \frac{1.40 \text{ kg}\cdot\text{cm/s} + 0.28 \text{ kg}\cdot\text{cm/s}}{0.600 \text{ kg}} = \frac{1.68 \text{ kg}\cdot\text{cm/s}}{0.600 \text{ kg}} = 2.80 \text{ cm/s}$$

$$v_{2,f} = \boxed{2.80 \text{ cm/s}} \text{ to the right}$$

10. $m_1 = 0.400 \text{ kg}$
 $v_{1,f} = -0.70 \times 10^{-2} \text{ m/s}$
 $m_2 = 0.600 \text{ kg}$
 $v_{2,f} = 2.80 \times 10^{-2} \text{ m/s}$

$$KE_f = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2$$

$$KE_f = \frac{1}{2} (0.400 \text{ kg})(-0.70 \times 10^{-2} \text{ m/s})^2 + \frac{1}{2} (0.600 \text{ kg})(2.80 \times 10^{-2} \text{ m/s})^2$$

$$KE_f = 9.8 \times 10^{-6} \text{ kg}\cdot\text{m}^2/\text{s}^2 + 2.35 \times 10^{-4} \text{ kg}\cdot\text{m}^2/\text{s}^2$$

$$KE_f = \boxed{2.45 \times 10^{-4} \text{ J}}$$

13. $m_1 = 8.0 \text{ g}$
 $m_2 = 2.5 \text{ kg}$
 $v_{2,i} = 0 \text{ m/s}$
 $h = 6.0 \text{ cm}$
 $g = 9.81 \text{ m/s}^2$

$$KE_i = PE_f$$

$$\frac{1}{2} m v_f^2 = mgh$$

$$v_f = \sqrt{2gh} = \sqrt{(2)(9.81 \text{ m/s}^2)(0.060 \text{ m})} = 1.1 \text{ m/s}$$

$$v_{1,i} = \frac{(m_1 + m_2)v_f - m_2 v_{2,i}}{m_1} = \frac{(0.0080 \text{ kg} + 2.5 \text{ kg})(1.1 \text{ m/s}) - (2.5 \text{ kg})(0 \text{ m/s})}{0.0080 \text{ kg}}$$

$$v_{1,i} = \frac{(2.5 \text{ kg})(1.1 \text{ m/s})}{0.0080 \text{ kg}} = \boxed{340 \text{ m/s}}$$

Givens

14. $m_1 = 8.0 \text{ g} = 0.0080 \text{ kg}$

$$m_2 = 2.5 \text{ kg}$$

$$g = 9.81 \text{ m/s}^2$$

$$h = 6.0 \text{ cm} = 0.060 \text{ m}$$

Solutions

$$KE_{\text{lowest}} = KE_i = PE_f = mgh$$

$$KE_{\text{lowest}} = (m_1 + m_2)gh$$

$$KE_{\text{lowest}} = (0.0080 \text{ kg} + 2.5 \text{ kg})(9.81 \text{ m/s}^2)(0.060 \text{ m})$$

$$KE_{\text{lowest}} = \boxed{1.5 \text{ J}}$$