

Two-Dimensional Motion and Vectors

Additional Practice A

Givens

1. $\Delta t_x = 7.95 \text{ s}$
 $\Delta y = 161 \text{ m}$
 $d = 226 \text{ m}$

Solutions

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(226 \text{ m})^2 - (161 \text{ m})^2} = \sqrt{5.11 \times 10^4 \text{ m}^2 - 2.59 \times 10^4 \text{ m}^2}$$

$$\Delta x = \sqrt{2.52 \times 10^4 \text{ m}^2} = 159 \text{ m}$$

$$\Delta x = \boxed{159 \text{ m}}$$

$$v = \frac{\Delta x}{\Delta t_x} = \frac{159 \text{ m}}{7.95 \text{ s}} = \boxed{20.0 \text{ m/s}}$$

2. $d_1 = 5.0 \text{ km}$
 $\theta_1 = 11.5^\circ$
 $d^2 = 1.0 \text{ km}$
 $\theta_2 = -90.0^\circ$

$$\Delta x_{tot} = d_1(\cos \theta_1) + d_2(\cos \theta_2) = (5.0 \text{ km})(\cos 11.5^\circ) + (1.0 \text{ km})[\cos(-90.0^\circ)]$$

$$\Delta x_{tot} = 4.9 \text{ km}$$

$$\Delta y_{tot} = d_1(\sin \theta_1) + d_2(\sin \theta_2) = (5.0 \text{ km})(\sin 11.5^\circ) + (1.0 \text{ km})[\sin(-90.0^\circ)]$$

$$= 1.0 \text{ km} - 1.0 \text{ km}$$

$$\Delta y_{tot} = 0.0 \text{ km}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(4.9 \text{ km})^2 + (0.0 \text{ km})^2}$$

$$d = \boxed{4.9 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left(\frac{0.0 \text{ km}}{4.9 \text{ km}} \right) = \boxed{0.0^\circ, \text{ or due east}}$$

3. $\Delta x = 5 \text{ jumps}$
 1 jump = 8.0 m
 $d = 68 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2$$

$$\Delta y = \sqrt{d^2 - \Delta x^2} = \sqrt{(68 \text{ m})^2 - [(5)(8.0 \text{ m})]^2} = \sqrt{4.6 \times 10^3 \text{ m}^2 - 1.6 \times 10^3 \text{ m}^2}$$

$$\Delta y = \sqrt{3.0 \times 10^3 \text{ m}^2} = 55 \text{ m}$$

$$\text{number of jumps northward} = \frac{55 \text{ m}}{8.0 \text{ m/jump}} = 6.9 \text{ jumps} = \boxed{7 \text{ jumps}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta x}{\Delta y} \right) = \tan^{-1} \left[\frac{(5)(8.0 \text{ m})}{55 \text{ m}} \right] = \boxed{36^\circ \text{ west of north}}$$

4. $\Delta x = 25.2 \text{ km}$
 $\Delta y = 21.3 \text{ km}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(25.2 \text{ km})^2 + (21.3 \text{ km})^2}$$

$$d = \sqrt{635 \text{ km}^2 + 454 \text{ km}^2} = \sqrt{1089 \text{ km}^2}$$

$$d = \boxed{33.00 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{21.3 \text{ km}}{25.2 \text{ km}} \right)$$

$$\theta = \boxed{42.6^\circ \text{ south of east}}$$

Givens

5. $\Delta y = -483 \text{ m}$
 $\Delta x = 225 \text{ m}$

Solutions

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{-483}{225} \right) = -65.0^\circ = \boxed{65.0^\circ \text{ below the waters surface}}$$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(225 \text{ m})^2 + (-483 \text{ m})^2}$$

$$d = \sqrt{5.06 \times 10^4 \text{ m}^2 + 2.33 \times 10^5 \text{ m}^2} = \sqrt{2.84 \times 10^5 \text{ m}^2}$$

$$d = \boxed{533 \text{ m}}$$

6. $v = 15.0 \text{ m/s}$
 $\Delta t_x = 8.0 \text{ s}$
 $d = 180.0 \text{ m}$

$$d^2 = \Delta x^2 + \Delta y^2 = (v\Delta t_x)^2 + (v\Delta t_y)^2$$

$$d^2 = v^2(\Delta t_x^2 + \Delta t_y^2)$$

$$\Delta t_y = \sqrt{\left(\frac{d}{v}\right)^2 - \Delta t_x^2} = \sqrt{\left(\frac{180.0 \text{ m}}{15.0 \text{ m/s}}\right)^2 - (8.0 \text{ s})^2} = \sqrt{144 \text{ s}^2 - 64 \text{ s}^2} = \sqrt{8.0 \times 10^1 \text{ s}^2}$$

$$\Delta t_y = \boxed{8.9 \text{ s}}$$

7. $v = 8.00 \text{ km/h}$
 $\Delta t_x = 15.0 \text{ min}$
 $\Delta t_y = 22.0 \text{ min}$

$$d = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(v\Delta t_x)^2 + (v\Delta t_y)^2}$$

$$= v\sqrt{\Delta t_x^2 + \Delta t_y^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{(15.0 \text{ min})^2 + (22.0 \text{ min})^2}$$

$$d = (8.00 \text{ km/h}) \left(\frac{1 \text{ h}}{60 \text{ min}} \right) \sqrt{225 \text{ min}^2 + 484 \text{ min}^2}$$

$$d = \left(\frac{8.00 \text{ km}}{60 \text{ min}} \right) \sqrt{709 \text{ min}^2} = \boxed{3.55 \text{ km}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y}{\Delta x} \right) = \tan^{-1} \left(\frac{v\Delta t_y}{v\Delta t_x} \right) = \tan^{-1} \left(\frac{\Delta t_y}{\Delta t_x} \right) = \tan^{-1} \left(\frac{22.0 \text{ min}}{15.0 \text{ min}} \right)$$

$$\theta = \boxed{55.7^\circ \text{ north of east}}$$

Additional Practice B

1. $d = (5)(33.0 \text{ cm})$
 $\Delta y = 88.0 \text{ cm}$

$$\theta = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left[\frac{88.0 \text{ cm}}{(5)(33.0 \text{ cm})} \right] = \boxed{32.2^\circ \text{ north of west}}$$

$$\Delta x = d(\cos \theta) = (5)(33.0 \text{ cm})(\cos 32.2^\circ) = \boxed{1.40 \times 10^2 \text{ cm to the west}}$$

2. $\theta = 60.0^\circ$
 $d = 10.0 \text{ m}$

$$\Delta x = d(\cos \theta) = (10.0 \text{ m})(\cos 60.0^\circ) = \boxed{5.00 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (10.0 \text{ m})(\sin 60.0^\circ) = \boxed{8.66 \text{ m}}$$

3. $d = 10.3 \text{ m}$
 $\Delta y = -6.10 \text{ m}$

Finding the angle between d and the x -axis yields,

$$\theta_1 = \sin^{-1} \left(\frac{\Delta y}{d} \right) = \sin^{-1} \left(\frac{-6.10 \text{ m}}{10.3 \text{ m}} \right) = -36.3^\circ$$

The angle between d and the negative y -axis is therefore,

$$\theta = -90.0 - (-36.3^\circ) = -53.7^\circ$$

$$\theta = \boxed{53.7^\circ \text{ on either side of the negative } y\text{-axis}}$$

$$d^2 + \Delta x^2 + \Delta y^2$$

$$\Delta x = \sqrt{d^2 - \Delta y^2} = \sqrt{(10.3 \text{ m})^2 - (-6.10 \text{ m})^2} = \sqrt{106 \text{ m}^2 - 37.2 \text{ m}^2} = \sqrt{69 \text{ m}^2}$$

$$\Delta x = \boxed{\pm 8.3 \text{ m}}$$

Givens

4. $d = (8)(4.5 \text{ m})$
 $\theta = 35^\circ$

$$\Delta x = d(\cos \theta) = (8)(4.5 \text{ m})(\cos 35^\circ) = \boxed{29 \text{ m}}$$

$$\Delta y = d(\sin \theta) = (8)(4.5 \text{ m})(\sin 35^\circ) = \boxed{21 \text{ m}}$$

5. $v = 347 \text{ km/h}$
 $\theta = 15.0^\circ$

$$v_x = v(\cos \theta) = (347 \text{ km/h})(\cos 15.0^\circ) = \boxed{335 \text{ km/h}}$$

$$v_y = v(\sin \theta) = (347 \text{ km/h})(\sin 15.0^\circ) = \boxed{89.8 \text{ km/h}}$$

6. $v = 372 \text{ km/h}$
 $\Delta t = 8.7 \text{ s}$
 $\theta = 60.0^\circ$

$$d = v\Delta t = (372 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})(8.7 \text{ s}) = 9.0 \times 10^2 \text{ m}$$

$$\Delta x = d(\cos \theta) = (9.0 \times 10^2 \text{ m})(\cos 60.0^\circ) = \boxed{450 \text{ m east}}$$

$$\Delta y = d(\sin \theta) = (9.0 \times 10^2 \text{ m})(\sin 60.0^\circ) = \boxed{780 \text{ m north}}$$

7. $d = 14\,890 \text{ km}$
 $\theta = 25.0^\circ$
 $\Delta t = 18.5 \text{ h}$

$$v_{avg} = \frac{d}{\Delta t} = \frac{1.489 \times 10^4 \text{ km}}{18.45 \text{ h}} = \boxed{805 \text{ km/h}}$$

$$v_x = v_{avg}(\cos \theta) = (805 \text{ km/h})(\cos 25.0^\circ) = \boxed{730 \text{ km/h east}}$$

$$v_y = v_{avg}(\sin \theta) = (805 \text{ km/h})(\sin 25.0^\circ) = \boxed{340 \text{ km/h south}}$$

8. $v_i = 6.0 \times 10^2 \text{ km/h}$
 $v_f = 2.3 \times 10^3 \text{ km/h}$
 $\Delta t = 120 \text{ s}$
 $\theta = 35^\circ$ with respect to horizontal

$$a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}$$

$$a = \frac{(2.3 \times 10^3 \text{ km/h} - 6.0 \times 10^2 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})}{1.2 \times 10^2 \text{ s}}$$

$$a = \frac{(1.7 \times 10^3 \text{ km/h})\left(\frac{1 \text{ h}}{3600 \text{ s}}\right)(10^3 \text{ m/km})}{1.2 \times 10^2 \text{ s}}$$

$$a = 3.9 \text{ m/s}^2$$

$$a_x = a(\cos \theta) = (3.9 \text{ m/s}^2)(\cos 35^\circ) = \boxed{3.2 \text{ m/s}^2 \text{ horizontally}}$$

$$a_y = a(\sin \theta) = (3.9 \text{ m/s}^2)(\sin 35^\circ) = \boxed{2.2 \text{ m/s}^2 \text{ vertically}}$$

Additional Practice C

1. $\Delta x_1 = 250.0 \text{ m}$
 $d_2 = 125.0 \text{ m}$
 $\theta_2 = 120.0^\circ$

$$\Delta x_2 = d_2(\cos \theta_2) = (125.0 \text{ m})(\cos 120.0^\circ) = -62.50 \text{ m}$$

$$\Delta y_2 = d_2(\sin \theta_2) = (125.0 \text{ m})(\sin 120.0^\circ) = 108.3 \text{ m}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 250.0 \text{ m} - 62.50 \text{ m} = 187.5 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 0 \text{ m} + 108.3 \text{ m} = 108.3 \text{ m}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(187.5 \text{ m})^2 + (108.3 \text{ m})^2}$$

$$d = \sqrt{3.516 \times 10^4 \text{ m}^2 + 1.173 \times 10^4 \text{ m}^2} = \sqrt{4.689 \times 10^4 \text{ m}^2}$$

$$d = \boxed{216.5 \text{ m}}$$

$$\theta = \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{108.3 \text{ m}}{187.5 \text{ m}}\right) = \boxed{30.01^\circ \text{ north of east}}$$

Givens

$$2. \nu = 3.53 \times 10^3 \text{ km/h}$$

$$\Delta t_1 = 20.0 \text{ s}$$

$$\Delta t_2 = 10.0 \text{ s}$$

$$\theta_1 = 15.0^\circ$$

$$\theta_2 = 35.0^\circ$$

Solutions

$$\Delta x_1 = \nu \Delta t_1 (\cos \theta_1)$$

$$\Delta x_1 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (20.0 \text{ s}) (\cos 15.0^\circ) = 1.89 \times 10^4 \text{ m}$$

$$\Delta y_1 = \nu \Delta t_1 (\sin \theta_1)$$

$$\Delta y_1 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (20.0 \text{ s}) (\sin 15.0^\circ) = 5.08 \times 10^3 \text{ m}$$

$$\Delta x_2 = \nu \Delta t_2 (\cos \theta_2)$$

$$\Delta x_2 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (10.0 \text{ s}) (\cos 35.0^\circ) = 8.03 \times 10^3 \text{ m}$$

$$\Delta y_2 = \nu \Delta t_2 (\sin \theta_2)$$

$$\Delta y_2 = (3.53 \times 10^3 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) (10.0 \text{ s}) (\sin 35.0^\circ) = 5.62 \times 10^3 \text{ m}$$

$$\Delta y_{tot} = \Delta y_1 + \Delta y_2 = 5.08 \times 10^3 \text{ m} + 5.62 \times 10^3 \text{ m} = \boxed{1.07 \times 10^4 \text{ m}}$$

$$\Delta x_{tot} = \Delta x_1 + \Delta x_2 = 1.89 \times 10^4 \text{ m} + 8.03 \times 10^3 \text{ m} = \boxed{2.69 \times 10^4 \text{ m}}$$

$$d = \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(2.69 \times 10^4 \text{ m})^2 + (1.07 \times 10^4 \text{ m})^2}$$

$$d = \sqrt{7.24 \times 10^8 \text{ m}^2 + 1.11 \times 10^8 \text{ m}^2} = \sqrt{8.35 \times 10^8 \text{ m}^2}$$

$$d = \boxed{2.89 \times 10^4 \text{ m}}$$

$$\theta = \tan^{-1} \left(\frac{\Delta y_{tot}}{\Delta x_{tot}} \right) = \tan^{-1} \left(\frac{1.07 \times 10^4 \text{ m}}{2.69 \times 10^4 \text{ m}} \right)$$

$$\theta = \boxed{21.7^\circ \text{ above the horizontal}}$$

$$3. \Delta x_1 + \Delta x_2 = 2.00 \times 10^2 \text{ m}$$

$$\Delta y_1 + \Delta y_2 = 0$$

$$\theta_1 = 30.0^\circ$$

$$\theta_2 = -45.0^\circ$$

$$\nu = 11.6 \text{ km/h}$$

$$\Delta y_1 = d_1 (\sin \theta_1) = -\Delta y_2 = -d_2 (\sin \theta_2)$$

$$d_1 = -d_2 \left(\frac{\sin \theta_2}{\sin \theta_1} \right) = -d_2 \left[\frac{\sin(-45.0^\circ)}{\sin 30.0^\circ} \right] = 1.41 d_2$$

$$\Delta x_1 = d_1 (\cos \theta_1) = (1.41 d_2) (\cos 30.0^\circ) = 1.22 d_2$$

$$\Delta x_2 = d_2 (\cos \theta_2) = d_2 [\cos(-45.0^\circ)] = 0.707 d_2$$

$$\Delta x_1 + \Delta x_2 = d_2 (1.22 + 0.707) = 1.93 d_2 = 2.00 \times 10^2 \text{ m}$$

$$d_2 = \boxed{104 \text{ m}}$$

$$d_1 = (1.41) d_2 = (1.41)(104 \text{ m}) = \boxed{147 \text{ m}}$$

$$\nu = 11.6 \text{ km/h} = (11.6 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) (10^3 \text{ m/km}) = 3.22 \text{ m/s}$$

$$\Delta t_1 = \frac{d_1}{\nu} = \left(\frac{147 \text{ m}}{3.22 \text{ m/s}} \right) = 45.7 \text{ s}$$

$$\Delta t_2 = \frac{d_2}{\nu} = \left(\frac{104 \text{ m}}{3.22 \text{ m/s}} \right) = 32.3 \text{ s}$$

$$\Delta t_{tot} = \Delta t_1 + \Delta t_2 = 45.7 \text{ s} + 32.3 \text{ s} = \boxed{78.0 \text{ s}}$$

Givens

4. $v = 925 \text{ km/h}$
 $\Delta t_1 = 1.50 \text{ h}$
 $\Delta t_2 = 2.00 \text{ h}$
 $\theta_2 = 135^\circ$

Solutions

$$\begin{aligned}d_1 &= v\Delta t_1 = (925 \text{ km/h})(10^3 \text{ m/km})(1.50 \text{ h}) = 1.39 \times 10^6 \text{ m} \\d_2 &= v\Delta t_2 = (925 \text{ km/h})(10^3 \text{ m/km})(2.00 \text{ h}) = 1.85 \times 10^6 \text{ m} \\ \Delta x_1 &= d_1 = 1.39 \times 10^6 \text{ m} \\ \Delta y_1 &= 0 \text{ m} \\ \Delta x_2 &= d_2(\cos \theta_2) = (1.85 \times 10^6 \text{ m})(\cos 135^\circ) = -1.31 \times 10^6 \text{ m} \\ \Delta y_2 &= d_2(\sin \theta_2) = (1.85 \times 10^6 \text{ m})(\sin 135^\circ) = 1.31 \times 10^6 \text{ m} \\ \Delta x_{tot} &= \Delta x_1 + \Delta x_2 = 1.39 \times 10^6 \text{ m} + (-1.31 \times 10^6 \text{ m}) = 0.08 \times 10^6 \text{ m} \\ \Delta y_{tot} &= \Delta y_1 + \Delta y_2 = 0 \text{ m} + 1.31 \times 10^6 \text{ m} = 1.31 \times 10^6 \text{ m} \\ d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(0.08 \times 10^6 \text{ m})^2 + (1.31 \times 10^6 \text{ m})^2} \\ d &= \sqrt{6 \times 10^9 \text{ m}^2 + 1.72 \times 10^{12} \text{ m}^2} = \sqrt{1.73 \times 10^{12} \text{ m}^2} \\ d &= \boxed{1.32 \times 10^6 \text{ m} = 1.32 \times 10^3 \text{ km}} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{1.31 \times 10^6 \text{ m}}{0.08 \times 10^6 \text{ m}}\right) = 86.5^\circ = 90.0^\circ - 3.5^\circ \\ \theta &= \boxed{3.5^\circ \text{ east of north}}\end{aligned}$$

5. $v = 57.2 \text{ km/h}$
 $\Delta t_1 = 2.50 \text{ h}$
 $\Delta t_2 = 1.50 \text{ h}$
 $\theta_2 = 30.0^\circ$

$$\begin{aligned}d_1 &= v\Delta t_1 = (57.2 \text{ km/h})(2.50 \text{ h}) = 143 \text{ km} \\d_2 &= v\Delta t_2 = (57.2 \text{ km/h})(1.50 \text{ h}) = 85.8 \text{ km} \\ \Delta x_{tot} &= d_1 + d_2(\cos \theta_2) = 143 \text{ km} + (85.8 \text{ km})(\cos 30.0^\circ) = 143 \text{ km} + 74.3 \text{ km} = 217 \text{ km} \\ \Delta y_{tot} &= d_2(\sin \theta_2) = (85.8 \text{ km})(\sin 30.0^\circ) = 42.9 \text{ km} \\ d &= \sqrt{(\Delta x_{tot})^2 + (\Delta y_{tot})^2} = \sqrt{(217 \text{ km})^2 + (42.9 \text{ km})^2} \\ d &= \sqrt{4.71 \times 10^4 \text{ km}^2 + 1.84 \times 10^3 \text{ km}^2} = \sqrt{4.89 \times 10^4 \text{ km}^2} \\ d &= \boxed{221 \text{ km}} \\ \theta &= \tan^{-1}\left(\frac{\Delta y_{tot}}{\Delta x_{tot}}\right) = \tan^{-1}\left(\frac{42.9 \text{ km}}{217 \text{ km}}\right) = \boxed{11.2^\circ \text{ north of east}}\end{aligned}$$

Additional Practice D

1. $v_x = 9.37 \text{ m/s}$
 $\Delta y = -2.00 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x} \\ \Delta x &= v_x \sqrt{\frac{2\Delta y}{a_y}} = (9.37 \text{ m/s}) \sqrt{\frac{(2)(-2.00 \text{ m})}{(-9.81 \text{ m/s}^2)}} = 5.98 \text{ m}\end{aligned}$$

The river is 5.98 m wide.

2. $\Delta x = 7.32 \text{ km}$
 $\Delta y = -8848 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\begin{aligned}\Delta t &= \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x} \\ v_x &= \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{(-9.81 \text{ m/s}^2)}{(2)(-8848 \text{ m})}} (7.32 \times 10^3 \text{ m}) = \boxed{172 \text{ m/s}}\end{aligned}$$

No. The arrow must have a horizontal speed of 172 m/s, which is much greater than 100 m/s.

Givens

3. $\Delta x = 471 \text{ m}$
 $v_i = 80.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

Solutions

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(471 \text{ m})^2}{(2)(80.0 \text{ m/s})^2} = -1.70 \times 10^2 \text{ m}$$

The cliff is $1.70 \times 10^2 \text{ m}$ high.

4. $v_x = 372 \text{ km/h}$
 $\Delta x = 40.0 \text{ m}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(40.0 \text{ m})^2}{(2) \left[(372 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) \right]^2}$$

$$\Delta y = -0.735 \text{ m}$$

The ramp is 0.735 m above the ground.

5. $\Delta x = 25 \text{ m}$
 $v_x = 15 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$
 $h = 25 \text{ m}$

$$\Delta t = \frac{\Delta x}{v_x}$$

$$\Delta y = \frac{1}{2} a_y (\Delta t)^2 = \frac{a_y (\Delta x)^2}{2v_x^2} = \frac{(-9.81 \text{ m/s}^2)(25 \text{ m})^2}{(2)(15 \text{ m/s})^2}$$

$$\Delta y = h - h' = -14 \text{ m}$$

$$h' = h - \Delta y = 25 \text{ m} - (-14 \text{ m})$$

$$= \boxed{39 \text{ m}}$$

6. $\ell = 420 \text{ m}$
 $\Delta y = \frac{-\ell}{2}$
 $\Delta x = \ell$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$\Delta t = \sqrt{\frac{2\Delta y}{a_y}} = \frac{\Delta x}{v_x}$$

$$v_x = \sqrt{\frac{a_y}{2\Delta y}} \Delta x = \sqrt{\frac{(-9.81 \text{ m/s}^2)}{(2)(-210 \text{ m})}} (420 \text{ m}) = \boxed{64 \text{ m/s}}$$

7. $\Delta y = -2.45 \text{ m}$
 $v = 12.0 \text{ m/s}$
 $a_y = -g = -9.81 \text{ m/s}^2$

$$v_y^2 = 2a_y \Delta y$$

$$v^2 = v_x^2 + v_y^2 = v_x^2 + 2a_y \Delta y$$

$$v_x = \sqrt{v^2 - 2a_y \Delta y} = \sqrt{(12.0 \text{ m/s})^2 - (2)(-9.81 \text{ m/s}^2)(-2.45 \text{ m})}$$

$$v_x = \sqrt{144 \text{ m}^2/\text{s}^2 - 48.1 \text{ m}^2/\text{s}^2}$$

$$= \sqrt{96 \text{ m}^2/\text{s}^2}$$

$$v_x = \boxed{9.8 \text{ m/s}}$$

Givens

8. $\Delta y = -1.95 \text{ m}$

$$v_x = 3.0 \text{ m/s}$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Solutions

$$v_y^2 = 2a_y \Delta y$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_x^2 + 2a_y \Delta y}$$

$$v = \sqrt{(3.0 \text{ m/s})^2 + (2)(-9.81 \text{ m/s}^2)(-1.95 \text{ m})}$$

$$v = \sqrt{9.0 \text{ m}^2/\text{s}^2 + 38.3 \text{ m}^2/\text{s}^2} = \sqrt{47.3 \text{ m}^2/\text{s}^2} = \boxed{6.88 \text{ m/s}}$$

$$\theta = \tan^{-1} \left(\frac{v_y}{v_x} \right) = \tan^{-1} \left(\frac{\sqrt{2a_y \Delta y}}{v_x} \right) = \tan^{-1} \left(\frac{\sqrt{(2)(-9.81 \text{ m/s}^2)(-1.95 \text{ m})}}{3.0 \text{ m/s}} \right)$$

$$\theta = \boxed{64^\circ \text{ below the horizontal}}$$

Additional Practice E

1. $\Delta x = 201.24 \text{ m}$

$$\theta = 35.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$\Delta y = v_i (\sin \theta) \Delta t + \frac{1}{2} a_y (\Delta t)^2 = v_i (\sin \theta) + \frac{1}{2} a_y \Delta t = 0$$

$$\Delta x = v_i (\cos \theta) \Delta t$$

$$\Delta t = \frac{\Delta x}{v_i (\cos \theta)}$$

$$v_i (\sin \theta) = -\frac{1}{2} a_y \left[\frac{\Delta x}{v_i (\cos \theta)} \right]$$

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(201.24 \text{ m})}{(2)(\sin 35.0^\circ)(\cos 35.0^\circ)}}$$

$$v_i = \boxed{45.8 \text{ m/s}}$$

2. $\Delta x = 9.50 \times 10^2 \text{ m}$

$$\theta = 45.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(9.50 \times 10^2 \text{ m})}{(2)(\sin 45.0^\circ)(\cos 45.0^\circ)}}$$

$$v_i = 96.5 \text{ m/s}$$

At the top of the arrow's flight:

$$v = v_x = v_i (\cos \theta) = (96.5 \text{ m/s})(\cos 45.0^\circ) = \boxed{68.2 \text{ m/s}}$$

3. $\Delta x = 27.5 \text{ m}$

$$\theta = 50.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(27.5 \text{ m})}{(2)(\sin 50.0^\circ)(\cos 50.0^\circ)}}$$

$$v_i = \boxed{16.6 \text{ m/s}}$$

4. $\Delta x = 44.0 \text{ m}$

$$\theta = 45.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

Using the derivation shown in problem 1,

$$\text{a. } v_i = \sqrt{\frac{-a_y \Delta x}{2 (\sin \theta) (\cos \theta)}} = \sqrt{\frac{-(-9.81 \text{ m/s}^2)(44.0 \text{ m})}{(2)(\sin 45.0^\circ)(\cos 45.0^\circ)}}$$

$$v_i = \boxed{20.8 \text{ m/s}}$$

- b.** At maximum height, $v_{y,f} = 0$ m/s

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y \Delta y_{\max} = 0$$

$$y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-v_i^2 (\sin \theta)^2}{2a_y} = \frac{-(20.8 \text{ m/s})^2 (\sin 45.0^\circ)^2}{(2)(-9.81 \text{ m/s}^2)} = 11.0 \text{ m}$$

The brick's maximum height is 11.0 m.

c. $y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-(20.8 \text{ m/s})^2}{(2)(-9.81 \text{ m/s}^2)} = 22.1 \text{ m}$

The brick's maximum height is 22.1 m.

- 5.** $\Delta x = 76.5$ m

$$\theta = 12.0^\circ$$

$$a_y = -g = -9.81 \text{ m/s}^2$$

At maximum height, $v_{y,f} = 0$ m/s.

$$v_{y,f}^2 = v_{y,i}^2 + 2a_y \Delta y_{\max} = 0$$

$$y_{\max} = \frac{-v_{y,i}^2}{2a_y} = \frac{-v_i^2 (\sin \theta)^2}{2a_y}$$

Using the derivation for v_i^2 from problem 1,

$$\Delta y_{\max} = \left[\frac{-a_y \Delta x}{2(\sin \theta)(\cos \theta)} \right] \frac{-(\sin \theta)^2}{2a_y} = \frac{\Delta x (\sin \theta)}{4(\cos \theta)} = \frac{\Delta x (\tan \theta)}{4}$$

$$\Delta y_{\max} = \frac{(76.5 \text{ m})(\tan 12.0^\circ)}{4} = 4.07 \text{ m}$$

- 6.** $v_{\text{runner}} = 5.82$ m/s

$$v_{i,\text{ball}} = 2v_{\text{runner}}$$

In x -direction,

$$v_{i,\text{ball}}(\cos \theta) = 2v_{\text{runner}}(\cos \theta) = v_{\text{runner}}$$

$$2(\cos \theta) = 1$$

$$\theta = \cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$$

- 7.** $v_i = 8.42$ m/s

$$\theta = 55.2^\circ$$

$$\Delta t = 1.40$$
 s

$$a_y = -g = -9.81 \text{ m/s}^2$$

For first half of jump,

$$\Delta t_1 = \frac{1.40 \text{ s}}{2} = 0.700 \text{ s}$$

$$\Delta y = v_i (\sin \theta) \Delta t_1 + \frac{1}{2} a_y (\Delta t_1)^2 = (8.42 \text{ m/s})(\sin 55.2^\circ)(0.700 \text{ s}) + \frac{1}{2}(-9.81 \text{ m/s}^2)(0.700 \text{ s})^2$$

$$\Delta y = 4.84 \text{ m} - 2.40 \text{ m} = 2.44 \text{ m}$$

The fence is 2.44 m high.

$$\Delta x = v_i (\cos \theta) \Delta t$$

$$\Delta x = (8.42 \text{ m/s})(\cos 55.2^\circ)(1.40 \text{ s}) = 6.73 \text{ m}$$

- 8.** $v_i = 2.2$ m/s

$$\theta = 21^\circ$$

$$\Delta t = 0.16$$
 s

$$a_y = -g = -9.81 \text{ m/s}^2$$

$$\Delta x = v_i (\cos \theta) \Delta t = (2.2 \text{ m/s})(\cos 21^\circ)(0.16 \text{ s}) = 0.33 \text{ m}$$

Maximum height is reached in a time interval of $\frac{\Delta t}{2}$

$$\Delta y_{\max} = v_i (\sin \theta) \left(\frac{\Delta t}{2}\right) + \frac{1}{2} a_y \left(\frac{\Delta t}{2}\right)^2$$

$$\Delta y_{\max} = (2.2 \text{ m/s})(\sin 21^\circ) \left(\frac{0.16 \text{ s}}{2}\right) + \frac{1}{2}(-9.81 \text{ m/s}^2) \left(\frac{0.16 \text{ s}}{2}\right)^2$$

$$\Delta y_{\max} = 6.3 \times 10^{-2} \text{ m} - 3.1 \times 10^{-2} \text{ m} = 3.2 \times 10^{-2} \text{ m} = 3.2 \text{ cm}$$

The flea's maximum height is 3.2 cm.

Additional Practice F

Givens

1. $\mathbf{v}_{se} = 126 \text{ km/h north}$
 $\mathbf{v}_{gs} = 40.0 \text{ km/h east}$

Solutions

$$v_{ge} = \sqrt{v_{gs}^2 + v_{se}^2} = \sqrt{(40.0 \text{ km/h})^2 + (126 \text{ km/h})^2}$$

$$v_{ge} = \sqrt{1.60 \times 10^3 \text{ km}^2/\text{h}^2 + 1.59 \times 10^4 \text{ km}^2/\text{h}^2}$$

$$v_{ge} = \sqrt{1.75 \times 10^4 \text{ km}^2/\text{h}^2} = \boxed{132 \text{ km/h}}$$

$$\theta = \tan^{-1} \left(\frac{v_{se}}{v_{gs}} \right) = \tan^{-1} \left(\frac{126 \text{ km/h}}{40.0 \text{ km/h}} \right) = \boxed{72.4^\circ \text{ north of east}}$$

2. $\mathbf{v}_{we} = -3.00 \times 10^2 \text{ km/h}$
 $\mathbf{v}_{pw} = 4.50 \times 10^2 \text{ km/h}$
 $\Delta x = 250 \text{ km}$

$$v_{pe} = v_{pw} + v_{we} = 4.50 \times 10^2 \text{ km/h} - 3.00 \times 10^2 \text{ km/h} = 1.50 \times 10^2 \text{ km/h}$$

$$\Delta t = \frac{\Delta x}{v_{pe}} = \frac{250 \text{ km}}{1.50 \times 10^2 \text{ km/h}} = \boxed{1.7 \text{ h}}$$

3. $\mathbf{v}_{tw} = 9.0 \text{ m/s north}$
 $\mathbf{v}_{wb} = 3.0 \text{ m/s east}$
 $\Delta t = 1.0 \text{ min}$

$$\mathbf{v}_{tb} = \mathbf{v}_{tw} + \mathbf{v}_{wb}$$

$$v_{tb} = \sqrt{v_{tw}^2 + v_{wb}^2} = \sqrt{(9.0 \text{ m/s})^2 + (3.0 \text{ m/s})^2} = \sqrt{81 \text{ m}^2/\text{s}^2 + 9.0 \text{ m}^2/\text{s}^2}$$

$$v_{tb} = \sqrt{9.0 \times 10^1 \text{ m}^2/\text{s}^2}$$

$$v_{tb} = 9.5 \text{ m/s}$$

$$\Delta x = v_{tb} \Delta t = (9.5 \text{ m/s})(1.0 \text{ min}) \left(\frac{60 \text{ s}}{1 \text{ min}} \right) = \boxed{570 \text{ m}}$$

$$\theta = \tan^{-1} \left(\frac{v_{wb}}{v_{tw}} \right) = \tan^{-1} \left(\frac{3.0 \text{ m/s}}{9.0 \text{ m/s}} \right) = \boxed{18^\circ \text{ east of north}}$$

4. $\mathbf{v}_{sw} = 40.0 \text{ km/h forward}$
 $\mathbf{v}_{fw} = 16.0 \text{ km/h forward}$
 $\Delta x = 60.0 \text{ m}$

$$\mathbf{v}_{sf} = \mathbf{v}_{sw} - \mathbf{v}_{fw} = 40.0 \text{ km/h} - 16.0 \text{ km/h} = 24.0 \text{ km/h toward fish}$$

$$\Delta t = \frac{\Delta x}{v_{sf}} = \frac{60.0 \text{ m}}{(24.0 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right)} = \boxed{9.00 \text{ s}}$$

5. $\mathbf{v}_{1E} = 90.0 \text{ km/h}$
 $\mathbf{v}_{2E} = -90.0 \text{ km/h}$
 $\Delta t = 40.0 \text{ s}$

$$\mathbf{v}_{12} = \mathbf{v}_{1E} - \mathbf{v}_{2E}$$

$$v_{12} = 90.0 \text{ km/h} - (-90.0 \text{ km/h}) = 1.80 \times 10^2 \text{ km/h}$$

$$\Delta x = v_{12} \Delta t = (1.80 \times 10^2 \text{ km/h}) \left(\frac{1 \text{ h}}{3600 \text{ s}} \right) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right) (40.0 \text{ s}) = 2.00 \times 10^3 \text{ m} = 2.00 \text{ km}$$

The two geese are initially 2.00 km apart

6. $\mathbf{v}_{me} = 18.0 \text{ km/h forward}$
 $\mathbf{v}_{re} = 0.333 \mathbf{v}_{me}$
 $\quad = 6.00 \text{ km/h forward}$
 $\Delta x = 12.0 \text{ m}$

$$\mathbf{v}_{mr} = \mathbf{v}_{me} - \mathbf{v}_{re}$$

$$v_{mr} = 18.0 \text{ km/h} - 6.0 \text{ km/h} = 12.0 \text{ km/h}$$

$$\Delta t = \frac{\Delta x}{v_{mr}} = \frac{12.0 \text{ m}}{(12.0 \text{ km/h}) \left(\frac{3600 \text{ s}}{1 \text{ h}} \right) \left(\frac{1 \text{ km}}{10^3 \text{ m}} \right)}$$

$$\Delta t = \boxed{3.60 \text{ s}}$$