Chapter 1: Alternate Problem Set in *Mastering Physics*

70. **INTERPRET:** First convert 1 foot per nanosecond to units of m/s. Then compare it to the accepted value for the speed of light c.

DEVELOP: We know that 1 ft = 0.3048 m, 1 nanosecond = 1 ns = 10^{-9} s, and c = 2.99792458 $\times 10^8$ m/s. The percent error in v is $\frac{v-c}{c}\times 100=\left(\frac{v}{c}-1\right)\times 100$.

EVALUATE: The speed v in m/s is $v=\frac{1\text{ ft}}{1\text{ ns}}=\frac{0.3048\text{ m}}{1\times 10^{-9}\text{ s}}=3.048\times 10^8\text{ m/s}.$

The percent error in v is $\left(\frac{v}{c}-1\right)\times 100=\left(\frac{3.048\times 10^8\text{ m/s}}{2.99792458\times 10^8\text{ m/s}}-1\right)(100)$ = 1.7%.

ASSESS: The percent error is positive because the approximate value v is greater than c. For rough calculations, the approximate answer should be adequate, but for more detailed work, a more accurate value is needed.

71. **INTERPRET:** Express the given mass of water using the appropriate SI prefix without an explicit power of 10.

DEVELOP: Use 1 kg = 10^3 g.

EVALUATE: 14 $\times 10^{15}$ kg = 14 $\times 10^{15} \times 10^3$ g = 14 $\times 10^{18}$ g. From Table 1.1, we know that 10^{18} = exa = E, so 14 $\times 10^{15}$ kg = 14 $\times 10^{18}$ g = 14 Eg.

ASSESS: Although we could write the result as 1.4 $\times 10^{19}$ g, this would not be helpful because there is no SI prefix for 10^{19}.

72. **INTERPRET:** We know the length s of the subtended arc and the radius r of the circle, and we want to find the angle θ subtended by this arc.

DEVELOP: We know that $\theta=\frac{s}{r}$ when θ is in radians. In this case, s = 11.2 cm and r = 8.16 cm.

EVALUATE: $\theta=\frac{s}{r}=\frac{11.2\text{ cm}}{8.16\text{ cm}}=1.37255\text{ rad},$ which rounds to 1.37 rad. We convert this angle to degrees using the fact that π rad = 180°.

$$\theta=(1.37255\text{ rad})\left(\frac{180°}{\pi\text{ rad}}\right)=78.6°.$$

ASSESS: We know that 1 radian is equal to 57.3°, so 1.37255 rad = (1.37255)(57.3°) = 78.6°, which agrees with our result above.

73. **INTERPRET:** We know the arc length s and the angle θ subtended by this arc length, and we want to find the radius r of the circle.

DEVELOP: We know that $\theta=\frac{s}{r}$ when θ is in radians, so $r = s/\theta$. In this case, s = 2.1 km and θ = 35°. We must convert 35° to radians.

EVALUATE: $\theta=(35°)\left(\frac{\pi\text{ rad}}{180°}\right)=0.61087\text{ rad}.$

Now calculate r: $r=\frac{s}{\theta}=\frac{2.1\text{ km}}{0.61087\text{ rad}}=3.4\text{ km}.$

Assess: The radian is a dimensionless quantity, so the quantity km/rad is just written as km, a standard metric unit of length.

74. **Interpret:** We need to convert 18 m/s to units of mi/h.
Develop: Use 1 mi = 1.609 km, 1 km = 10^3 m = 1000 m, 1 h = 60 min, 1 min = 60 s.
Evaluate: First convert hr to s: 1 h = 60 min = (60)(60 s) = 3600 s. Now make the total conversion.

$$(18\ \text{m/s})\left(\frac{1\ \text{km}}{1000\ \text{m}}\right)\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)=40\ \text{mi/h}.$$

Assess: Example 1.1 in the text showed that 65 mi/h = 29 m/s. Since 18 m/s is 62.1% of 29 m/s, 40 mi/h should be 62.1% of 65 mi/h.
(0.621)(65 mi/h) = 40 mi/h, which checks.

75. **Interpret:** We want to convert 10 m^2/L to units of ft^2/gal.
Develop: Use quantities from Appendix C: 1 L = 10^{-3} m^3, 1 gal = 3.785 $\times 10^{-3}$ m^3, 1 ft = 0.3048 m.
Evaluate: Make the conversion:

$$(1\ \text{m}^2/\text{L})\left(\frac{1\ \text{L}}{10^{-3}\ \text{m}^3}\right)\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right)\left(\frac{3785\times10^{-3}\ \text{m}^3}{1\ \text{gal}}\right)\left(\frac{1\ \text{ft}}{0.3048\ \text{m}}\right)^2=407\ \text{ft}^2/\text{gal},$$ which rounds to 400 ft^2/gal to 1 significant figure.

Assess: According to the information on typical labels of house point, a coverage rate of 400 ft^2/gal is not unusual for ordinary wall paints.

76. **Interpret:** First convert 100 km/h to units of mi/h, and then compare this result with 65 mi/h.
Develop: Use 1 mi = 1.609 km.
Evaluate: First make the conversion: $\left(\frac{100\ \text{km}}{\text{h}}\right)\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right)=62\ \text{mi/h}.$

You are driving at 65 mi/h, so you should slow down to 62 mi/h. Thus you should slow down by 3 mi/h.
Assess: It is reasonable that the speed limit in Canada would be about the same as it is in the United States for the same type of road. And this is just what we found, since the speed limit in Canada (100 km/h = 62 mi/h) is just about the same as that in the United States (65 mi/h).

77. **Interpret:** Convert 1 km/h to units of m/s.
Develop: Use 1 km = 1000 m and 1 h = 60 min = 60 (60 s) = 3600 s.
Evaluate: Do the conversion: $\left(\frac{1\ \text{km}}{\text{h}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)=0.278\ \text{m/s}.$

Assess: Our result says that a km/h is approximately ¼ m/s.

78. **Interpret:** A braid contains 10^5 strands. Assume that all the strands are the same size, that there is no space between them, and that a typical braid has a circular cross-section that is 3 cm in diameter.
Develop: Each braid contains 10^5 strands of hair, so
cross-sectional area of a braid = (10^5)(cross-sectional area of a single strand).
The area of a circle is πR^2. If R is the radius of the braid and r the radius of a single strand, then we have
$\pi R^2 = (10^5)\pi r^2$.
In this case, we have R = 1.5 cm.
Evaluate: Solving for r gives

$$r=\frac{R}{\sqrt{10^5}}=\frac{1.5\ \text{cm}}{\sqrt{10^5}}=0.00474\ \text{cm}.$$

The diameter d of a strand is
$d = 2r = 2(0.00474\ \text{cm}) = 0.0095\ \text{cm} = 0.095\ \text{mm} \approx 0.1\ \text{mm}$.
Assess: According to our result, 10 hairs placed side-by-side should be about 1 mm wide. This result seems fairly reasonable.

79. **INTERPRET:** The amount of energy used per year depends on the rate at which people use energy and the world population.

DEVELOP: The world population at the beginning of 2015 was about 7.22 billion people. We also used $1\text{ y} = 3.156\times10^{7}$ s, 1 kW = 1000 W = 1000 J/s. The total energy E used for a year is

E = (rate of use per person)(number of people)(time of energy use)

EVALUATE: $E = (2000\text{ J/s})(7.22\times10^{9})(3.156\times10^{7}\text{ s}) = 4.56\times10^{20}$ J. Since we know to the average rate of energy consumption (2 kW per person) to only one significant figure, we should round our answer to one significant figure. Therefore the coefficient 4.56 rounds to 5, so $E = 5\times10^{20}\text{ J} = 500\times10^{18}\text{ J} = 500$ EJ.

ASSESS: As population increases and the per capita use of energy also increases, yearly the energy use will become considerably greater than the value found since it depends on the product of population and the per capita energy use.

Chapter 2: Alternate Problem Set in *Mastering Physics*

97. **INTERPRET:** We know the initial velocity of your car and the distance it travels during 6.0 s with constant acceleration. We want to find the final velocity of your car at the end of the 6.0 s. The one-dimensional kinematics equations for constant acceleration apply to the motion of your car.

DEVELOP: First use the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ to find the acceleration a of the car. Then use the equation $v = v_0 + at$ to find the final speed v of your car. Let x_0 be the point where your car begins to accelerate at $t = 0$ s, so $x - x_0 = 130$ m. We know that $v_0 = 75$ km/h and $t = 6.0$ s. Use standard SI units.

EVALUATE: First convert 75 km/h to units of m/s:

$$v_0 = \left(75\frac{\text{km}}{\text{h}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right) = 20.833\text{ m/s}$$

NOW find the acceleration:

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

130 m = 0 m + (20.833 m/s)(6.0 s) + (1/2)a(6.0 s)^2

$a = 0.2778\text{ m/s}^2$

Now use $v = v_0 + at$ to find v:

$v = v_0 + at$ = 20.833 m/s + (0.2778 m/s^2)(6.0 s) = 22.50 m/s.

Now convert this result to km/h:

$$v = \left(22.5\frac{\text{m}}{\text{s}}\right)\left(\frac{3600\text{ s}}{1\text{ h}}\right)\left(\frac{1\text{ km}}{1000\text{ m}}\right) = 81\text{ km/h.}$$

ASSESS: Check by using the equation $v^2 = v_0^2 + 2a(x - x_0)$ to find v:

v^2 = (20.833 m/s)^2 + 2(0.2778 m/s^2)(130 m – 0 m)

v = 22.50 m/s, which is the same as we just found.

98. **INTERPRET:** We know the initial and final speeds of the rocket and its acceleration, and we want to find its height and the time it took to reach that height. The constant-acceleration kinematics formulas apply. Assume the rocket starts from rest.

DEVELOP: Use the formula $v^2 = v_0^2 + 2a(x - x_0)$ to find the height and $v = v_0 + at$ to find the time. Let $x_0 = 0$ at the point when the rocket starts its motion.

EVALUATE: **(a)** Using $v^2 = v_0^2 + 2a(x - x_0)$ gives

(2800 m/s)^2 = (0 m/s)^2 + 2(46 m/s^2)(x – 0 m)

x = 85,000 m = 85 km.

(b) Using $v = v_0 + at$ gives

2800 m/s = 0 m/s + (46 m/s^2)t

t = 60.87 s, which rounds to 61 s.

ASSESS: We can check using the formula $x = x_0 + v_0 t + \frac{1}{2}at^2$:

x = (1/2)(46 m/s^2)(60.87 s)^2 = 85,000 m = 85 km, so the answers to parts **(a)** and **(b)** are consistent.

99. **INTERPRET:** The rock falls with constant downward acceleration due to gravity. The elapsed time is the time for it to reach the water since we are neglecting the speed of sound.

DEVELOP: Use $x = x_0 + v_0 t + \frac{1}{2}at^2$ with downward positive. We know that $v_0 = 0$ m/s, $a = g = 9.8$ m/s^2, and $x - x_0 = 0$ m.

EVALUATE: The equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ gives
95 m = 0 m/s + (1/2)(9.8 m/s^2)t^2
$t = 4.4$ s.

ASSESS: For a well 95 m deep, the elapsed time would be slightly greater than 4.4 s since it takes time for the sound to go from the water to the top of the well. With a sound speed of 344 m/s, this time would be $t = x/v$ = (95 m)/(344 m/s) = 0.27 s, which is about a quarter of a second. This is rather small compared to 4.4 s, but it is large enough to introduce some error in the calculated result.

100. **INTERPRET:** Calculate the speed of the rock when it reaches a height of 6.5 m to see if it is great enough to dislodge the Frisbee. The acceleration of the rock is g downward.

DEVELOP: Call upward positive and use the formula $v^2 = v_0^2 + 2a(x - x_0)$ to find v when the rock is 6.5 m above the ground. It starts at a height of 1.3 m, so $x - x_0$ = 6.5 m – 1.3 m = 5.2 m.

EVALUATE: The equation $v^2 = v_0^2 + 2a(x - x_0)$ gives
v^2 = (12 m/s)^2 + 2(–9.8 m/s^2)(5.2 m)
$v = 6.5$ m/s.
The speed is greater than 3 m/s, so the rock will dislodge the Frisbee.

ASSESS: We can use $v^2 = v_0^2 + 2a(x - x_0)$ to find the minimum initial speed the rock would need to dislodge the Frisbee.
(3 m/s)^2 = v_0^2 + 2(–9.8 m/s^2)(5.2 m)
$v_0 = 11$ m/s.
Therefore our answer is reasonable since the rock started at 12 m/s.

101. **INTERPRET:** Both planes are traveling at constant speed. Just as they pass each other, they have been flying for the same amount of time and the sum of the distances they have traveled is equal to the distance between San Francisco and New York.

DEVELOP: At constant speed, $x - x_0 = vt$. First use the New York-bound plane to find the time at which they pass each other. Then use the sum of their distances traveled to find the speed of the San Francisco-bound plane.

EVALUATE: For the New York-bound plane, use $x = vt$ to find t:
2800 km = (1100 km/h)t
$t = 2.545$ h.
When they pass, the San Francisco-bound plane has traveled a distance
x = 4600 km – 2800 km = 1800 km. Using $x = vt$ for that plane to find v gives
1800 km = v(2.545 h)
$v = 710$ km/h.

ASSESS: As they pass, the New York-bound plane has flown 2800 km while the San Francisco-bound plane has flown 1800 km. Therefore the San Francisco-bound plane should have a smaller speed than the New York-bound plane, which agrees with our result of 710 km/h for the San Francisco-bound plane and 1100 km/h for the New York-bound plane.

102. **INTERPRET:** The stopping time is reduced by 55%, so the stopping distance and acceleration will also be reduced. The initial speed is the same with and without the antilock brakes. We assume constant acceleration in both cases.

DEVELOP: Use the equation $v = v_0 + at$ to find the acceleration and the equation $x = x_0 + v_0 t + \frac{1}{2}at^2$ to find the stopping distance. If t is the stopping time without the antilock brakes and t_B is the time with the, then $t_B = 0.45t$. Let the positive direction be that in which the car is moving. We know that v_0 is the same in both cases, and also $x - x_0$ is the stopping distance.

EVALUATE: With the ordinary brakes:
$v = v_0 + at$
0 m/s = $v_0 + at$
$a = -v_0/t$

Using this result, we find that the stopping distance is

$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$

$$x - x_0 = v_0 t + \tfrac{1}{2}\left(-\frac{v_0}{t}\right)t^2 = \frac{v_0 t}{2}.$$

With antilock brakes:

We follow the same procedure, which gives

$$x - x_0 = \frac{v_0 t_B}{2}.$$

Since $t_B = 0.45t$, we find that the stopping distance with antilock brakes is 0.45 times the distance with ordinary brakes. Therefore the stopping distance has been reduced by 55% by the antilock brakes.

ASSESS: Check using $v^2 = v_0^2 + 2a(x - x_0)$ to solve for $x - x_0$. We use $a = -v_0/t$ and $v =$, which gives

$$(0\ \text{m/s})^2 = v_0^2 + 2(-v_0/t)(x - x_0)$$

$$x - x_0 = v_0 t/2.$$

From this result, we can see that if t is reduced by 55%, $x - x_0$ will also be reduced by 55%, which agrees with our result.

103. **INTERPRET:** We know the initial and final velocities of the puck and the distance it travels while stopping. We want its acceleration, assumed to be constant, so the constant-acceleration kinematics formulas apply.

DEVELOP: Use $v^2 = v_0^2 + 2a(x - x_0)$ with the positive direction the same as the direction in which the puck is moving and $x - x_0$ the distance the puck moves while stopping, which is 87 cm = 0.87 m. The initial velocity is 32 m/s and the final velocity is zero.

EVALUATE: Using $v^2 = v_0^2 + 2a(x - x_0)$ gives

$$(0\ \text{m/s})^2 = (32\ \text{m/s})^2 + 2a(0.87\ \text{m})$$

$$a = -590\ \text{m/s}^2.$$

The magnitude is 590 m/s^2.

ASSESS: The acceleration is negative since it is directed opposite to the positive direction of the initial velocity. This result is reasonable since the puck is slowing down.

104. **INTERPRET:** We know the initial and final velocities of the jetliner and its stopping distance. We want to find the time it takes to stop, assuming constant acceleration.

DEVELOP: Use $v^2 = v_0^2 + 2a(x - x_0)$ to find the acceleration, where $x - x_0$ = 890 m = 0.890 km (the stopping distance). Take the $+x$ direction to be the direction of motion of the plane. Then use $v = v_0 + at$ to find the time t.

EVALUATE: Using $v^2 = v_0^2 + 2a(x - x_0)$ gives

$$(0\ \text{km/h})^2 = (220\ \text{km/h})^2 + 2a(0.890\ \text{km})$$

$$a = -0.2719 \times 10^4\ \text{km/h}^2.$$

Now use $v = v_0 + at$:

$$0\ \text{km/h} = 220\ \text{km/h} + (-0.2719 \times 10^4\ \text{km/h}^2)t$$

$$t = 8.09 \times 10^{-3}\ \text{h} = 29\ \text{s}.$$

ASSESS: Check using $x = x_0 + v_0 t + \tfrac{1}{2}at^2$.

$$x - x_0 = (220\ \text{km/h})(8.09 \times 10^{-3}\ \text{h}) + (1/2)(-0.2719 \times 10^4\ \text{km/h}^2)(8.09 \times 10^{-3}\ \text{h})^2$$

which gives 0.890 km = 890 km, in agreement with the given information in the problem.

105. **INTERPRET:** We know the initial velocity of the object, the total distance it falls, and its constant downward acceleration of g. We want to find how far it moves during the final second of its fall.

DEVELOP: First use $x = x_0 + v_0 t + \tfrac{1}{2}at^2$ to find the time for the object to fall 288 m. Then use the same equation to find the distance it falls in 1.00 s less than that time. Finally subtract that distance from 288 m to find how far it falls in the final second of its motion. Call downward positive, let $x - x_0$ be the distance the object has fallen in each case, and use g = 9.81 m/s^2.

EVALUATE: The time to fall 288 m starting from rest is

$x = x_0 + v_0 t + \frac{1}{2}at^2$

288 m = (0 m/s)t + (1/2)(9.81 m/s^2)t^2

t = 7.663 s.

The final second begins at 7.663 s – 1.00 s = 6.663 s. The distance the object falls in the first 6.663 s is

$x = x_0 + v_0 t + \frac{1}{2}at^2$

288 m = (0 m/s)(0.663 s) + (1/2)(9.81 m/s^2)(0.663 s)^2 = 217.7 m.

The distance it falls during the last 1.00 s is

288 m – 217.7 s = 70.3 m.

ASSESS: The distance the object falls during the first second is

x = (1/2)gt^2 = (1/2)(9.8 m/s^2)(1.0 s)^2 = 4.9 m.

This distance is much less than the distance it falls during its final second, which is reasonable since the object speeds up as it falls and therefore falls farther during each succeeding second.

106. **INTERPRET:** The passenger moves upward at constant velocity while the ball is in free fall after it is thrown. At the instant the ball is caught, the passenger and the ball have the same position. We want the speed with which the passenger threw the ball relative to the balloon. This speed must be added to the speed of the balloon to get the initial speed of the ball relative to the ground. While in flight, the ball has a downward acceleration g.

DEVELOP: The formula $x = x_0 + v_0 t + \frac{1}{2}at^2$ applies to the passenger and the ball, but the passenger has no acceleration. The initial speed of the ball relative to the balloon is v, and relative to the ground it is v + 10.0 m/s. Call upward positive, and let $x - x_0$ be the change in position of the person and the ball.

EVALUATE: Use $x = x_0 + v_0 t + \frac{1}{2}at^2$ to find the position of the passenger when the ball is caught. The acceleration of zero.

$x - x_0$ = (10.0 m/s)(2.40 s) = 24.0 m.

This is also the position of the ball, so

$x = x_0 + v_0 t + \frac{1}{2}at^2$

24.0 m = (v + 10.0 m/s)(2.40 s) + (1/2)(–9.81 m/s^2)(2.40 s)^2

v = 11.8 m/s.

ASSESS: The ball and person are at the same position when the ball is caught, but they have not traveled the same distance. The balloon and passenger have traveled 24.0 m, but the ball has traveled more than this because it went up and then came down to meet the passenger.

Chapter 3: Alternate Problem Set in *Mastering Physics*

93. **INTERPRET:** Find the sum of 3 given vectors.
DEVELOP: Use components. The x-component of the sum is equal to the sum of the x-components of the 3 vectors, and likewise for the y-components. The components are: $A_x = 3.0$ m, $A_y = 0$ m, $B_x = 0$ m, $B_y = 4.0$ m, $C_x = -C\cos\theta = -(5.0 \text{ m})(3/5) = -3.0$ m, $C_y = -C\sin\theta = -(5.0 \text{ m})(4/5) = -4.0$ m.
EVALUATE: The x-component of the sum is
$A_x + B_x + C_x = 3.0 \text{ m} + 0 \text{ m} + (-3.0 \text{ m}) = 0$ m
$A_y + B_y + C_y = 0 \text{ m} + 4.0 \text{ m} + (-4.0 \text{ m}) = 0$ m.
All the components of the sum are zero, so the sum is the zero vector $\vec{0}$.
ASSESS: To check, do a graphical sum to see if your answer is reasonable.

94. **INTERPRET:** We know the magnitude and one component of a vector and want to find the other component.
DEVELOP: The magnitude is $A = \sqrt{A_x^2 + A_y^2}$, where $A_x = 5.00$, $A_y = b$, and $A = 7.07$.
EVALUATE: Using $A = \sqrt{A_x^2 + A_y^2}$ gives
$7.07 = \sqrt{(5.00)^2 + b^2}$
$b = \pm 5.00$.
ASSESS: The vector $\vec{A}$ points into the first quadrant (for $b = +5.00$) or into the fourth quadrant (for $b = -5.00$).

95. **INTERPRET:** Find the direction and magnitude of the velocity vector (the speed) using the given components.
DEVELOP: The formulas $\tan\theta = \dfrac{v_y}{v_x}$ and $v = \sqrt{v_x^2 + v_y^2}$ both apply, where v is the speed, $v_x = -14.0$ m/s, and $v_y = -12.0$ m/s.
EVALUATE: **(a)** $\tan\theta = \dfrac{v_y}{v_x} = \dfrac{-12.0 \text{ m/s}}{-14.0 \text{ m/s}} = 0.857$, so $\theta = 40.6°$. Both components are negative, so the vector points into the third quadrant and makes an angle of 40.6° below the $-x$-axis. The positive angle from the $+x$-axis is $180° + 40.6° = 220.6°$.
(b) The speed is $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(-14.0 \text{ m/s})^2 + (-12.0 \text{ m/s})^2} = 18.4$ m/s.
ASSESS: The components are close to being equal, so the angle with respect to the x-axis should be close to 45°, which agrees with our result of 40.6°. The magnitude is greater than both of the components, as it must be.

96. **INTERPRET:** The problem involves relative velocities. The velocity of the plane relative to the ground must be directly southward. The plane heads 9.7° west of east to counteract the 160-mile per hour wind that is blowing toward the east.
DEVELOP: The velocity of the plane relative to the air is 950 km/h at 9.7° west of south. The velocity $\vec{v}$ of the plane relative to the ground is directly south. Since the wind has no southward component, the southward component of the plane's velocity relative to the air, which is (950 km/h) cos(9.7°), must be equal to the magnitude of $\vec{v}$. First find this magnitude v. Then use it to find the time for the plane to fly 1500 km. Since the plane has uniform velocity, the formula $x = vt$ applies to its southward motion.
EVALUATE: Find the magnitude of the southward velocity v.
$v = (950 \text{ km/h})\cos(9.7°) = 936.4$ km/h.
Now use $x = vt$ to find the time to fly 1500 km:
$1500 \text{ km} = (936.4 \text{ km/h})t$
$t = 1.60 \text{ h} = (1.6)(60 \text{ min}) = 96$ min.

ASSESS: Check by finding the magnitude v of the vector $\vec{v}$ using the information given in the problem.
$v^2 + (160\ \text{km/h})^2 = (950\ \text{km/h})^2$
$v = 936.4$ km/h, which agrees with the value we found above.

97. **INTERPRET:** This is a projectile problem. We know the vertical distance the shingle falls, the horizontal distance it travels while falling, and its initial vertical velocity, which is zero. We want to find the time it is in the air and its initial speed.

DEVELOP: The shingle falls 8.82 m with no initial vertical velocity, so we use the equation $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$ to find the time t that it is in the air. In this case, we know that $y - y_0 = 8.82$ m and $a_y = g$ downward. It travels 15.0 m horizontally with no horizontal acceleration, so we can use $x = v_x t$ to find v_x.

EVALUATE: Using $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$ with the positive direction downward gives

$8.82\ \text{m} = (0\ \text{m/s})t + \frac{1}{2}(9.81\ \text{m/s}^2)t^2$

$t = 1.34$ s.

Now we use $x = v_x t$ to find its horizontal speed v_x.

$x = v_x t$

$15.0\ \text{m} = v_x(1.34\ \text{s})$

$v_x = 11.2$ m/s.

ASSESS: The horizontal velocity is constant at 15.0 m/s. The average vertical velocity is (8.82 m)/(1.34 s) = 6.58 m/s. Since the horizontal velocity is greater than the average vertical velocity, the shingle should travel a greater distance horizontally than it falls vertically, which is in fact what we have. So our answer is reasonable.

98. **INTERPRET:** This is a projectile problem. The arrow keeps moving with a constant horizontal velocity until it has fallen 1.5 m starting from rest vertically.

DEVELOP: First find the time to fall 1.5 m using the equation $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$ for the vertical motion, with the positive direction upward and $a_y = -g$. Then use the equation $x = v_x t$ to find the distance x traveled while falling.

EVALUATE: For the vertical motion, $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$ with $y - y_0 = -1.5$ m and $a_y = -9.8\ \text{m/s}^2$, which gives us

$-1.5\ \text{m} = (0\ \text{m/s})t - \frac{1}{2}(9.8\ \text{m/s}^2)t^2$

$t = 0.553$ s.

For the horizontal motion, we use $x = v_x t$ which gives

$x = v_x t = (41\ \text{m/s})(0.553\ \text{s}) = 23$ m.

ASSESS: The average vertical velocity is (1.5 m)/(0.553 s) = 2.7 m/s. The constant horizontal velocity of 41 m/s is much greater than the average vertical velocity, so the arrow travels much farther horizontally (23 m) than it does vertically (1.5 m), which is physically reasonable.

99. **INTERPRET:** This is a projectile problem on the Moon and on Earth. We know the horizontal range of the golf ball on the Moon, where g is less than on Earth, and we want to find the range on Earth. We assume, quite reasonably, that the initial speed and take-off angle are the same on Earth and the Moon.

DEVELOP: The horizontal range x is given by $x = \frac{v_0^2}{g}\sin 2\theta_0$. This formula will apply on both Earth and the Moon if there is negligible air resistance. On Earth $g = 9.81\ \text{m/s}^2$ but on the Moon it is $1.62\ \text{m/s}^2$ (from Appendix E). If we take the ratio of the range on Earth to the range on the Moon, the common factors of v_0 and $\sin 2\theta_0$ will cancel out.

EVALUATE: Taking the ration x_E/x_M and canceling common factors gives

$x_E/x_M = g_M/g_E$

$x_E = (g_M/g_E)x_M = (1.62\ \text{m/s}^2)(1.30\ \text{km})/(9.81\ \text{m/s}^2) = 0.215\ \text{km} = 215$ m.

ASSESS: Since g on the Moon is less than g on Earth, the ball on the Moon will remain in flight longer than on Earth and will therefore have more time to travel horizontally. Thus the horizontal range on the Moon (1.30 km) is greater than it is on Earth (0.215 km), which agrees with our result.

100. **INTERPRET:** For circular motion, the acceleration of the car is its centripetal acceleration. We know the speed of the car and its acceleration, and we want to find the radius of the curve.

DEVELOP: The centripetal acceleration is $a = v^2/r$, where $a = 2g$ and $v = 177$ km/h. First convert the speed to units of m/s.

EVALUATE: First convert units.

$$v = \left(177\frac{\text{km}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1000\text{ m}}{1\text{ km}}\right) = 49.17\text{ m/s}.$$

Now solve $a = v^2/r$ for r and use $a = 2g$.
$r = v^2/a = (49.17\text{ m/s})^2/[2(9.81\text{ m/s}^2)] = 123\text{ m}$.
ASSESS: A radius of 123 m (≈400 ft) is reasonable for a highway curve.

101. **INTERPRET:** The x-motion of the object is at constant speed, and the y-motion has constant acceleration but zero initial speed. We want to find out how long the acceleration must last so that the x- and y-distances are equal.
DEVELOP: In the x-direction, $x - x_0 = v_x t$. In the y-direction, $v_{y0} = 0$, so we have
$y - y_0 = \frac{1}{2}a_y t^2$.
EVALUATE: Equate the two distances, and cancel common factors, giving
$v_x t = \frac{1}{2}a_y t^2$
$4.5\text{ m/s} = \frac{1}{2}(0.50\text{ m/s}^2)t$
$t = 18\text{ s}$.
ASSESS: After 18 s, the y-distance will be greater than the x-distance due to the acceleration in the y-direction. Before 18 s, the x-distance will be greater than the y-distance because the object has initial speed in the x-direction but none in the y-direction.

102. **INTERPRET:** Over horizontal ground, the maximum height is reached if we throw the ball at 45° above the horizontal. For the maximum vertical height, we should throw the ball vertically upward. Both cases involve projectile motion.

DEVELOP: The horizontal range x is $x = \frac{v_0^2}{g}\sin 2\theta_0$. For the maximum vertical height, we use the equation $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ to find $y - y_0$.

EVALUATE: For the maximum range x_{max}, $\theta_0 = 45°$, so $x_{\text{max}} = \frac{v_0^2}{g}\sin 2\theta_0 = \frac{v_0^2}{g}\sin[2(45°)] = \frac{v_0^2}{g}$.

Therefore the maximum initial speed v_0 that we can throw is given by $v_0^2 = gx_{\text{max}}$.

Now find the maximum vertical height using this maximum speed. We use the equation $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$, with $v_y = 0$ m/s at the highest point and $a = -g$.
$(0\text{ m/s})^2 = gx_{\text{max}} + 2(-g)(y - y_0)$

$$y - y_0 = \frac{x_{\text{max}}}{2}.$$

ASSESS: The maximum vertical height is only one-half the maximum horizontal range. It is reasonable that, with the same initial speed, the ball can travel farther horizontally than vertically because with the vertical motion, gravity opposes all of the motion, whereas with the horizontal motion, gravity opposes only the vertical motion but not the horizontal motion.

Chapter 4: Alternate Problem Set in *Mastering Physics*

80. **INTERPRET:** We know the force on the train and its acceleration. Use Newton's second law to find the acceleration of the train.

DEVELOP: Use $F_{net} = ma$ to find m. The only horizontal external force acting on the train is 3.8 MN = 3.8×10^6 N.

EVALUATE: $F_{net} = ma$.

$3.8 \times 10^6 \text{ N} = m(2.5 \text{ m/s}^2)$

$m = 1.5 \times 10^6$ kg.

ASSESS: The train must have a large mass since a large force (3.8 MN) gives it an acceleration of only 2.5 m/s^2, so our answer is reasonable.

81. **INTERPRET:** In both cases, we know the external force and the acceleration of the object of interest, and we want to find its mass. Newton's second applies.

DEVELOP: In part **(a)**, we just look at the locomotive and apply $F_{net} = ma$ to it to find its mass. In part **(b)**, we consider the entire train (including the locomotive) as a single system and apply $F_{net} = ma$ to it. In both cases, the only horizontal external force acting is 350 kN = 350×10^3 N.

EVALUATE: **(a)** We look at just the locomotive.

$F_{net} = ma$

$350 \times 10^3 \text{ N} = m(1.9 \text{ m/s}^2)$

$m = 184 \times 10^5 \text{ kg} = 184 \times 10^3 \text{ kg} = 184 \times 10^6 \text{ g} = 184$ Mg, which rounds to 180 Mg.

(b) Now we look at the entire train.

$F_{net} = ma$

$350 \times 10^3 \text{ N} = m(0.082 \text{ m/s}^2)$

$m = 4.27 \times 10^6 \text{ kg} = 4.27 \times 10^9$ g.

This is the mass of the locomotive plus the rest of the train, so the mass m_T of the rest of the train is

$m_T = 4.27 \times 10^9 \text{ g} - 184 \times 10^6 \text{ g} = 4.08 \times 10^9 \text{ g} = 4.08$ Gg, which rounds to 4.1 Gg.

ASSESS: When the same force is applied, the acceleration of the entire train is much less than the acceleration of just the locomotive. Therefore the rest of the train must be much more massive than the locomotive, which agrees with our results.

82. **INTERPRET:** If the speed and stopping distance of a car are both doubled, we want to see how this affects the force required to stop the car. Apply Newton's second law to the car. Assuming the acceleration is constant, we can use the constant-acceleration kinematics formulas.

DEVELOP: First use the equation $v^2 = v_0^2 + 2a(x - x_0)$ to find the acceleration of the car. Then apply $F_{net} = ma$ to it. The stopping distance, d, is $x - x_0$.

EVALUATE: At the lower speed v_0:

$v^2 = v_0^2 + 2a(x - x_0)$

$(0 \text{ m/s})^2 = v_0^2 + 2a_1 d$

$a_1 = -v_0^2/2d$

At double the speed, $v_0 \to 2v_0$ and $d \to 2d$. The same kinematics equation now gives

$(0 \text{ m/s})^2 = (2v_0)^2 + 2a_2(2d)$

$a_2 = -(4v_0^2/4d) = -v_0^2/d = 2a_1$

Therefore the acceleration of the car doubles. Applying Newton's second law to the car tells us that $F_{\text{net}} = ma$, so since the acceleration doubles, the force required to stop the car also doubles.
ASSESS: If the stopping distance did not change, then the force to stop the car would have to quadruple because the acceleration would be 4 times as great.

83. **INTERPRET:** Since you can bare lift the concrete block on Earth, the maximum force you are able to exert on it is equal to its weight. On Mars that same force would lift a more massive block because g_M on Mars is less than g on Earth.
DEVELOP: On Earth, the maximum force is the weight of the block, $w = mg$. On Mars, the force is the same but $g_M < g$, so the mass you can lift is greater. From Appendix E we find that $g_M = 3.71\ \text{m/s}^2$.
EVALUATE: On Earth your maximum force is
$w = mg = (35\ \text{kg})(9.8\ \text{m/s}^2) = 343\ \text{N}$.
On Mars your force is still 343 N, but m is different.
$w = mg_M$
$343\ \text{N} = m(3.71\ \text{m/s}^2)$
$m = 92\ \text{kg}$.
ASSESS: We have found that you can lift a greater mass on Mars than on Earth, but you cannot lift a heavier weight. On both planets, your muscles exert a maximum force of 343 N.

84. **INTERPRET:** Gravity and the air-resistance act on the parachutist. Since she descends with a steady speed, her acceleration is zero. We know the force the air exerts on her, and this must be equal to her weight.
DEVELOP: The net force on the parachutist is $F_{\text{net}} = mg - F_{\text{air}}$, and this must be equal to zero, so $mg = F_{\text{air}}$.
EVALUATE: Using $mg = F_{\text{air}}$ we have
$m(9.8\ \text{m/s}^2) = 510\ \text{N}$
$m = 52\ \text{kg}$.
ASSESS: A 52-kg person would weigh about 110 lb, which is a reasonable weight. Notice that the speed of 40 km/h is not necessary for the solution.

85. **INTERPRET:** During acceleration, the person in the elevator experiences two vertical forces: gravity, $F_g = mg$, downward and the upward normal force n of the scale (which reads his apparent weight) inside the elevator. Newton's second law applies to the person.
DEVELOP: The person's actual weight is $F_g = mg$ and his apparent weight during acceleration is n, which is 85% of his actual weight, so $n = 0.85mg$. The elevator is accelerating downward, so it is convenient to call downward positive. Newton's second law applied to the person is then
$F_{\text{net}} = mg - n = ma_y$.
We use it to find the acceleration a_y of the person (and the elevator). Then we use the kinematics formula $v_y = v_{y0} + a_y t$ to find the time t for which the acceleration lasted. The elevator started from rest, so $v_{y0} = 0$ m/s.
EVALUATE: First use Newton's second law to find the acceleration a_y.
$F_{\text{net}} = mg - n = ma_y$
$mg - 0.85mg = ma_y$
$a_y = 0.15g$.
Now use this acceleration to find the time t that it lasts.
$v_y = v_{y0} + a_y t$
$4.3\ \text{m/s} = 0\ \text{m/s} + (0.15)(9.8\ \text{m/s}^2)t$
$t = 2.9\ \text{s}$.
ASSESS: If the elevator were accelerating upward, the normal force n would be grater than the person's actual weight mg. Note that the sign of the acceleration does not tell us which way the elevator is accelerating. A downward acceleration could apply to an elevator that was moving upward but slowing down or to an elevator that was moving downward but speeding up.

86. **INTERPRET:** Hooke's law applies to the spring. We know the spring constant and the distance the spring has stretched, so we can find the force that is stretching the spring.

DEVELOP: Use $F_s = kx$ to find the force F_s.

EVALUATE: Hooke's law gives the magnitude of the spring force.

$F_s = kx = (220\text{ N/m})(0.16\text{ m}) = 35\text{ N}.$

ASSESS: Note that the 16 cm is distance the spring has stretched, *not* the length of the spring.

87. **INTERPRET:** The elevator is moving upward but slowing down, so its acceleration is downward. If the elevator accelerates downward with an acceleration of magnitude greater than g, the feet of the passengers will not touch the floor because their acceleration would be g. So the maximum downward acceleration the elevator can have with the passengers on the floor is g. We want to find the initial speed of the elevator if it has this acceleration and stops in a given time.

DEVELOP: We know that $a_y = g$ downward for the elevator, and that it stops in 0.53 s. We can use $v_y = v_{y0} + a_y t$ to find the initial speed v_{y0}. Call upward positive.

EVALUATE: Using $v_y = v_{y0} + a_y t$ we have

$0\text{ m/s} = v_{y0} + (-9.8\text{ m/s}^2)(0.53\text{ s})$

$v_{y0} = 5.2\text{ m/s}.$

ASSESS: The passengers in the elevator also have a downward acceleration of g, so they are in free fall. This is *not* a well-designed elevator!

88. **INTERPRET:** Apply Newton's second law since the crates will accelerate. The 950-N force accelerates both crates, but the spring force accelerates the lighter crate. Hooke's law applies for the spring.

DEVELOP: We can use Newton's second law for the entire two-crate system to find the acceleration of both crates. Then we can apply the same law to the rear crate, where the external horizontal force is due to the spring.

Two-crate system: The mass of the two-crate system is $m_1 + m_2$, so

$F_{net} = (m_1 + m_2)a.$

Rear crate: The only external horizontal force is that of the spring, which has magnitude $F_s = kx$. Calling m_1 the mass of the rear crate, Newton's second law gives

$F_{net} = F_s = m_1 a.$

EVALUATE: First find the acceleration.

$F_{net} = (m_1 + m_2)a$

$950\text{ N} = (490\text{ kg} + 640\text{ kg})a$

$a = 0.8407\text{ m/s}^2.$

Now look at the rear crate.

$F_{net} = F_s = m_1 a$

$kx = m_1 a$

$(8100\text{ N/m})x = (490\text{ kg})(0.8407\text{ m/s}^2)$

$x = 0.051\text{ m} = 5.1\text{ cm}.$

ASSESS: If the masses were reversed, the acceleration would be the same, but the spring would stretch more because it would be accelerating the more massive crate. In that case, the distance stretched would be

$(8100\text{ N/m})x = (640\text{ kg})(0.8407\text{ m/s}^2)$

$x = 0.066\text{ m} = 6.6\text{ cm}.$

89. **INTERPRET:** The acceleration of the plane is greater than g, so the book will be in free fall. We want to find the acceleration of the book as observed inside the plane.

DEVELOP: Relative to Earth, the plane accelerates downward at 11.8 m/s^2 and the book accelerates downward at 9.8 m/s^2.

EVALUATE: Relative to the plane, the acceleration of the book is

$11.8\text{ m/s}^2 - 9.8\text{ m/s}^2 = 2.0\text{ m/s}^2$ upward.

ASSESS: A person in the plane sees the book accelerating upward at 2.0 m/s^2 while a person on the ground sees it accelerating downward at 9.8 m/s^2.

Chapter 5: Alternate Problem Set in *Mastering Physics*

81. **INTERPRET:** From Newton's second law, we know that the vector sum of the external forces acting on an object is equal to its mass times its acceleration. In this case, we forces cause a known acceleration. We know one of the forces and want to find the other one.

DEVELOP: We apply $\vec{F}_{\text{net}} = m\vec{a}$, which in this case gives $\vec{F}_1 + \vec{F}_2 = m\vec{a}$. Call $\vec{F}_1 = \left(4.0\hat{i} + 1.7\hat{j}\right)\text{N}$ and $\vec{F}_2 = F_x\hat{i} + F_y\hat{j}$. We know that $\vec{a} = \left(0.91\hat{i} - 0.27\hat{j}\right)\text{m/s}^2$ and $m = 3.1$ kg, and we want to find F_x and F_y.

EVALUATE: Using $\vec{F}_1 + \vec{F}_2 = m\vec{a}$ gives

$$\left(4.0\hat{i} + 1.7\hat{j}\right)\text{N} + F_x\hat{i} + F_y\hat{j} = (3.1\text{ kg})\left(0.91\hat{i} - 0.27\hat{j}\right)\text{m/s}^2.$$

Collecting coefficients of the unit vectors on the left-hand side gives

$$(4.0\text{ N} + F_x)\ \hat{i} + (1.7\text{ N} + F_y)\ \hat{j} = 2.8\text{ N}\,\hat{i} - 0.84\text{ N}\,\hat{j}.$$

Equating coefficients on the left to corresponding coefficients on the right gives

$4.0\text{ N} + F_x = 2.8\text{ N}$

$F_x = -1.2\text{ N}$

$1.7\text{ N} + F_y = -0.84$

$F_y = -2.5\text{ N}$

Therefore the unknown force vector is

$$\vec{F}_2 = \left(-1.2\hat{i} - 2.5\hat{j}\right)\text{N}.$$

ASSESS: Both components of the unknown vector are negative, so it points into the third quadrant.

82. **INTERPRET:** The acceleration of the glider is the same as the acceleration due to gravity on a certain planet in our solar system. We can determine the glider's acceleration using the angle at which the air track is tilted above the horizontal.

DEVELOP: Since only gravity acts parallel to the air track, the acceleration of the glider is $a = g\sin\theta$. We can calculate the acceleration a and compare it with the values of g for various planets found in Appendix E.

EVALUATE: $a = g\sin\theta = (9.81\text{ m/s}^2)\sin(22.4°) = 3.74\text{ m/s}^2$. From Appendix E, this value is closest to that of Mars, where $g_{\text{Mars}} = 3.71\text{ m/s}^2$, so the planet must be Mars.

ASSESS: The acceleration we found is also very close to that of Mercury, which is 3.70 m/s^2.

83. **INTERPRET:** The blocks are accelerating, so we apply Newton's second law to each of the blocks. Only gravity is causing the system to accelerate since there is no friction. Both blocks have the same magnitude acceleration and the magnitude of the tension T is the same on each block.

DEVELOP: We apply Newton's second law to each block. In each case, it is best to take the positive direction to be the same as the direction of the acceleration. In each case, we apply $F_{\text{net}} = ma$ parallel to the surface of the incline. The only forces acting on each block are gravity down the incline and the tension in the rope.

Right-hand block: It is accelerating down the plane, so we take downward as positive. Its mass is $M = 7.1$ kg. Newton's second law gives

$Mg\sin 20° - T = Ma$ (Eq. 1)

Left-hand block: It is accelerating up the plane, so we take upward as positive and call its mass m. This gives

$T - mg\sin 60° = ma$ (Eq. 2)

EVALUATE: Now combine equations (1) and (2) to find m.

$Mg\sin 20° - mg\sin 60° = Ma + ma$

$$m = \frac{Mg\sin 20° - Ma}{a + g\sin 60°} = \frac{(7.1\text{ kg})(9.8\text{ m/s}^2) - (7.1\text{ kg})(0.64\text{ m/s}^2)}{0.64\text{ m/s}^2 + (9.8\text{ m/s}^2)\sin 60°} = 2.1\text{ kg}.$$

ASSESS: Another way to approach this problem is to visualize it as a single object of mass $m + M$ that is being accelerated on a frictionless horizontal surface by a force of $Mg\sin 20°$ in one direction and $mg\sin 60°$ in the opposite direction, with the acceleration in the direction of the $Mg\sin 20°$ force. Newton's second law for this system gives $Mg\sin 20° - mg\sin 60° = (m + M)a$. This is the same result we got by combining equations (1) and (2), but this new approach requires a bit less work.

84. **INTERPRET:** If the tension T in the cable held Earth in orbit, that tension would have to be equal to the centripetal force on the Earth. This force depends on the mass of Earth, it orbital speed and orbital radius (the distance to the Sun).

DEVELOP: Use $T = m\dfrac{v^2}{r}$ and solve for T. Appendix E gives the following values:

$v = 29.8\text{ km/s} = 2.98\times 10^4\text{ m/s}$, $r = 149.6\times 10^6\text{ km} = 1.496\times 10^{11}\text{ m}$, and

$m = 5.97\times 10^{24}\text{ kg}$.

EVALUATE: $T = m\dfrac{v^2}{r} = \dfrac{(5.97\times 10^{24}\text{ kg})(2.98\times 10^4\text{ m/s})^2}{1.496\times 10^{11}\text{ m}} = 3.54\times 10^{22}\text{ N}.$

INTERPRET: Our result is the gravitational force that the Sun exerts on Earth to hold it in orbit, so it must be a *very* large force, as we have found.

85. **INTERPRET:** We know the banking angle and the speed of the plane and want to find the radius of the turn. We can use the same result as for a car on a banked curve since the same principles are involved.

DEVELOP: The formula $\tan\theta = v^2/rg$ applies. We can solve for r. We must convert $v = 490$ km/h to units of m/s, and we know that $\theta = 28°$.

EVALUATE: Convert 490 km/h: $v = \left(490\dfrac{\text{km}}{\text{h}}\right)\left(\dfrac{1\text{ h}}{3600\text{ s}}\right)\left(\dfrac{1000\text{ m}}{1\text{ km}}\right) = 136.1\text{ m/s}.$

Solving $\tan\theta = v^2/rg$ for r gives

$$r = \frac{v^2}{g\tan\theta} = \frac{(136.1\text{ m/s})^2}{(9.8\text{ m/s}^2)\tan 28°} = 3.6\times 10^3\text{ m} = 3.6\text{ km}.$$

ASSESS: It may seem that 3.6 km is a very large radius of curvature, but the plane's speed is 490 km/h, which is about 300 mi/h. This speed is much faster than that of a car on a highway, so a large radius is necessary.

86. **INTERPRET:** We know the initial speed of the block and the distance it goes up the ramp. From this we can find its acceleration, and from that we can find the inclination angle of the ramp. Only gravity acts on the block parallel to the surface of the ramp.

DEVELOP: First use $v^2 = v_0^2 + 2a(x - x_0)$ to find the acceleration. We know that $v = 0$ m/s at the highest point, $v_0 =$ 2.2 m/s, and $x - x_0 = 43\text{ cm} = 0.43\text{ m}$. Using the acceleration, we know that $a = g\sin\theta$ because only gravity acts parallel to the surface of the ramp.

EVALUATE: Find the acceleration.

$v^2 = v_0^2 + 2a(x - x_0)$

$(0\text{ m/s})^2 = (2.2\text{ m/s})^2 + 2a(0.43\text{ m})$

$a = 5.628\text{ m/s}^2$.

Now find θ.

$a = g\sin\theta$

$5.628\text{ m/s}^2 = (9.8\text{ m/s}^2)\sin\theta$

$\theta = 35°$.

ASSESS: This is quite a steep ramp, so it would not be very practical for heavy objects or for people using wheel chairs.

87. **INTERPRET:** The analysis of this problem is like that for a car on a banked curve. We know the radius her turn and the angle her body makes with the vertical, and we want to find her speed.

DEVELOP: Use $\tan\theta = v^2/rg$ to solve for v. We know that r = 5.0 m and θ = 21°.

EVALUATE: Solving for v gives

$$v = \sqrt{rg\tan\theta} = \sqrt{(5.0\text{ m})(9.8\text{ m/s}^2)\tan 21°} = 4.3\text{ m/s}.$$

ASSESS: At a brisk walk a person moves at around 1 m/s. The skater's speed is over 4 times as great, but this is reasonable for an ice skater.

88. **INTERPRET:** As the children sled down the hill, gravity speeds them up but friction with the snow opposes their motion. After entering the level stretch, friction slows them down until they stop. We know the angle θ of the hill above the horizontal and the distance they slide on it. On the level stretch, we know the stopping distance. We are looking for the coefficient of friction between the sled and the snow assuming that it is the same on the hill and on the level stretch. Newton's second law and the constant-acceleration kinematics formulas apply to the motion on the slope and on the level stretch. The final speed at the bottom of the hill is equal to the initial speed just as they enter the level stretch.

DEVELOP: <u>On the slope</u>: Apply Newton's second law parallel to the surface of the slope, taking the positive direction downward since that is the direction of the acceleration. The external forces acting on the sled are kinetic friction f_k, the normal force n, and gravity mg. There is no acceleration perpendicular to the slope, so

$n = mg\cos\theta$.

Along the surface of the slope, Newton's second law gives

$mg\sin\theta - f_k = ma_1$

where a_1 is the acceleration on the slope. For kinetic friction, we know that

$f_k = \mu_k n = \mu_k mg\cos\theta$.

Putting this into Newton's second law gives

$mg\sin\theta - \mu_k mg\cos\theta = ma_1$.

To find the speed at the bottom of the hill, we use $v^2 = v_0^2 + 2a(x - x_0)$, where $v_0 = 0$ and $x - x_0 = 41$ m.

<u>On the level stretch</u>: Only friction acts horizontally, so $f_k = ma_2$, where a_2 is the acceleration on the level stretch. The vertical forces balance, so $n = mg$. Therefore Newton's second law gives

$\mu_k mg = ma_2$

$a_2 = \mu_k g$.

We know the stopping distance is 110 m, so we use $v^2 = v_0^2 + 2a(x - x_0)$. In this case, $v = 0$, $a = a_2$, $x - x_0 = 110$ m, and v_0 is equal to the speed at the bottom of the hill.

EVALUATE: <u>On the slope</u>: From Newton's second law, we found that

$mg\sin\theta - \mu_k mg\cos\theta = ma_1$.

$(9.8\text{ m/s}^2)\sin 25° - \mu_k(9.8\text{ m/s}^2)\cos 25° = a_1$

$4.142\text{ m/s}^2 - \mu_k(8.882\text{ m/s}^2) = a_1$ (Eq. 1)

Use $v^2 = v_0^2 + 2a(x - x_0)$ to relate v to a_1.

$v^2 = (0\text{ m/s})^2 + 2a_1(41\text{ m})$

$v^2 = (82\text{ m})a_1$.

<u>On the level stretch</u>: Using Newton's second law, we saw that $a_2 = \mu_k g$ in a direction opposite to the velocity. Now apply $v^2 = v_0^2 + 2a(x - x_0)$, where

$v_0^2 = (82\text{ m})a_1$ from our previous work and $a = a_2 = \mu_k g$.

Using the above quantities, the formula $v^2 = v_0^2 + 2a(x - x_0)$ gives

$(0\text{ m/s})^2 = (82\text{ m})a_1 - 2\mu_k(9.8\text{ m/s}^2)(110\text{ m})$

$(110\text{ m})(9.8\text{ m/s}^2)\mu_k = (41\text{ m})a_1$.

Now use a_1 from equation (1).

$(110\text{ m})(9.8\text{ m/s}^2)\mu_k = (41\text{ m})[4.142\text{ m/s}^2 - \mu_k(8.882\text{ m/s}^2)]$

Solving for μ_k gives

$\mu_k = 0.12$.

ASSESS: According to the text, a waxed ski on snow has $\mu_k \approx 0.04$, and our coefficient of friction is 0.12, which is about 3 times as great. This result is reasonable because a sled on snow would slide easily, but certainly not as easily as a waxed ski.

89. **INTERPRET:** The limitation on the box's time on the ramp determines what its acceleration down the ramp must be. This acceleration then determines what the inclination angle θ should be since we know the coefficient of kinetic friction between the box and the ramp. Newton's second law and the constant-acceleration kinematics formulas apply to the box.

DEVELOP: The initial velocity of the box is zero and it must slide 5.40 m in no more than 3.30 s. We use the equation $x = x_0 + v_0t + \frac{1}{2}at^2$ to find the acceleration. Next apply Newton's second law to the box. The external forces acting on it are gravity mg, the normal force n, and the kinetic friction force f_k. Forces perpendicular to the ramp surface cancel, so $n = mg\cos\theta$. Call down the ramp positive since that is the direction of the acceleration. Therefore

$mg\sin\theta - f_k = ma$.

We also know that $f_k = \mu_k n = \mu_k mg\cos\theta$, so, after canceling m, the previous equation becomes

$g\sin\theta - \mu_k g\cos\theta = a$.

We use the acceleration from our work with kinematics.

EVALUATE: Find the acceleration using $x = x_0 + v_0t + \frac{1}{2}at^2$, where $x - x_0 = 5.40$ m, $v_0 = 0$ m/s, and $t = 3.30$ s. This gives

$5.40\text{ m} = \frac{1}{2}a(3.30\text{ s})^2$

$a = 0.9917\text{ m/s}^2$.

Now use our results from Newton's second law, $g\sin\theta - \mu_k g\cos\theta = a$, to find θ.

$(9.8\text{ m/s}^2)\sin\theta - (0.37)(9.8\text{ m/s}^2)\cos\theta = 0.9917\text{ m/s}^2$

$\sin\theta - 0.37\cos\theta = 0.1012$.

To solve this equation, we use the identity $\sin^2\theta + \cos^2\theta = 1$ to put the equation only in terms of $\cos\theta$. This gives

$\pm\sqrt{1-\cos^2\theta} - 0.37\cos\theta = 0.1012$.

Separate the square root, square both sides of the equation, and collect terms. The result is

$1.1369\cos^2\theta + 0.07489\cos\theta - 0.98976 = 0$.

Use the quadratic formula, $\cos\theta = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, to solve for $\cos\theta$. The result gives two solutions. One is negative and is therefore not allowed since $0 < \theta < 90°$. The other solution is $\cos\theta = 0.90069$, which gives $\theta = 26°$.

ASSESS: The acceleration is rather low ($\approx g/10$), so the inclination angle can be 26°, which is fairly large. If there were no friction, the acceleration would be $g\sin 26° = 4.3\text{ m/s}^2$, considerably greater than 0.99 m/s^2.

90. **INTERPRET:** As the block slides up the ramp, kinetic friction is down the ramp, but as it slides down the ramp, friction is up the ramp. The normal force and the force of gravity are the same in both cases. Apply Newton's second law on the way up and on the way down.

DEVELOP: In both cases (up and down), $f_k = \mu_k n$, where n is the normal force, which is $n = mg\cos\theta$ in both cases. Using the given value of μ_k, we have

$f_k = (\frac{3}{5}\tan\theta)(mg\cos\theta) = \frac{3}{5}mg\sin\theta$.

Apply Newton's second law along the surface of the ramp. Use the kinematics equation $v_y^2 = v_{y0}^2 + 2a_y(y - y_0)$ as needed.

EVALUATE: Going up: The acceleration, friction, and gravity component are downward. Applying Newton's second law along the ramp surface, with downward positive, gives

$mg\sin\theta + \frac{3}{5}mg\sin\theta = ma_{up}$

$a_{up} = \frac{8}{5}g\sin\theta$ (this acceleration is down the ramp).

Now find the maximum distance $x - x_0 = d$ up the ramp, calling upward positive.

$$v^2 = v_0^2 + 2a(x - x_0)$$

$$(0 \text{ m/s})^2 = v_0^2 + 2(-\tfrac{8}{5} g \sin\theta)d$$

$$d = \frac{5v_0^2}{16g\sin\theta}.$$

Going down: Call downward positive and apply Newton's second law.

$mg \sin\theta - f_k = ma$

$mg \sin\theta - \mu_k mg\cos\theta = ma_{\text{down}}$

Using the given value for μ_k, this becomes

$g \sin\theta - (\tfrac{3}{5}\tan\theta)g\cos\theta = a_{\text{down}}$

$a_{\text{down}} = \tfrac{2}{5} g \sin\theta$.

Now find v at the bottom, with $x - x_0 = d = \dfrac{5v_0^2}{16g\sin\theta}$ as we just found. Using $v^2 = v_0^2 + 2a(x - x_0)$, we have

$$v^2 = 0^2 + 2\left(\tfrac{2}{5}g\sin\theta\right)\left(\frac{5v_0^2}{16g\sin\theta}\right) = \frac{v_0^2}{4}$$

$$v = \frac{v_0}{2}.$$

Assess: If the ramp angle were $\theta = 30°$, μ_k would be $\tfrac{3}{5}\tan 30° = 0.35$, which is a reasonable coefficient of friction.

Chapter 6: Alternate Problem Set in *Mastering Physics*

94. **INTERPRET:** We want the coefficient of kinetic friction between the box and the floor. Since the speed of the box is constant, the horizontal forces on it must balance, so the work done by the push must be equal to the magnitude of the work done by friction.

DEVELOP: Use the formula for work when the force and displacement are along the same direction, $W = F_x \Delta x$. The normal force n is $n = mg$, and $f_k = \mu_k n = \mu_k mg$. The forces balance, so $W_{\text{push}} = |W_{\text{friction}}| = \mu_k mg\,\Delta x$.

EVALUATE: Using $W_{\text{push}} = \mu_k mg\,\Delta x$ gives

$490\text{ J} = \mu_k(50\text{ kg})(9.8\text{ m/s}^2)(4.8\text{ m})$

$\mu_k = 0.21.$

ASSESS: A coefficient of kinetic friction of 0.21 is reasonable for typical surfaces such as those in this problem. We also note that the push does positive work on the box while friction does negative work, making the net work zero.

95. **INTERPRET:** We know the work done on the meteorite and the average force on it. We want to find the distance over which this force acted. We can use the definition of work.

DEVELOP: Use $W = F\Delta r\cos\theta$. In this case, the force is upward and the displacement is downward, so $\theta = 180°$. We know that the magnitude of the force on the meteorite is 190 MN and the work it does on the meteorite is –140 MJ.

EVALUATE: Using $W = F\Delta r\cos\theta$ gives

$-140\text{ MJ} = (190\text{ MN})(\Delta r)(\cos 180°) = -(190\text{ MN})\,\Delta r$

$\Delta r = 0.74\text{ m} = 74\text{ cm}.$

ASSESS: Earth does negative work on the meteorite but the meteorite does positive work on the Earth. Note that it was not necessary to convert the work and force to standard SI units because both of them were expressed using the mega prefix, which canceled out. Only convert units when it is necessary!

96. **INTERPRET:** We know the mass and speed of each vehicle, so we can calculate their kinetic energies and compare them.

DEVELOP: Use $K = \frac{1}{2}mv^2$. To compare the kinetic energies, divide the kinetic of the car by that of the truck.

EVALUATE: Compare the kinetic energies, using subscript "t" for the truck and "c" for the car.

$$\frac{K_c}{K_t} = \frac{\frac{1}{2}m_c v_c^2}{\frac{1}{2}m_t v_t^2} = \frac{(950\text{ kg})(114\text{ km/h})^2}{(31{,}600\text{ kg})(14\text{ km/h})^2} = 2.0\,.$$

$K_c = 2.0K_t.$

ASSESS: The mass of the car is only 3% that of the truck, but its speed is about 8 times as great. Since kinetic energy depends on the *square* of the speed but only the first power of the mass, the greater speed of the car more than makes up for its smaller mass to give it a greater kinetic energy than the truck. Notice also that we did not have to convert km/h to SI units since the units cancel when taking the ratio. Only convert units when it is necessary!

97. **INTERPRET:** We want to find out if the kinetic energy of a car traveling at 30 mi/h is the same as if it were dropped from 40 ft.

DEVELOP: Use $K = \frac{1}{2}mv^2$ to get the kinetic energy at 30 mi/h. Then use the work-kinetic energy theorem, $W_{\text{net}} = \Delta K$, to find the height from which the car would have to be dropped to have this amount of kinetic energy when it hit the ground. The net work is $W_{\text{net}} = mg\,\Delta y$. The initial kinetic energy is zero, and the final kinetic energy is the energy at 30 mi/h. We first convert the speed to units of m/s. The instructor's original statement applied to any car, regardless of mass, so we just us m for the mass of the car.

EVALUATE: First convert units.

$$\left(30\frac{\text{mi}}{\text{h}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)=13.41\text{ m/s}.$$

The kinetic energy at that speed is

$K = \frac{1}{2}mv^2 \; = \; \frac{1}{2}m(13.41\text{ m/s})^2 = (89.9\text{ m}^2/\text{s}^2)m.$

Now use $W_{\text{net}} = \Delta K$ to find the height:

$W_{\text{net}} = \Delta K \; = K_2 - K_1$

$mg\,\Delta y \; = (89.9\text{ m}^2/\text{s}^2)m - 0\text{ J}$

$\Delta y \; = 9.2\text{ m}.$

Converting this result to feet gives

$$(9.2\text{ m})\left(\frac{1\text{ ft}}{0.3048\text{ m}}\right)=30\text{ ft}.$$

Therefore the instructor's comment was not quite correct.

ASSESS: If the car were dropped from 40 ft (12.2 m), its speed would be

$mg\,\Delta y \; = \; K = \frac{1}{2}mv^2$

$v=\sqrt{2g\Delta y}=\sqrt{2(9.8\text{ m/s}^2)(12.2\text{ m})} \; = 15.5\text{ m/s} = 35\text{ mi/h}.$

The figure the instructor used was 30 mi/h, which is not too different from 35 mi/h, so he was not far off.

98. **INTERPRET:** We know the work done by the lawnmower and the time for this work, and we want to find the power output of the mower. We can use the definition of average power.

DEVELOP: The average power is $\bar{P}=\dfrac{\Delta W}{\Delta t}$, where $\Delta W \; = 9.4$ MJ and $\Delta t \; = 1$ h. We also know that 1 hp = 746 W.

EVALUATE: (a) $\bar{P}=\dfrac{\Delta W}{\Delta t}=\dfrac{9.4\times10^6\text{ J}}{3600\text{ s}}=2.6\times10^3\text{ W} = 2.6\text{ kW}.$

(b) Convert 2.6 kW to units of horsepower.

$$(2600\text{ W})\left(\frac{1\text{ hp}}{746\text{ W}}\right)=3.5\text{ hp}.$$

ASSESS: Ten 100-W light bulbs consume 1 kW of power, so this lawnmower output is equivalent to 26 such light bulbs in terms of power consumption.

99. **INTERPRET:** We know the force exerted on the train and the work done by that force. We want to find the distance through which the force acts. We can use the definition of work.

DEVELOP: Since the force is in the same direction as the displacement, we use $W = F_x\Delta x$, where $F_x = 4.4$ MN = 4.4×10^6 N and $W = 0.79$ TJ = 0.79×10^{12} J.

EVALUATE: Using $W = F_x\Delta x$ gives

$0.79\times10^{12}\text{ J} = (4.4\times10^6\text{ N})\,\Delta x$

$\Delta x \; = 1.8\times10^5\text{ m} = 180\text{ km}.$

ASSESS: It is reasonable that a coal mine would be 180 km (≈110 mi) from a power plant.

100. **INTERPRET:** We can use the given vector components to calculate the dot product of the vectors. We can then use that result to find the angle between the vectors.

DEVELOP: We can express the scalar product either $\vec{A}\cdot\vec{B}=AB\cos\theta$ or as $\vec{A}\cdot\vec{B}=A_xB_x + A_yB_y$. For part **(a)** we use the second form since we know the vector components. For part **(b)** we use the first form since we want the angle θ between the two vectors. We also know that $A=\sqrt{A_x^2 + A_y^2}$ for any vector.

EVALUATE: **(a)** Using the components, we have

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y = a(-a) + a(2a) = a^2.$

(b) In the formula $\vec{A}\cdot\vec{B} = AB\cos\theta$, we use the result of part (a), along with the formula $A=\sqrt{A_x^2+A_y^2}$ for both vectors.

$\vec{A}\cdot\vec{B} = AB\cos\theta = \sqrt{A_x^2+A_y^2}\sqrt{B_x^2+B_y^2}\cos\theta$

$a^2 = \sqrt{a^2+a^2}\sqrt{(-a)^2+(2a)^2}\cos\theta = \left(a\sqrt{2}\right)\left(a\sqrt{5}\right)\cos\theta = \left(a^2\sqrt{10}\right)\cos\theta$

$\theta = \cos^{-1}\left(1/\sqrt{10}\right) = 71.6°.$

ASSESS: In part **(a)** we found that the scalar product of the vectors was positive, so the angle between them must be less than 90°. This agrees with our result in **(b)** of $\theta = 71.6°$.

101. **INTERPRET:** We know the force vector acting on an object, the work done by this force during a displacement, and the initial position vector of the object. We also know that the *x*- and *y*-components of the final position vector are both zero. We want to find the *z*-component of the final position vector.

DEVELOP: Use $W = \vec{F}\cdot\Delta\vec{r} = \vec{F}\cdot(\vec{r}_2-\vec{r}_1)$. We know that $\vec{r}_1 = (16\hat{i}+31\hat{j})\,\text{m}$, $\vec{F} = (67\hat{i}+23\hat{j}+55\hat{k})\,\text{N}$, and $W =$ 622 J. We can express $\vec{r}_2$ as $\vec{r}_2 = z\,\hat{k}$, where we want to find *z*.

EVALUATE: Using $W = \vec{F}\cdot(\vec{r}_2-\vec{r}_1)$ gives us

622 J = $(67\hat{i}+23\hat{j}+55\hat{k})\,\text{N}\cdot[z\,\hat{k} - (16\hat{i}+31\hat{j})\,\text{m}]$

622 J = (67 N)(–16 m) + (23 N)(–31 m) + (55 N)(*z*)

z = 44 m.

ASSESS: The object begins in the *xy*-plane and ends up on the +*z*-axis at 44 m from the origin.

102. **INTERPRET:** Spring B stretches twice as much as spring A, and in so doing the work done on spring B is 3 times the work to stretch spring A. We want to compare the spring constants of springs A and B.

DEVELOP: The work to stretch a spring a distance *x* is $W = \frac{1}{2}kx^2$.

Spring A: $W_\text{A} = \frac{1}{2}k_\text{A}x_\text{A}^2$.

Spring B: $W_\text{B} = \frac{1}{2}k_\text{B}x_\text{B}^2$, where $x_\text{B} = 2x_\text{A}$ and $W_\text{B} = 3W_\text{A}$.

EVALUATE: Take the ratio of the two works using the fact that $W_\text{B} = 3W_\text{A}$.

$$\frac{W_\text{B}}{W_\text{A}} = \frac{\frac{1}{2}k_\text{B}x_\text{B}^2}{\frac{1}{2}k_\text{A}x_\text{A}^2} = \frac{k_\text{B}(2x_\text{A})^2}{k_\text{A}x_\text{A}^2} = 3$$

$$\frac{4k_\text{B}x_\text{A}^2}{k_\text{A}x_\text{A}^2} = 3$$

$k_\text{B} = \frac{3}{4}k_\text{A}$.

ASSESS: If springs A and B had the same spring constants, the work to stretch B twice as far as A would be 4 times the work to stretch A because *W* is proportional to x^2. Since it required only 3 times as much work, B must have a smaller spring constant than A, as we found. So A is the stiffer of the two springs.

103. **INTERPRET:** The car travels at constant speed so the power delivered by the drive wheels must be equal to the power against gravity, which acts down the incline. We want the angle θ of the incline above the horizontal.

DEVELOP: Use $\overline{P} = \vec{F}\cdot\vec{v}$. This gives

$P_\text{car} = P_\text{grav} = \overline{P} = \vec{F}_\text{g}\cdot\vec{v} = F_\text{g}v\sin\theta = mgv\sin\theta.$

We know that P_car = 35 kW = 35,000 W, m = 1700 kg, and v = 75 km/h, and we want θ.

EVALUATE: First convert 75 km/h to units of m/s.

$$\left(75\frac{\text{km}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1000\ \text{m}}{1\ \text{km}}\right) = 20.83\ \text{m/s}.$$

Now use $P_{car} = mgv \sin\theta$ to find θ.

35,000 W = (1700 kg)(9.8 m/s^2)(20.83 m/s) sin θ

θ = 5.8°.

ASSESS: An incline at 5.8° is a 10% grade because tan 58° = 0.10. This is a fairly steep grade. Highway grades rarely get this steep except for some mountain roads.

Chapter 7: Alternate Problem Set in *Mastering Physics*

72. **INTERPRET:** You have gravitational potential energy due to the altitude of the plane. We know your mass and potential energy and want to find your altitude.

DEVELOP: We use $U = mg\Delta y$, where Δy is the altitude of the plane (which we want to find). We know U and your mass m.

EVALUATE: Using $U = mg\Delta y$ gives

7.0×10^6 J = (65 kg)(9.8 m/s^2) Δy

Δy = 11,000 m = 11 km.

ASSESS: An altitude of 11 km is 6.8 mi, which is 36,000 ft. Jetliners typically cruise at such an altitude, so our answer is physically reasonable.

73. **INTERPRET:** This problem involves the elastic potential energy of a spring. The spring is initially stretched an unknown distance. We give it two additional stretches and know the total potential energy in each case. From this information we want to find the spring constant and the initial distance the spring was stretched.

DEVELOP: The elastic potential of an ideal spring is $U = \frac{1}{2}kx^2$. Call x_0 the initial distance the spring was stretched. When we stretch it 2.5 cm beyond x_0, its stretched length is x_0 + 2.5 cm and the stored energy is 18 J, so $U_{2.5} = \frac{1}{2}k\left(x_0 + 2.5\text{ cm}\right)^2 = 18\text{ J}$. When we stretch an additional 2.5 cm, its stretched length is x_0 + 5.0 cm and the stored energy is 25 J, so $U_{5.0} = \frac{1}{2}k\left(x_0 + 5.0\text{ cm}\right)^2 = 25\text{ J}$. We can solve these two equations simultaneously for k and x_0.

EVALUATE: It is easier to solve part (b) first.

(b) If we take the ratio of the potential energies, the common factor of k will cancel, leaving x_0 as the only unknown. This gives

$$\frac{U_{5.0}}{U_{2.5}} = \frac{\frac{1}{2}k\left(x_0 + 5.0\text{ cm}\right)^2}{\frac{1}{2}k\left(x_0 + 2.5\text{ cm}\right)^2} = \frac{25\text{ J}}{18\text{ J}} = 1.389$$

$(x_0 + 5.0\text{ cm})^2 = 1.389(x_0 + 2.5\text{ cm})^2$

We could square the quantities in parentheses, collect terms, and apply the quadratic formula. However it is somewhat easier if we just take the square root of both sides of the equation, giving

$$\pm\left(x_0 + 5.0\text{ cm}\right) = \sqrt{1.389}\left(x_0 + 2.5\text{ cm}\right).$$

There are two possible solutions, one using the plus sign and the other using the minus sign. As you can show, use of the minus sign leads to a value of x_0 that is negative, which is not physical. So we use only the plus sign, which gives

$$x_0 + 5.0\text{ cm} = \sqrt{1.389}\left(x_0 + 2.5\text{ cm}\right).$$

Solving for x_0 using a little algebra gives

$$x_0 = \frac{5.0\text{ cm} - 2.5\text{ cm}\sqrt{1.389}}{\sqrt{1.389} - 1} = 11.5\text{ cm}.$$

We can now use this value for x_0 to solve for k.

(a) Using $U_{2.5} = \frac{1}{2}k\left(x_0 + 2.5\text{ cm}\right)^2 = 18\text{ J}$, we have

$$\tfrac{1}{2}k\left(11.5\text{ cm} + 2.5\text{ cm}\right)^2 = 18\text{ J}$$

Since the right-hand side is in joules, we must convert the cm to units of m, giving

$\frac{1}{2}k\left(0.14\ \text{m}\right)^2 = 18\ \text{J}$

$k = 1.83 \times 10^3$ N/m, which rounds to $k = 1.8$ kN/m.

ASSESS: To check, use the equation $U_{5.0} = \frac{1}{2}k\left(x_0 + 5.0\ \text{cm}\right)^2$ to calculate $U_{5.0}$.

$U_{5.0} = \frac{1}{2}k\left(11.5\ \text{cm} + 5.0\ \text{cm}\right)^2 = \frac{1}{2}(1.837\times10^3\ \text{N/m})\left(0.165\ \text{m}\right)^2 = 25\ \text{J}$. So our values of k and x_0 check.

74. **INTERPRET:** This problem involves the conservation of mechanical energy. The initial elastic potential energy of the arrow is converted to gravitational potential energy at its highest point. We want to find the effective spring constant k of the bow.

DEVELOP: Conservation of mechanical energy is $K + U = K_0 + U_0$. In this case, $U_0 = \frac{1}{2}kx^2$, $K_0 = K = 0$, and $U = mg\Delta y$. We want to find k.

EVALUATE: Since $U = U_0$, we have

$mg\Delta y = \frac{1}{2}kx^2$

$(0.120\ \text{kg})(9.8\ \text{m/s}^2)(92\ \text{m}) = \frac{1}{2}k(0.71\ \text{m})^2$

$k = 430$ N/m.

ASSESS: It takes 4.3 N (≈2 lb) to stretch the bow by 1.0 cm. This is a fairly stiff "spring," which is reasonable for a bow.

75. **INTERPRET:** We model the molecule as a spring, so this problem involves elastic potential energy. We know the spring constant and displacement, and want the potential energy.

DEVELOP: Use $U = \frac{1}{2}kx^2$, where $k = 1.9$ kN/m and $x = 1.6$ pm. We must convert to standard SI units.

EVALUATE: In SI units, $k = 1.9$ kN/m = 1900 N/m and $x = 1.6$ pm $= 1.6 \times 10^{-12}$ m.

$U = \frac{1}{2}kx^2 = \frac{1}{2}(1900\ \text{N/m})(1.6 \times 10^{-12}\ \text{m})^2 = 2.4 \times 10^{-21}\ \text{J}$.

This is a very small amount of energy, so it is convenient to express it in units of electronvolts (eV). We use the fact that 1 eV $= 1.602 \times 10^{-19}$ J, from Appendix C.

$$U = (2.4\times10^{-21}\ \text{J})\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = 15\times10^{-3}\ \text{eV} = 15\ \text{meV}.$$

ASSESS: Energies at the atomic level are normally in the electronvolt range, so our answer seems reasonable.

76. **INTERPRET:** This problem involves elastic potential energy, but the spring is not ideal and does not obey Hooke's law. We know that the force is $F = -kx - bx^3$ and want to find b.

DEVELOP: Since the force varies with x and does not obey Hooke's law, we must integrate to find the potential energy. So we use $\Delta U = -\int_{x_1}^{x_2} F(x)dx$. In this case, we have $x_1 = 0$, $x_2 = 43.0$ cm = 0.430 m, and $F = -kx - bx^3$, where $k = 172$ N/m. When $x = 43.0$ cm, $\Delta U = 16.7$ J.

EVALUATE: $\Delta U = -\int_{x_1}^{x_2} F(x)dx = -\int_0^{0.430\ \text{m}}\left(-kx - bx^3\right)dx = \left.\dfrac{kx^2}{2} + \dfrac{bx^4}{4}\right|_{0\ \text{m}}^{0.430\ \text{m}}$

$$16.7\ \text{J} = \frac{(172\ \text{N/m})(0.430\ \text{m})^2}{2} + \frac{b(0.430\ \text{m})^4}{4}$$

$b = 93.4\ \text{N/m}^3$.

ASSESS: If this spring were ideal, the energy stored at $x = 4.30$ cm would be

$U = \frac{1}{2}kx^2 = \frac{1}{2}(172\ \text{N/m})(0.430\ \text{m})^2 = 15.9$ J. For this spring, we know that the stored energy is 16.7 J. The percent difference is $\dfrac{16.7\ \text{J} - 15.9\ \text{J}}{15.9\ \text{J}}\times100 = 5.0\ \%$. This is a measurable difference, but not extremely large. For small x, the difference would be even less because $x^4 << x^2$ for $x << 1$. But for larger x the difference would be greater as x^4 becomes important compared to x^2.

77. **INTERPRET:** We are dealing with a potential energy due to a force of the form $F = ax^{-n}$. We want to find limits on the value of n so that the potential energy is finite as $x \to \infty$.

DEVELOP: We use $\Delta U = -\int_{x_1}^{x_2} F(x)dx$. In this case, we have $F(x) = ax^{-n}$, $x_1 = x_0$, and $x_2 \to \infty$. We want to find n so that ΔU is finite.

EVALUATE: Using the conditions of this problem, we have

$$\Delta U = \lim_{x_2\to\infty}\left(-\int_{x_0}^{x_2} ax^{-n}dx\right) = -a\lim_{x_2\to\infty}\left[\frac{1}{-n+1}\left(x_2^{-n+1} - x_0^{-n+1}\right)\right].$$

We want to find n such that ΔU is finite as $x_2 \to \infty$. Since x_0 is finite, we need only look at the limit on x_2, which is $-a\lim_{x_2\to\infty}\left[\frac{1}{-n+1}\left(x_2^{-n+1}\right)\right]$. If $n < 1$, the exponent $-n + 1$ is negative, so the limit is infinity as $x_2 \to \infty$. If $n > 1$, the exponent $-n + 1$ is negative, so the limit is zero as $x_2 \to \infty$. If $n = 1$, $-n + 1 = 0$, so the $-n + 1$ in the denominator is infinite. Therefore the only possibility is $n > 1$.

ASSESS: We have just shown that a finite potential energy occurs only if F is of the form $F = a/x^n$, where $n > 1$. This would *exclude* forces such as $F = a/x^{1/2} = a/\sqrt{x}$. Even though this force goes to zero as x approaches infinity, the potential energy does not. Just because the force approaches zero as x approaches infinity, it does *not* necessarily follow that the potential energy approaches zero.

78. **INTERPRET:** In this problem, the gravitational potential energy of the water is converted to electrical energy, but only with an 85% efficiency. We want the power output, which is the *rate* at which the conversion occurs.

DEVELOP: The average power is $\bar{P} = \dfrac{\Delta U}{\Delta t}$, where ΔU is 85% of the potential energy of the water in the reservoir. The potential energy is $mg\Delta y$, so $\Delta U = 0.85mg\Delta y$. The time for this conversion is 8.0 h = (8.0)(3600 s), Δy = 140 m, and m = 8.4 Tg = 8.4×10^{12} g = 8.4×10^9 kg.

EVALUATE: $\bar{P} = \dfrac{\Delta U}{\Delta t} = \dfrac{0.85mg\Delta y}{\Delta t} = \dfrac{(0.85)(8.4\times10^9\text{ kg})(9.8\text{ m/s}^2)(140\text{ m})}{(8.0)(3600\text{ s})}$

$\bar{P} = 3.4\times10^8$ W = 340×10^6 W = 340 MW.

ASSESS: The height of 140 m (≈460 ft) of the reservoir above the generating station is reasonable, as is the amount of water (8.4×10^9 kg) per 8.0 h, so our answer should be reasonable, which it is.

79. **INTERPRET:** We are dealing with energy conservation involving one conservative force (the spring) and one nonconservative force (friction). The spring gives its elastic potential to the block, which then has kinetic energy, all of which is converted to internal energy by friction with the surface. Therefore all the initial elastic potential energy is converted to internal energy. We can calculate the initial energy in the spring, and we know the stopping distance of the block. We want the coefficient of kinetic friction between the block and the surface.

DEVELOP: All the initial elastic potential energy is converted to internal energy by friction, and the initial and final kinetic energies are zero. Therefore

$\Delta U = -\Delta E_{\text{int}} = -f_k d$. In this case, $U_0 = \frac{1}{2}kx^2$, $U = 0$, $f_k = \mu_k mg$. This gives

$\frac{1}{2}kx^2 = \mu_k mgd$. We know k, x, m, and d, and we want to find μ_k.

EVALUATE: Putting in the appropriate numbers gives

$\frac{1}{2}kx^2 = \mu_k mgd$

$\frac{1}{2}(344\text{ N/m})(0.185\text{ m})^2 = \mu_k(1.52\text{ kg})(9.81\text{ m/s}^2)(1.48\text{ m})$

$\mu_k = 0.267.$

ASSESS: A coefficient of kinetic friction of 0.267 is physically for ordinary surfaces such as those in this problem. In this case, all of the elastic potential energy went into internal energy of the system (block, air, surface). Note that we never needed to find the initial speed of the block; we could do the problem in one step.

80. **INTERPRET:** This problem involves the conservation of energy with one conservative force (gravity) and one nonconservative force (friction). The initial gravitational potential energy of the child is all converted into internal energy by friction on the level stretch. We know the coefficient of kinetic friction and the stopping distance, and we want to find the initial height h of the child.

DEVELOP: The original potential energy is $U_0 = mgh$ with the reference level at the base of the hill. The work done by friction on the level stretch is $-f_k d$, so $\Delta U = -\Delta E_{\text{int}} = -f_k d$. This gives $mgh = \mu_k mgd$, so $h = \mu_k d$.

EVALUATE: Using the given numbers gives

$h = \mu_k d = (0.27)(19\text{ m}) = 5.1\text{ m}$.

ASSESS: We did not need to use g or the mass of the child. The units are correct since μ_k is a dimensionless number.

81. **INTERPRET:** When a force acts on an object through a distance x, it gives the object a known velocity. The force converts potential energy into kinetic energy, which depends on the velocity. We want to find an equation for the force as a function of x.

DEVELOP: We know that $F_x = -dU/dx$. A loss of potential energy (dU) results in a gain of kinetic energy (dK), so $dU = -dK$. Therefore $F_x = -dU/dx = -(-dK/dx) = dK/dx$. The kinetic energy is $K = \frac{1}{2}mv^2$, $F_x = d(\frac{1}{2}mv^2)/dx$. We use the given expression for $v(x)$.

EVALUATE: We are given that $v = x^{2/3}\sqrt{\dfrac{3a}{2m}}$, so $v^2 = \dfrac{3a}{2m}x^{4/3}$. Therefore

$$F_x = \frac{dK}{dx} = \frac{d}{dx}\left(\tfrac{1}{2}mv^2\right) = \frac{d}{dx}\left[\tfrac{1}{2}m\left(\frac{3a}{2m}x^{4/3}\right)\right] = \left(\frac{3a}{4}\right)\left(\frac{4}{3}x^{1/3}\right) = ax^{1/3}.$$

ASSESS: Check units: since we have found that $F_x = ax^{1/3}$, a should have SI units of $\text{N/m}^{1/3}$. Using the given equation for v, $v = x^{2/3}\sqrt{\dfrac{3a}{2m}}$, squaring gives

$v^2 = x^{4/3}(3a/2m)$. Solving for a gives $a = \dfrac{2}{3}\dfrac{mv^2}{x^{4/3}}$. The quantity mv^2 has the same units as kinetic energy, which are $\text{J} = \text{N}\cdot\text{m}$, and x has units of meters (m). Thus the units of a are $\dfrac{\text{N}\cdot\text{m}}{\text{m}^{4/3}} = \text{N/m}^{1/3}$, as they should be. Since the units are correct, we can be somewhat confident that our answer is correct. If the units were wrong, we would be *sure* that our answer was wrong.

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76. **INTERPRET:** This problem involves the acceleration due to gravity on a planet (call it Planet X). We know its mass and radius compared to those of Earth and want to find the acceleration due to gravity at its surface.

DEVELOP: Use the equation $a = GM/R^2$. Call R_X the radius of planet X and M_X its mass. Planet X is 5.50% smaller than Earth, so its radius is $R_X = R_E - 0.0550R_E = 0.945R_E$. Its mass is 12.0% greater than that of Earth, so $M_X = M_E + 0.120M_E = 1.12M_E$. At the surface of Earth, $g_E = GM_E/R_E^2$ and at the surface of planet X it is $g_X = GM_X/R_X^2$. If we take the ratio of the two accelerations, common factors will cancel, giving

$$\frac{g_X}{g_E} = \frac{GM_X/R_X^2}{GM_E/R_E^2} = \frac{M_X R_E^2}{M_E R_X^2} = \frac{(1.12M_E)R_E^2}{M_E(0.945R_E)^2} = 1.254\,.$$

EVALUATE: $g_X = 1.254g_E = (1.254)(9.81\ \text{m/s}^2) = 12.3\ \text{m/s}^2$.

ASSESS: Planet X is more massive than Earth and slightly smaller. Both of these characteristics will make the acceleration due to gravity at its surface larger than on Earth, which is what our results show so they seem reasonable.

77. **INTERPRET:** This problem involves the gravitational force between two identical spheres. We know the force of attraction and the masses of the spheres and want to find their separation.

DEVELOP: The gravitational force is $F = \dfrac{Gm_1m_2}{r^2}$. We can treat them as point-masses since they are uniform spheres. We know that $m_1 = m_2 = 8.65$ kg and $F = 0.275\ \mu\text{N} = 0.275\times10^{-6}$ N, and we want to find r.

EVALUATE: Solving the gravitational force for r gives

$$r = \sqrt{\frac{Gm_1m_2}{F}} = \sqrt{\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(8.65\ \text{kg})^2}{0.275\times10^{-6}\ \text{N}}} = 0.135\ \text{m} = 13.5\ \text{cm}.$$

ASSESS: The spheres should be quite close together for the force to be appreciable since the gravitational force is weak. This is consistent with your answer of 13.5 cm.

78. **INTERPRET:** Here we are dealing with an orbital problem with Jupiter as the central body. We know the orbital period and want to find the orbital radius of Io.

DEVELOP: For a circular orbit, the orbital period is given by $T^2 = \dfrac{4\pi^2r^3}{GM}$, where M is the mass of the central body. From Appendix E we have $M = 1900\times10^{24}$ kg. Solving for r gives $r = \left(\dfrac{GMT^2}{4\pi^2}\right)^{1/3}$. We must express T in seconds.

EVALUATE: Converting 1.77 d to units of seconds gives

$T = (1.77\ \text{d})\left(\dfrac{24\ \text{h}}{1\ \text{d}}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right) = 1.529\times10^5\ \text{s}$. Calculating r gives

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1900\times10^{24}\ \text{kg})(1.529\times10^5\ \text{s})^2}{4\pi^2}\right]$$

$r = 4.22\times10^8\ \text{m} = 4.22\times10^5$ km.

ASSESS: Our result agrees with the value given in Appendix E.

79. **INTERPRET:** We want to find what the gravitational acceleration at Earth's surface would be if our planet were smaller but had the same mass as now.

DEVELOP: The present acceleration at the surface is $g = GM/R^2$. The new radius is $0.850R$, so the new acceleration would be $g_{\text{new}} = GM/(0.850R)^2$.

EVALUATE: Take the ratio of the two accelerations.

$$\frac{g_{\text{new}}}{g} = \frac{GM/(0.850R)^2}{GM/R^2} = \frac{1}{(0.850)^2}$$

$g_{\text{new}} = (9.81\ \text{m/s}^2)/(0.850)^2 = 13.6\ \text{m/s}^2$.

ASSESS: We found that g would be greater than $9.81\ \text{m/s}^2$. If the size of Earth decreased, all the mass would be concentrated closer together, so the gravitational force, and hence g, at the surface would increase, which agrees with our result.

80. **INTERPRET:** This problem involves the gravitational attraction between two objects, the water and the person. When you are below the water tank, the gravity of the water pulls upward on you, so you weigh less than usual. We know the amount by which your weight is reduced and your distance from the water, and we want to find the mass of the water.

DEVELOP: The gravitational attraction is given by $F = \dfrac{Gm_1m_2}{r^2}$. Without the water tank, you weight mg. When you are 15.0 m below the tank, you weigh 1 part in 10 million (which is $1/10^7$) less than mg, so then gravitational pull of the water on you must be 1.00×10^{-7} of your ordinary weight, or $1.00 \times 10^{-7}mg$. Therefore $\dfrac{Gmm_{\text{w}}}{r^2} = 1.00 \times 10^{-7}mg$, where m_{w} is the mass of the water. We want to find m_{w}.

EVALUATE: Canceling m and solving for m_{w} gives

$$m_{\text{w}} = \frac{\left(1.00\times10^{-7}\right)gr^2}{G} = \frac{\left(1.00\times10^{-7}\right)(9.8\ \text{m/s}^2)(15.0\ \text{m})^2}{6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = 3.31\times10^6\ \text{kg}.$$

ASSESS: The gravitational force is weak, so we need a lot of water to have an appreciable effect on your weight. The density of water is $1000\ \text{kg/m}^3$, so the size of such a sphere would be given by $(1000\ \text{kg/m}^3)(4/3\ \pi r^3)$ $=3.31 \times 10^6$ kg, which gives $r = 9.2$ m. This is a reasonable size for a tank of water.

81. **INTERPRET:** This problem involves the acceleration due to gravity. We know its value at the surface of a white dwarf, as well as the star's mass, and we want to find the radius of the star.

DEVELOP: Use $a = GM/R^2$, where $a = 8.91\ \text{Mm/s}^2 = 8.91 \times 10^6\ \text{m/s}^2$, and $M = 1.15M_{\text{Sun}} = (1.15)(1.99 \times 10^{30}\ \text{kg}) = 2.28 \times 10^{30}$ kg (using Appendix E).

EVALUATE: Solving for R gives

$$R = \sqrt{\frac{GM}{a}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.289\times10^{30}\ \text{kg}\right)}{8.91\times10^6\ \text{m/s}^2}} = 4.14\times10^6\ \text{m} = 4.14\ \text{Mn}.$$ Now compare this radius with that of Earth, using Appendix E.

$$\frac{R}{R_{\text{E}}} = \frac{4.14\times10^6\ \text{m}}{6.37\times10^6\ \text{m}} = 0.650.$$

The radius of the white dwarf is only 65.0% that of Earth.

ASSESS: This white dwarf is enormously dense because it has slightly more mass than the Sun squeezed into a sphere just 65% the radius of Earth. This characteristic is typical of all white dwarfs.

82. **INTERPRET:** In this problem, we have two satellites in circular orbits about the same planet. We know how their periods compare and want to know how their orbital radii compare.

DEVELOP: For a circular orbit, $T^2 = \dfrac{4\pi^2r^3}{GM}$. We know that $T_{\text{A}} = 2^{3/2}T_{\text{B}}$ and want to find r_{A} in terms of r_{B}.

EVALUATE: Take the ratio of T_A^2 to T_B^2. Common factors will cancel out.

$$\frac{T_A^2}{T_B^2}=\frac{\dfrac{4\pi^2 r_A^3}{GM}}{\dfrac{4\pi^2 r_B^3}{GM}}=\left(\frac{r_A}{r_B}\right)^3.$$

Using the fact that $T_A = 2^{3/2}T_B$, we get

$$\frac{\left(2^{3/2}T_B\right)^2}{T_B^2}=2^3=\left(\frac{r_A}{r_B}\right)^3.$$

Taking the cube root of both sides and solving for r_A gives

$r_A = 2.00r_B$.

ASSESS: Since $T_A > T_B$, r_A should be greater than r_B, which is in fact what we do find, so our answer seems reasonable.

83. **INTERPRET:** In this case we have a comet in an elliptical orbit around the Sun. We know its distance at perihelion (closest approach to the Sun) and want to find its speed at that point. We also know its speed when it is at the distance of Neptune from the sun. We can apply energy conservation.

DEVELOP: Energy conservation gives $U_0 + K_0 = U + K$. Let the initial state be at perihelion and the final state be at Neptune's distance. Also let m be the comet's mass, M the Sun's mass, the subscript p be perihelion and N be at Neptune's distance. The gravitational potential energy is $U = -GmM/r$ and the kinetic energy is $K=\frac{1}{2}mv^2$.

Energy conservation gives $-\dfrac{GmM}{r_p}+\frac{1}{2}mv_p^2=-\dfrac{GmM}{r_N}+\frac{1}{2}mv_N^2$. We know $v_N = 4.48$ km/s, $r_N = 4.50\times10^9$ km (from Appendix E), and $r_p = 8.79\times10^7$ km, and we want to find v_p.

EVALUATE: Solve the energy equation for v_p.

$$v_p^2=v_N^2+2GM\left(\frac{1}{r_p}-\frac{1}{r_N}\right)$$

$$v_p^2=\left(4.48\times10^3\ \text{m/s}\right)^2+2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)\left(\frac{1}{8.79\times10^{10}\ \text{m}}-\frac{1}{4.50\times10^{12}\ \text{m}}\right)$$

$v_p = 5.46\times10^4$ m/s = 54.6 km/s.

ASSESS: The comet should be moving fastest at perihelion, which agrees with our result of $v_p = 54.6$ km/s and $v_N =$ 4.48 km/s.

84. **INTERPRET:** In this problem we want to compare the actual height reached by a rocket with the height we would calculate assuming constant acceleration $g = 9.81$ m/s^2. Knowing the actual height, energy conservation will give us the initial speed.

DEVELOP: We know the rocket reaches a height of 95.0 km above Earth's surface. This height is large enough that we are concerned about the variation in g with altitude. We can use energy conservation to find the initial speed of the rocket. Then we can use that speed with the constant-acceleration formula to calculate the expected height if g remains constant. Energy conservation gives $U_0 + K_0 = U + K$. The gravitational potential energy is $U = -GmM/r$ and the kinetic energy is $K=\frac{1}{2}mv^2$. Let the initial state be at Earth's surface and the final state at a maximum height of $h = 95.0$ km = 95,000 m. $K = 0$ at the maximum height. Energy conservation gives $-\dfrac{GmM_E}{R_E}+\frac{1}{2}mv_0^2=-\dfrac{GmM_E}{R_E+h}$. From Appendix E, we have $R_E = 6.37\times10^6$ m and $M_E = 5.97\times10^{24}$ kg. Canceling m and solving for v_0^2 gives us

$v_0^2=2GM_E\left(\dfrac{1}{R_E}-\dfrac{1}{R_E+h}\right)$. Next use the equation $v^2=v_0^2+2a\left(x-x_0\right)$ to calculate the maximum height $x - x_0 = h$, with v_0^2 equal to the value we just found.

EVALUATE: Calculate v_0^2 first.

$$v_0^2 = +2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.97\times10^{24}\ \text{kg}\right)\left(\frac{1}{6.37\times10^6\ \text{m}} - \frac{1}{6.37\times10^6\ \text{m} + 95{,}000\ \text{m}}\right)$$

$v_0^2 = 1.8372\times10^6\ \text{m}^2/\text{s}^2$.

Now use this result in the equation $v^2 = v_0^2 + 2a\left(x - x_0\right)$ to find h.

$0 = 1.8372\times10^6\ \text{m}^2/\text{s}^2 + 2(-9.81\ \text{m/s}^2)\text{h}$

$\text{h} = 9.3637\times10^4\ \text{m} = 93.637\ \text{km}$.

The percent error is $\dfrac{95.0\ \text{km} - 93.637\ \text{km}}{95.0\ \text{km}}\times100 = 1.43\%$. The constant-acceleration formula gives an answer that is too low.

ASSESS: It is reasonable that the constant-acceleration formula is too low because it assumes a constant acceleration of $g = 9.81\ \text{m/s}^2$. But in fact g decreases with altitude, so the rocket will actually go higher than calculated because the pull of gravity on it is getting weaker as it gets higher.

85. **INTERPRET:** If the comet is moving fast enough, it will escape the Sun's gravity and never return. So we need to calculate the escape speed of the sun at Earth's orbital distance.

DEVELOP: The escape speed is $v = \sqrt{\dfrac{2GM}{r}}$. From Appendix E we have M = mass of sun = 1.99×10^{30} kg and r = Earth-Sun distance = 1.50×10^{11} m.

EVALUATE: $v = \sqrt{\dfrac{2GM}{r}} = \sqrt{\dfrac{2\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{1.50\times10^{11}\ \text{m}}}$

$v = 4.21\times10^4\ \text{m/s} = 42.1\ \text{km/s}$.

ASSESS: If the speed of the comet is greater than or equal to 42.1 km/s, the comet will escape the Sun and never return.

Chapter 9: Alternate Problem Set in *Mastering Physics*

98. **INTERPRET:** This is a center of mass problem in which we know the location of the center of mass of a double-star system and want to compare the masses of the stars.

DEVELOP: The location of the center of mass of two objects is $x_{\rm cm} = \dfrac{m_1x_1 + m_2x_2}{m_1 + m_2}$. Let star A be at the origin and star B be at $x_{\rm B}$. We know that the center of mass of the system is at $x_{\rm cm} = \frac{2}{3}x_{\rm B}$.

EVALUATE: Using the center of mass formula gives

$$\tfrac{2}{3}x_{\rm B} = \frac{0 + m_{\rm B}x_{\rm B}}{m_{\rm A} + m_{\rm B}}$$

$m_{\rm B} = 2m_{\rm A}$

Star B is twice as massive as star A.

ASSESS: The center of mass should be closer to the more massive star, which is B. We know the center of mass is $\frac{2}{3}$ of the way from A to B, so our result is reasonable.

99. **INTERPRET:** This problem involves the conservation of momentum. We know the initial momentum of the kernel, the final mass and velocity of the first piece, and the final velocity of the second piece. We want the mass of the second piece.

DEVELOP: An explosion involves only internal forces, so the momentum of the system is conserved. The initial momentum is zero, so the final momentum must also be zero, which means that the pieces must move in opposite directions. Therefore $0 = m_1v_1 + m_2v_2$. We know m_1, v_1, and v_2, and we want to find m_2.

EVALUATE: Using $v_1 = 47$ cm/s, $v_2 = -67$ cm/s, and $m_1 = 91$ mg, we get

$0 = (47\text{ cm/s})(91\text{ mg}) + m_2(-67\text{ cm/s})$

$m_2 = 64$ mg.

ASSESS: The more massive piece should have the smaller speed. We found that the 64-mg piece moves at 67 cm/s and the 91-mg piece moves at 47 cm/s, which agrees with our expectation.

100. **INTERPRET:** In this problem, the explosion creates internal kinetic energy. We know the initial kinetic energy and the final internal kinetic energy. We want to find out how the speeds of the two explosion fragments compare to the original speed of the object.

DEVELOP: The kinetic energy of a system is $K = K_{\rm cm} + K_{\rm int}$. The explosion does not change $K_{\rm cm}$ but it does produce $K_{\rm int}$. The two fragments have equal speeds, so they must have equal masses, m. We know this because in the center-of-mass reference frame, the momentum is zero. Since the speeds are equal, the masses must also be equal. The initial kinetic energy of the object is K, so

$K_{\rm i} = K = \frac{1}{2}(2m)v_0^2 = m v_0^2$.

The final internal kinetic energy is given as $K_{\rm ing} = 3K$. Since $K_{\rm cm}$ is not changed by the explosion, we have

$K_{\rm f} = K_{\rm cm} + K_{\rm int} = K + 3K$.

EVALUATE: The internal kinetic energy is measured relative to the center of mass, so each fragment has mass m and speed v in the center-of-mass frame. Therefore

$3K = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 = mv^2$.

But we saw that $K = m\,v_0^2$, so

$3(m\,v_0^2) = mv^2$

$v = v_0\sqrt{3}$.

Each fragment has speed $v_0\sqrt{3}$ relative to the center of mass of the object.

ASSESS: Relative to the lab the fragments have different speeds. The center of mass does not change its speed of v_0, so one goes forward at speed $v_0 + v_0\sqrt{3}$ and the other goes backward at $v_0 - v_0\sqrt{3}$.

101. **INTERPRET:** This problem requires the use of impulse. We know the impulse and the time for which the force acts, and we want to find the force (thrust) that gives this impulse.

DEVELOP: Use $J_x = F_x\Delta t$ since we have one-dimensional motion. We are given J_x and Δt , and we want to find F_x.

EVALUATE: Using $J_x = F_x\Delta t$ gives

$5.64\ \text{N}\cdot\text{s} = F_x(41.8\ \text{s})$

$F_x = 0.135\ \text{N} = 135\ \text{mN}$.

ASSESS: The force acts for a rather long time, so a small force can give a large impulse.

102. **INTERPRET:** This problem involves a one-dimensional elastic collision of an alpha particle with a massive target nucleus. We know how much of its kinetic energy the alpha particle gives to the target, and we want to find the mass of the target.

DEVELOP: The collision is along a straight line with the target initially stationary. We have information about the motion of the target, so we look at its speed after the collision. For an elastic collision, with the target initially at rest, we know that $v_{2\text{f}} = \dfrac{2m_1}{m_1 + m_2}v_{1\text{i}}$, where 1 is the alpha particle and 2 is the target. The target gets 7.80% of the alpha particle's kinetic energy, so $K_\text{T} = \frac{1}{2}m_\text{T}v_\text{T}^2 = 0.0780K_\alpha = (0.0780)\left(\frac{1}{2}m_\alpha v_\alpha^2\right)$.

Solving for v_T gives

$$v_\text{T} = \sqrt{\frac{(0.0780)m_\alpha}{m_\text{T}}}\,v_\alpha .$$

Putting this into the equation for $v_{2\text{f}}$ with $v_\text{T} = v_{2\text{f}}$ gives

$$v_\text{T} = \sqrt{\frac{(0.0780)m_\alpha}{m_\text{T}}}\,v_\alpha = \frac{2m_\alpha}{m_\alpha + m_\text{T}}v_\alpha .$$

Squaring, canceling, and rearranging gives

$(0.0780)\left(m_\alpha + m_\text{T}\right)^2 = 4m_\alpha m_\text{T}$.

EVALUATE: We know that $m_\alpha = 4.00$ u, so $(0.0780)\left(m_\alpha + m_\text{T}\right)^2 = 4m_\alpha m_\text{T}$ becomes

$(0.0780)(4.00\ \text{u} + m_\text{T})^2 = 16.0\ \text{u}\ m_\text{T}$.

Squaring and collecting terms gives

$m_\text{T}^2 + (8.00\ \text{u} - 205.13\ \text{u})m_\text{T} + 16.0\ \text{u}^2 = 0$.

Using the quadratic formula to solve this equation gives two solution. One is $m_\text{T} = 0.0821$ u, which is too small for a nucleus. The other solution is $m_\text{T} = 197$ u.

ASSESS: The target nucleus is described as being massive. From our results, $m_\text{T} = 197$ u, which is over 49 times the mass of the alpha particle, which does qualify as being a massive target.

103. **INTERPRET:** This problem involves finding the center of mass of three objects: the base of the glass, the sides, and the water. We first consider an empty glass and then a glass half-full of water.

DEVELOP: Call the y-axis vertically upward with the origin at the base of the glass. The formula $y_\text{cm} = \dfrac{m_1y_1 + m_2y_2 + m_3y_3}{m_1 + m_2 + m_3}$ gives the vertical location of the center of mass. Let m_1 be the mass of the base of the glass, m_2 the mass of the sides, and m_3 the mass of the water in the glass. We are given the masses of the base and sides, but will need to calculate the mass of the water using its density ρ, where $\rho = m/V$. We assume that the sides and base are uniform.

EVALUATE: **(a)** For the empty glass, $m_1 = 87.2$ g, $m_2 = 54.3$ g, $m_3 = 0$ g, $y_1 = 0$ cm, and $y_2 = 8.00$ cm. Putting these numbers into the center-of-mass formula gives

$$y_{\text{cm}} = \frac{(87.2\text{ g})(0\text{ cm}) + (54.3\text{ g})(8.00\text{ cm})}{87.2\text{ g} + 54.3\text{ g}} = 3.07\text{ cm}.$$

The center of mass is 3.07 cm above the base of the glass.

(b) First calculate the mass of water in the glass. The volume of the water is $V = \pi r^2 h$. Since the glass is half full, $h = 8.00$ cm. The mass of the water is

$\rho = m/V$

$m = \rho V = \rho \pi r^2 h.$

Putting in the given numbers, and using $\rho = 1.00\text{ g/cm}^3$ for water, we have

$m_3 = (1.00\text{ g/cm}^3)\pi(2.50\text{ cm})^2(8.00\text{ cm}) = 157.08$ g.

The center of mass of the water column is half its height, or 4.00 cm. We can use the center of mass we found in part (a), so the center-of-mass formula gives

$$y_{\text{cm}} = \frac{(87.2\text{ g} + 54.3\text{ g})(3.07\text{ cm}) + (157.08\text{ g})(4.00\text{ cm})}{87.2\text{ g} + 54.3\text{ g} + 157.08\text{ g}} = 3.56\text{ cm}.$$

The center of mass is 3.56 cm above the base of the glass.

ASSESS: The center of mass of just the water is 4.00 cm above the base, and of the glass alone it is 3.07 cm above the base. The mass of the water (157 g) is slightly larger than the mass of the glass (87.2 g + 54.3 g = 141.5 g), so the center of mass of the half-full glass should be almost midway between 4.00 cm and 3.07 cm, but slightly closer to the center of mass of the water. This is just what we have found. Midway between the two centers of mass would be (4.00 cm + 3.07 cm)/2 = 3.53 cm, and our result is 3.56 cm, which is slightly closer to the center of mass of the water than to that of the glass alone. So our result is reasonable.

104. **INTERPRET:** This problem involves momentum conservation along the horizontal. We know the initial momentum and the sprinter's final velocity relative to the cart. We want her final horizontal velocity relative to the ground.

DEVELOP: Momentum conservation tells us that the initial momentum must be equal to the final momentum of the system. Let the initial state be just before she starts to run and the final state be just as she leaves the cart. We can find the final velocity of the cart, v_c. Calling positive to the left, her velocity relative to the ground, v_s, will be $v_s = v_c - 9.3$ m/s. Momentum conservation gives us

$(m_s + m_c)v_0 = m_c v_c + m_s v_s = m_c v_c + m_s(v_c - 9.3\text{ m/s}).$

EVALUATE: Putting in the numbers and solving for v_c gives

$(55\text{ kg} + 240\text{ kg})(7.6\text{ m/s}) = (240\text{ kg})v_c + (55\text{ kg})(v_c - 9.3\text{ m/s})$

$v_c = 9.3$ m/s to the left.

She and the cart are both moving to the left at 9.3 m/s, so her velocity relative to the ground is 0.0 m/s.

ASSESS: Relative to the ground, the cart should have all the momentum just as she leaves it. We'll use this to check our answer. The initial momentum of the system is (240 kg + 55 kg)(7.6 m/s) = $2200\ \text{kg}\cdot\text{m/s}$. The momentum of the cart just as she leaves it is (240 kg)(9.3 m/s) = $2200\ \text{kg}\cdot\text{m/s}$. This is the same as the initial momentum, so our answer checks.

105. **INTERPRET:** This problem involves two principles: momentum conservation (during the explosion) and free-fall projectile motion (after the explosion). We know the time that each of the two pieces is in the air after the explosion, and we want to find the launch speed of the rocket.

DEVELOP: At the highest point, momentum conservation tells us that the vertical components of the velocities of the two pierces pieces must be equal just after the explosion since they have equal masses. Call their vertical speeds v_0. Both pieces undergo the same vertical displacement, h, after the explosion. Call downward positive with y_0 the highest point. Find h and then use it to find the launch speed v_L. During free-fall, we use the formula $y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$. Mechanical energy is conserved between the upward launch and the explosion.

EVALUATE: For the downward-moving piece:

$y = y_0 + v_{y0}t + \frac{1}{2}a_y t^2$

$h = v_0(2.87\text{ s}) + \frac{1}{2}(9.81\text{ m/s}^2)(2.87\text{ s})^2$

For the upward-moving piece:

$h = -v_0(5.12\ \text{s}) + \frac{1}{2}(9.81\ \text{m/s}^2)(5.12\ \text{s})^2$

Equate the two expressions for h and solve for v_0, giving

$v_0(2.87\ \text{s}) + \frac{1}{2}(9.81\ \text{m/s}^2)(2.87\ \text{s})^2 = -v_0(5.12\ \text{s}) + \frac{1}{2}(9.81\ \text{m/s}^2)(5.12\ \text{s})^2$

$v_0 = 11.16$ m/s.

Now find h using the downward-moving piece.

$h = (11.16\ \text{m/s})(2.87\ \text{s}) + \frac{1}{2}(9.81\ \text{m/s}^2)(2.87\ \text{s})^2$

$h = 77.344$ m.

Now use energy conservation to find the launch speed v_L.

$\frac{1}{2}mv_L^2 = mgh$

$v_L = \sqrt{2gh} = \sqrt{2(9.81\ \text{m/s}^2)(72.44\ \text{m})} = 37.6\ \text{m/s}.$

ASSESS: The two pieces will hit the ground with the same speed, but not at the same time and not necessarily at the same place. Both pieces have the same vertical displacement of $h = 77.3$ m, but they do not travel the same vertical distance. The upward-moving piece travels a greater vertical distance than the downward-moving piece. In addition, the pieces may have horizontal velocity components. These would remain constant but would not affect the time the pieces were in the air.

106. **INTERPRET:** We have an inelastic collision along a straight line. The two objects have equal speeds, and we know their relative masses. We want to know what fraction of the initial kinetic energy of the system is lost during the collision. Momentum is conserved during the collision.

DEVELOP: Let m be the smaller mass and let it be traveling to the left. The larger mass is $4.8m$ and it is moving to the right. Calling the right positive, momentum conservation gives

$(4.8\ m)v - mv = (5.8m)v_2$

$v_2 = 0.65517v$ after the collision.

The initial kinetic energy K_1 and the final kinetic energy K_2 are

$K_1 = \frac{1}{2}mv^2 + \frac{1}{2}(4.8m)v^2 = 2.9mv^2$

$K_2 = \frac{1}{2}(5.8m)v_2^2 = \frac{1}{2}(5.8m)(0.65517v)^2 = 1.245mv^2.$

EVALUATE: The percent lost is

$$\frac{K_1 - K_2}{K_1} \times 100 = \frac{2.9mv^2 - 1.245mv^2}{2.9mv^2} \times 100 = 57\%.$$

ASSESS: The lost kinetic energy does not disappear from the universe. It gets converted to internal energy. Note that while 57% of the kinetic energy is lost, none of the momentum is lost.

107. **INTERPRET:** In this problem we have a two-dimensional collision. It is inelastic since the two objects stick together. We know that the objects have the same mass and initial speeds, and we know the angle between their initial velocities. We want to find the speed of the composite object just after the collision.

DEVELOP: Let each object have mass m and initial speed v, and call the final speed of the composite object V. Let one of the objects be moving in the $+x$ direction. The other one is moving at 132° with the other object, so it has x- and y-components to its velocity, with $v_x = -v \sin 42°$ and $v_y = v \cos 42°$. Conservation of x- and y-momentum gives

$mv - mv \sin 42° = (2m)V_x$

$mv \cos 42° = (2m)V_y.$

EVALUATE: Solve for V_x:

$mv - mv \sin 42° = (2m)V_x$

$(1.00\ \text{m/s})(1 - \sin 42°) = 2V_x$

$V_x = 0.1654$ m/s

Solve for V_y:

$mv \cos 42° = (2m)V_y$

$2V_y = (1.00\ \text{m/s}) \cos 42°$

$V_y = 0.3716$ m/s.

The speed V is the magnitude of the velocity vector, so

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(0.1654 \text{ m/s})^2 + (0.3716 \text{ m/s})^2} = 0.407 \text{ m/s} = 40.7 \text{ cm/s}.$$

ASSESS: The collision is inelastic, so kinetic energy is not conserved, but moment is conserved. The initial kinetic energy is $2(\frac{1}{2}mv^2) = m(1.00 \text{ m/s})^2$. The final kinetic energy is $\frac{1}{2}(2m)V^2 = m(0.407 \text{ m/s})^2 = (0.166 \text{ m}^2/\text{s}^2)\, m$. The ratio of kinetic energies is $K_f/K_f = (0.166 \text{ m}^2/\text{s}^2)/(1.00 \text{ m}^2/\text{s}^2) = 0.166$, so most of the original kinetic energy is converted to internal energy.

Chapter 10: Alternate Problem Set in *Mastering Physics*

85. **INTERPRET:** For this problem, we need to relate angular and linear speeds. We know the radius of the blade and the linear speed of the teeth, and we want to find the angular speed of the blade.

DEVELOP: The tangential speed of the teeth is $v_t = 2.9$ m/s and the radius of the blade is $r = 12.5$ cm = 0.125 m. We know that $v_t = \omega r$.

EVALUATE: Using $v_t = \omega r$ gives

2.9 m/s = ω (0.125 m)

ω = 23.2 rad/s.

Converting to rpm (rev/min) gives

$$\omega = \left(23.2\frac{\text{rad}}{\text{s}}\right)\left(\frac{60\text{ s}}{1\text{ min}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = 220\text{ rev/min} = 220\text{ rpm}.$$

ASSESS: This is far less than the typical 3600 rpm of circular saws, but is still 3.7 rev/s.

86. **INTERPRET:** This problem involves angular motion with constant angular acceleration. We know the angle through which the turbine turns, as well as its initial and final angular speeds. We want to find its angular acceleration.

DEVELOP: For constant angular acceleration, we can use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$. We know that ω_0 = 0 rpm, ω = 3600 rpm, and $\theta - \theta_0$ = 32,000 rev, and we want to find α.

EVALUATE: Using $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ gives

$(3600\text{ rev/min})^2 = (0\text{ rev/min})^2 + 2\,\alpha\,(32{,}000\text{ rev})$

α = 202.5 rev/min^2.

Converting to units of rad/s^2 gives

$$\alpha = \left(202.5\frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)^2 = 0.35\text{ rad/s}^2.$$

ASSESS: The units in our conversion actually come out to be s^{-2}. However the radian is a dimensionless number, so rad/s^2 is the same as 1/s^2 = s^{-2}. Also, units of rev/min^2 are correct units of angular acceleration, but they are not as frequently used as rad/s^2, so we made the unit conversion.

87. **INTERPRET:** This problem involves torque. We know the torque due to the weight of the valve and want to finds its mass.

DEVELOP: Torque is given by $\tau = rF\sin\theta$, where θ is the angle between $\vec{r}$ and $\vec{F}$. In this case, r is 32 cm from the center of the wheel to the valve and is 24° below the horizontal. The force F is vertically downward, so the angle θ between $\vec{r}$ and $\vec{F}$ is 90° – 24° = 66°. The force F is the weight of the valve, mg. We know that the torque is τ = 72 mN$\cdot$m = 0.072 N$\cdot$m.

EVALUATE: Using $\tau = rF\sin\theta$ gives

0.072 N$\cdot$m = (0.32 m)m(9.8 m/s^2) sin 66°

m = 0.025 kg = 25 g.

ASSESS: Careful! In the formula $\tau = rF\sin\theta$, θ is *not* 24° since it is the angle between $\vec{r}$ and $\vec{F}$. Don't carelessly use any angle that happens to be given!

88. **INTERPRET:** This problem is about the rotational inertia of a wheel. We know the mass of the wheel and want to know the maximum rotational inertia it can possibly have.

DEVELOP: The rotational inertia is greatest when the mass is farthest from the axis. For a wheel, that would be when all the mass is at the rim. In that case, $I = mR^2$. In this case, $R = 41.0\text{ cm} = 0.410\text{ m}$.

EVALUATE: **(a)** The maximum rotational inertia is

$I = mR^2 = (3.67\text{ kg})(0.410\text{ m})^2 = 0.617\ \text{kg}\cdot\text{m}^2$.

(b) Bringing some of the mass closer to the center, away from the rim toward the axle, would decrease the rotational inertia.

ASSESS: The smallest rotational inertia would be zero if all the mass were at the axle.

89. **INTERPRET:** This problem involves a calculation using rotational inertia. Half the mass of the Frisbee is equally distributed over its rim and the other half over its area. We know the diameter and rotational inertia of the Frisbee and want to find its mass.

DEVELOP: The total rotational inertia is the sum of the individual ones, so $I = I_{\text{rim}} + I_{\text{area}}$. Calling m the mass of the Frisbee and R its radius, we have $I_{\text{rim}} = m_{\text{rim}}R^2 = (m/2)R^2 = \frac{1}{2}mR^2$ and $I_{\text{area}} = \frac{1}{2}m_{\text{area}}R^2 = \frac{1}{2}(m/2)R^2 = \frac{1}{4}mR^2$. Therefore the total rotational inertia is $I = \frac{1}{2}mR^2 + \frac{1}{4}mR^2 = \frac{3}{4}mR^2$.

EVALUATE: Using the result we just found gives

$I = \frac{3}{4}mR^2$

$1.17\ \text{g}\cdot\text{m}^2 = \frac{3}{4}m(0.120\text{ m})^2$

$m = 108\text{ g}$.

ASSESS: If all the mass were in the rim, the rotational inertia would be $I = mR^2 = (108\text{ g})(0.12\text{ m})^2 = 1.56\ \text{g}\cdot\text{m}^2$. It all the mass were in the area, I would be $I = \frac{1}{2}mR^2 = \frac{1}{2}(108\text{ g})(0.120\text{ m})^2 = 0.778\ \text{g}\cdot\text{m}^2$. Our result of $I = 1.17\ \text{g}\cdot\text{m}^2$ is midway between these two extremes, so it is reasonable.

90. **INTERPRET:** The problem involves the kinetic energy of a rolling sphere. We know the mass, linear speed, and total kinetic energy of the sphere and want to know if the sphere is hollow or solid.

DEVELOP: For a rolling object, the total kinetic energy is $K_{\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{2}I_{\text{cm}}\,\omega^2$ and $v = R\,\omega$. Therefore the total kinetic energy is $K_{\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{2}I_{\text{cm}}(v/R)^2$. We can express the rotational inertia for a uniform sphere as $I_{\text{cm}} = bMR^2$, where $b = \frac{2}{3}$ for a hollow sphere and $b = \frac{2}{5}$ for a solid sphere. We can write the kinetic energy as $K_{\text{total}} = \frac{1}{2}Mv^2 + \frac{1}{2}(bMR^2)(v/R)^2 = \frac{1}{2}Mv^2(1 + b)$.

EVALUATE: Now find b to see what type of sphere we have. Using the information given gives

$K_{\text{total}} = \frac{1}{2}Mv^2(1 + b)$

$42\text{ J} = \frac{1}{2}(2.4\text{ kg})(5.0\text{ m/s})^2(1 + b)$

$b = 0.40 = \frac{2}{5}$, so the sphere is solid.

ASSESS: If the sphere were hollow and had the same speed, b would be $\frac{2}{3}$, so the kinetic energy would be $K_{\text{total}} = \frac{1}{2}(2.4\text{ kg})(5.0\text{ m/s})^2(1 + \frac{2}{3}) = 50\text{ J}$, which is greater than 42 J. This is physically reasonable because more of the mass is farther from the rotation axis for a hollow sphere, so I and hence the kinetic energy would be greater than for a solid sphere.

91. **INTERPRET:** We must relate the rotational speed to the angular speed for a compact disc.

DEVELOP: The angular speed varies so that the tangential speed v_t is always constant at the point where information is being read from the CD. We use $v_t = \omega r$.

EVALUATE: **(a)** At $r = 6.00$ cm, $\omega = 207$ rpm. First convert ω to rad/s and then use $v_t = \omega r$.

$$\omega = \left(207\frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 21.677\text{ rad/s}$$

$v_t = \omega r = (21.677\text{ rad/s})(6.00\text{ cm}) = 130\text{ cm/s} = 1.30\text{ m/s}$.

(b) The tangential speed is always the same at the point where information is being read, so v_t = 130 cm/s at 3.75 cm from the center. Using $v_t = \omega r$ gives

130 cm/s = ω (3.75 cm)

ω = 34.683 rad/s.

Converting to rpm gives

$$\omega = \left(34.683 \frac{\text{rad}}{\text{s}}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right)\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 331 \text{ rpm}.$$

ASSESS: We just found that at r = 6.00 cm, ω = 207 rpm, and at r = 3.75 cm, ω = 331 rpm. For large r, ω is less than for small r. This is plausible since ω must keep v_t constant where information is being read.

92. **INTERPRET:** In this problem, two objects are connected by a string that passes over a pulley. We know the mass of one of the objects and the torque applied to the pulley to prevent it from turning. We want to find the mass of the other object.

DEVELOP: There is no motion, so the forces on each of the two objects must balance. This tells us that the tension in each part of the string is equal to the weight of the object hanging from it. The pulley does not turn, so the torques on it must balance. Call the unknown mass m. There are two cases to consider: the object m is just ready to move downward (m > 228 g) and the object m is just ready to move upward (m < 228 g).

<u>If m > 228 g</u>: The torque due to mg is just balanced by the torque due to the other object plus the applied 153- mN·m (0.153 N·m) torque. The lever arm for each tension is R = 7.50 cm = 0.0750 m.

<u>If m < 228 g</u>: The torque due to the 228-g object is just balanced by the torque due to mg plus the applied 153- mN·m torque. The lever arm for the tensions is still R.

EVALUATE: We balance the torques in each of the two cases.

<u>m > 228 g</u>: $mgR = (0.228 \text{ kg})gR + \tau_{\text{applied}}$

$m(9.81 \text{ m/s}^2)(0.0750 \text{ m}) = (0.228 \text{ kg})(9.81 \text{ m/s}^2)(0.0750 \text{ m}) + 0.153 \text{ N}\cdot\text{m}$

m = 0.436 kg = 436 g.

<u>m < 228 g</u>: $(0.228 \text{ kg})gR = mgR + \tau_{\text{applied}}$

$(0.228 \text{ kg})(9.81 \text{ m/s}^2)(0.0750 \text{ m}) = m(9.81 \text{ m/s}^2)(0.0750 \text{ m}) + 0.153 \text{ N}\cdot\text{m}$

m = 0.0200 kg = 20.0 g.

ASSESS: The 153- mN·m applied torque could be either clockwise or counterclockwise, depending on which way the system is ready to turn. Thus we get two possible answers for m.

93. **INTERPRET:** In this problem, we must use rotational dynamics. The torque due to the applied force gives the wheel an angular acceleration, starting from rest. We know the applied force F at the rim and its lever arm. We also know the angular speed of the wheel after it has turned through $\frac{1}{8}$ of a revolution. We want to find the mass of the wheel.

DEVELOP: The force is constant, so the angular acceleration is also constant. So we can use the equation $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$. We know that ω_0 = 0 rad/s, ω = 23 rpm, and $\theta - \theta_0 = \frac{1}{8}$ rev = $\pi/4$ rad, so we can find α. Then we can use this α in $\tau = I\alpha$ with $I = \frac{1}{2}mR^2$ (for a solid stone disk) to find m. When using $\tau = I\alpha$, α must be expressed in rad/s^2.

EVALUATE: First convert 23 rpm to units of rad/s.

$$\left(23 \frac{\text{rev}}{\text{min}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 2.40855 \text{ rad/s}.$$

Now use $\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$ to find α.

$(2.40855 \text{ rad/s})^2 = (0 \text{ rad/s})^2 + 2\alpha(\pi/4 \text{ rad})$

α = 3.6931 rad/s^2.

Now use $\tau = I\alpha$ to find m. The lever arm for the 68-N force is R = 0.44 m, so

$(68 \text{ N})(0.44 \text{ m}) = \frac{1}{2}m(0.44 \text{ m})^2(3.6931 \text{ rad/s}^2)$

m = 84 kg.

ASSESS: A mass of 84 kg (≈185 lb) is reasonable for a stone wheel that is nearly a meter in diameter.

94. **INTERPRET:** This problem involves rotational motion of the drum and linear motion of the dropping object. We know the mass of the drum and the acceleration of the dropping object, and we want to find the tension in the rope that is attached to the object and wrapped around the rim of the drum.

DEVELOP: Since we know the acceleration a_m of the dropping object, Newton's second law is appropriate to use. This acceleration is the same as the tangential acceleration a_t of the rim of the drum, so $a_m = a_t = R\alpha$. Newton's second law for the dropping object gives

$mg - T = ma_m$ (Eq. 1)

where T is the tension in the rope. The drum is in rotational motion, so we apply $\tau = I\alpha$, with $I = \frac{1}{2}MR^2$ for a solid cylinder of mass M and radius R. The lever arm for the tension is R, so $\tau = I\alpha$ gives

$TR = \frac{1}{2}MR^2\alpha = \frac{1}{2}MR^2(a_t/R) = \frac{1}{2}MR^2(a_m/R)$

$T = \frac{1}{2}Ma_m$ (Eq. 2)

Combining equations (1) and (2) to eliminate T gives

$mg = ma_m + \frac{1}{2}Ma_m$.

Solve for m for part (a), and then use equation (2) for part (b).

EVALUATE: **(a)** Solving for m and using the given numbers gives

$$m = \frac{Ma_m}{2(g-a_m)} = \frac{(17.5\text{ kg})(3.73\text{ m/s}^2)}{2(9.81\text{ m/s}^2 - 3.73\text{ m/s}^2)} = 5.37\text{ kg}.$$

(b) Equation (1) gives

$T = \frac{1}{2}Ma_m = \frac{1}{2}(17.5\text{ kg})(3.73\text{ m/s}^2) = 32.6\text{ N}$.

ASSESS: The tension should be less than the weight of the dropping object since it is accelerating downward. We found $T = 32.6$ N, and $mg = (5.37\text{ kg})(9.81\text{ m/s}^2) = 52.7$ N. So $T < mg$ as we expect, so our answer is reasonable.

Chapter 11: Alternate Problem Set in *Mastering Physics*

69. **INTERPRET:** This problem involves the relationship between angular motion and linear motion. We know the angular velocity of the wheels of a car and their diameter, and we want to find the car's linear velocity.

DEVELOP: For rolling motion, the forward speed of the car is equal to the tangential speed of the rim of the tires. So $v_{car} = v_t = \omega r$, where r is the radius of the tires and ω must be in radian measure. The direction of the angular velocity vector tells us the direction in which the wheels are turning.

EVALUATE: The speed of the car is

$v_{car} = v_t = \omega r = (61\ \text{s}^{-1})(0.31\ \text{m}) = 19$ m/s.

The angular velocity vector $\vec{\omega}$ points toward the east. Using the right-hand rule, if our thumb points toward the east, our fingers curl counterclockwise as viewed looking from east to west. This type of rolling motion would move the car toward the south, so it is going southward.

ASSESS: If the car is on slippery pavement (icy, for example) so that the wheels are slipping, it would have be true that $v_{car} = v_t$, but it would still be true that $v_t = \omega r$.

70. **INTERPRET:** We must use the vector nature of the angular velocity and angular acceleration. We know the angular acceleration vector, the initial angular velocity, and the time during which the angular acceleration is applied. We want to find the magnitude and direction of the final angular velocity.

DEVELOP: The angular acceleration is constant, so we can use $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t} = \dfrac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$. We know that $\vec{\alpha} = (0.419\hat{i} - 0.328\hat{j})\ \text{s}^{-2}$, $\vec{\omega}_i = 47.0\hat{j}$ rpm, and $\Delta t = 15.0$ s. We want the magnitude and direction of $\vec{\omega}_f$. We will need to convert 47.0 rpm to units of rad/s = s^{-1}.

EVALUATE: First convert units. $\left(47\dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 4.9218\ \text{rad/s} = 4.9218\ \text{s}^{-1}$.

Now use $\vec{\alpha} = \dfrac{\Delta\vec{\omega}}{\Delta t} = \dfrac{\vec{\omega}_f - \vec{\omega}_i}{\Delta t}$.

$$(0.419\hat{i} - 0.328\hat{j})\ \text{s}^{-2} = \frac{\vec{\omega}_f - 4.92\ \text{s}^{-1}\hat{j}}{15.0\ \text{s}}$$

$$\vec{\omega}_f = (15.0\ \text{s}^{-1})(0.419\hat{i} - 0.328\hat{j})\ \text{s}^{-2} + 4.92\ \text{s}^{-1}\hat{j} = 6.29\text{s}^{-1}\hat{i} - 4.92\ \text{s}^{-1}\hat{j} + 4.92\ \text{s}^{-1}\hat{j}$$

$$\vec{\omega}_f = 6.29\text{s}^{-1}\hat{i}.$$

So the magnitude of $\vec{\omega}_f$ is 6.29 s^{-1}, which is equal to 60.0 rpm.

(b) As we saw from our work in part (a), $\vec{\omega}_f$ is in the $+\hat{i}$ (or $+x$) direction.

ASSESS: As viewed by looking in the $+x$ direction, $\vec{\omega}_f$ is clockwise.

71. **INTERPRET:** This problem requires an angular momentum calculation. We know the angular momentum and linear speed of a ball going in a circle, and we want to find its mass.

DEVELOP: The angular momentum is $L = I\omega$, where $I = mr^2$ for the ball, and the linear speed of the ball is $v_t = \omega r$. Therefore

$L = I\omega = (mr^2)(v_t/r) = mrv_t$.

In this case, $r = 1.2\ \text{m} + 0.88\ \text{m} = 2.08$ m.

EVALUATE: Using the given numbers, we get

$L = mrv_t$

$410\ \text{kg}\cdot\text{m}^2/\text{s} = m(2.08\ \text{m})(27\ \text{m/s})$

$m = 7.3$ kg.

ASSESS: This ball weighs about 16 lb, which seems reasonable for a steel ball in an Olympic hammer throw.

72. **INTERPRET:** We must do an angular momentum calculation. We know the rotational inertia and angular velocity of the gymnast, and we want her angular momentum.

DEVELOP: Use $L = I\omega$, with ω in rad/s.

EVALUATE: First convert units.

$$\left(72.6\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 7.603\ \text{rad/s} = 7.603\ \text{s}^{-1}.$$

Now use $L = I\omega$ to find her angular momentum.

$L = I\omega = (62.4\ \text{kg}\cdot\text{m}^2)(7.603\ \text{s}^{-1}) = 474\ \text{kg}\cdot\text{m}^2/\text{s}$.

ASSESS: Check units: $(\text{kg}\cdot\text{m}^2)(\text{s}^{-1}) = (\text{kg}\cdot\text{m}^2)/\text{s} = \text{kg}\cdot\text{m}^2/\text{s}$, which are the correct units for angular momentum.

73. **INTERPRET:** This problem requires use of the conservation of angular momentum. We know the initial and final angular velocities, the rotational inertia of the wheel, and the distance from the rotation axis at which the clay drops. We want to find the mass of the clay.

DEVELOP: No external torques are involved, so the angular momentum ($I\omega$) of the clay-wheel system is conserved. We treat the clay as a point mass. The initial angular momentum of the wheel is equal to the final angular momentum of the system after the clay has stuck to the wheel, so $I_i\omega_i = I_f\omega_f$. $I_i = I_{\text{wheel}}$ and $I_f = I_{\text{wheel}} + I_{\text{clay}}$. $I_{\text{clay}} = mr^2$, where $r = 46.0$ cm = 0.460 m and m is what we want to find. Therefore

$I_{\text{wheel}}\,\omega_i = (I_{\text{wheel}} + mr^2)\,\omega_f$.

EVALUATE: Put in the known numbers.

$I_{\text{wheel}}\,\omega_i = (I_{\text{wheel}} + mr^2)\,\omega_f$

$(6.42\ \text{kg}\cdot\text{m}^2)(19.3\ \text{rpm}) = [6.42\ \text{kg}\cdot\text{m}^2 + m(0.460\ \text{m})^2](17.4\ \text{rpm})$

$m = 3.31$ kg.

ASSESS: In this case, we did not need to convert angular velocity units from rpm to rad/s because both angular velocities are expressed in rpm, so the units canceled. Only convert units when it is necessary!

74. **INTERPRET:** We use conservation of angular momentum since no external torques act on the skater-weight system. The skater increases his angular speed by pulling in his fists with identical weights in each hand. We have information about the skater's rotational inertia and angular speeds, and we want to find the mass of the weights in his hands.

DEVELOP: Conservation of angular momentum gives $I_i\omega_i = I_f\omega_f$. The rotational inertia is the sum of I of the skater and I of the two weights in his hands, so $I = I_s + I_w$. Initially his hands are out, so $I_w = 2mr^2$, where $r = 76.0$ cm = 0.760 m. We use a factor of 2 because there are two weights, each of mass m. When his hands are pulled in, $I_w = 0$ because the weights are at the rotation axis. We must realize that the skater's rotational inertia changes when he pulls in his hands. Angular momentum conservation now gives

$\left(I_{si} + 2mr_i^2\right)\omega_i = I_{sf}\omega_f$.

EVALUATE: Use the known numbers in $\left(I_{si} + 2mr_i^2\right)\omega_i = I_{sf}\omega_f$ and solve for m.

$[5.75\ \text{kg}\cdot\text{m}^2 + 2m(0.760\ \text{m})^2](3.10\ \text{rev/s}) = (4.22\ \text{kg}\cdot\text{m}^2)(6.17\ \text{rev/s})$

$m = 2.29$ kg.

ASSESS: It was not necessary to convert rev/s to rad/s since those units canceled. Look before converting!

75. **INTERPRET:** We need to do an angular momentum calculation for a moving ball. We know its speed and mass and want its angular momentum.

DEVELOP: The angular momentum of a moving object is $L = mvr$, where r is the perpendicular distance from the axis about which the angular momentum is calculated to the line along which the object is moving.

EVALUATE: (a) Using the given numbers gives

$L = mvr = (0.410\ \text{kg})(7.6\ \text{m/s}^2)(1.2\ \text{m}) = 3.7\ \text{kg}\cdot\text{m}^2/\text{s}$.

(b) Even though the ball is 11 m from the point of closest approach to the vertical axis, the value of r is still 1.2 m. Remember that r is not the distance from the axis to the ball; rather r is the distance from the axis to the line along which the ball is moving, and that is 1.2 m in both parts (a) and (b). Therefore $L = 3.7\ \text{kg}\cdot\text{m}^2/\text{s}$.

ASSESS: We should not be surprised that the answers to (a) and (b) are the same. The ball is rolling so there is no net external torque on it. Therefore its angular momentum is conserved.

76. **INTERPRET:** As the mouse walks from the outer edge to the center of the turntable, the angular velocity increases. This problem involves the conservation of angular momentum of the mouse-turntable system. We know the initial and final angular speeds of the turntable, plus its radius and rotational inertia, and we want to find the mass of the mouse. Treat the mouse as a point mass.

DEVELOP: No net torque acts on the mouse-turntable system, so its angular momentum is conserved, so $I_i\omega_i = I_f\omega_f$. As the mouse nears the center, the rotational inertia of the system decreases so its angular speed increases. Let the initial state be when the mouse is at the edge and let the final state be when the mouse is at the center. The initial rotational inertia of the mouse is $I_m = mR^2$, where R = 25.0 cm = 0.250 m, and its final rotational inertia is zero. Call I_t the rotational inertia of the turntable. Therefore

$I_i = I_t + I_m = I_t + mR^2$

$I_f = I_t$

Using $I_i\omega_i = I_f\omega_f$ we now have

$(I_t + mR^2)\omega_i = I_t\omega_f$.

In part (b), the work done by the mouse is equal to the gain in kinetic energy of the system, so $W = K_f - K_i = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$.

EVALUATE: (a) Using the given numbers, we have

$(I_t + mR^2)\omega_i = I_t\omega_f$.

$[0.0154\ \text{kg}\cdot\text{m}^2 + m(0.250\ \text{m})^2](22.0\ \text{rpm}) = (0.0154\ \text{kg}\cdot\text{m}^2)(23.7\ \text{rpm})$

m = 19.0 g.

(b) Using $W = \frac{1}{2}I_f\omega_f^2 - \frac{1}{2}I_i\omega_i^2$, we get

$W = \frac{1}{2}[I_t\omega_f^2 - (I_t + I_m)\omega_i^2] = \frac{1}{2}[I_t\omega_f^2 - (I_t + mR^2)\omega_i^2]$.

We want the work in SI units of joules, so we must convert rpm to rad/s.

$$\left(72.6\frac{\text{rev}}{\text{min}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 2.482\ \text{rad/s} = 2.482\ \text{s}^{-1}$$

and similarly 22.0 rpm = 2.304 s^{-1}. Putting in the numbers gives

$$W = \tfrac{1}{2}\{(0.0154\ \text{kg}\cdot\text{m}^2)(2.482\ \text{s}^{-1})^2 - [0.0154\ \text{kg}\cdot\text{m}^2 + (0.0190\ \text{kg})(0.250\ \text{m})^2](2.304\ \text{s}^{-1})^2\}$$

$W = 3.41\times10^{-3}$ J = 3.41 mJ.

ASSESS: In part (b) we had to convert rpm to rad/s because we needed work in joules. In this case the units did not cancel, so we had to do the conversion. In part (a) we did not need to do it because the rpm units canceled.

77. **INTERPRET:** This problem involves the conservation of angular momentum. As the dog walks in one direction, the turntable rotates in the opposite direction. We have information about the dog and turntable, and we want to find the angle through which the dog moved and the fraction of a circumference that the dog has walked.

DEVELOP: The angular momentum of the dog-turntable system is conserved since there is no net external torque on the system. As viewed by on observer at rest on the ground, the initial angular momentum is zero, so after the dog has begun to walk, the angular momentum is still zero. Therefore the dog and turntable have equal but opposite angular momentum, so $I_d\omega_d = I_t\omega_t$. Treating the dog as a point mass, $I_d = m_dR^2$, where R is the turntable radius. We need the angle, relative to the ground, through which the dog moves, so we use $\omega = \dfrac{\Delta\theta}{\Delta t}$. Applying $I_d\omega_d = I_t\omega_t$ gives $m_dR^2\dfrac{\Delta\theta_d}{\Delta t} = I_t\dfrac{\Delta\theta_t}{\Delta t}$. The dog and turntable move for the same time, so Δt is the same for both of them. Solving for the angle through which the dog has moved gives $\Delta\theta_d = \Delta\theta_t\dfrac{I_t}{m_dR^2}$.

EVALUATE: (a) Putting in the appropriate numbers gives

$\Delta\theta_{\rm d} = \Delta\theta_{\rm t}\dfrac{I_{\rm t}}{m_{\rm d}R^2} = \Delta\theta_{\rm d} = (113°)\dfrac{95.0\,{\rm kg}\cdot{\rm m}^2}{(17.0\,{\rm kg})(1.81\ {\rm m})^2} = 193°.$ This is the angle through which the dog moves as observed by someone at rest on the ground.

(b) The dog has moved in one direction through an angle of 193°, and at the same time the turntable has turned in the opposite direction through 113°. Therefore the dog has moved through an angle of 113° + 193° = 306° relative to the turntable. The fraction of the turntable's circumference that the dog has walked through is the same as the fraction that 306° is of 360°, which is (306°)/(360°) = 0.850. So the dog has walked 0.850, or 85.0%, of the circumference of the turntable.

ASSESS: The rotational inertia of the dog is $mR^2 = (17.0\text{ kg})(1.81\text{ m})^2 = 55.7\ \text{kg}\cdot\text{m}^2$, which is about 59% that of the turntable. Therefore the dog would have to move through a greater angle than the turntable to balance its angular momentum, which is what we find.

78. **INTERPRET:** This problem involves the cross product and dot product of two vectors. We know that their dot product is half the magnitude of their cross product, and we want to find the angle between the vectors.

DEVELOP: We know that $\vec{A}\cdot\vec{B} = AB\cos\theta$ and $|\vec{A}\times\vec{B}| = AB\sin\theta$. In this case, $\vec{A}\cdot\vec{B} = \frac{1}{2}|\vec{A}\times\vec{B}|$.

EVALUATE: For the given condition, we have

$AB\cos\theta = \frac{1}{2}AB\sin\theta$

$\cos\theta = \frac{1}{2}\sin\theta$

$\tan\theta = 2$

$\theta = \arctan 2 = 63.4°$.

ASSESS: Check by calculating both products. $AB\sin\theta = AB\sin 63.4° = 0.894AB$ and $AB\cos\theta = AB\cos 63.4° = 0.447AB = \frac{1}{2}(0.894AB)$, so our result checks.

Chapter 12: Alternate Problem Set in *Mastering Physics*

68. **INTERPRET:** In this problem we need to balance the three forces acting on a body. We know two of the forces and want to find the third force so that the body is in static equilibrium.

DEVELOP: For static equilibrium, $\sum \vec{F} = \vec{0}$. We know $\vec{F}_1$ and $\vec{F}_2$ and want to find $\vec{F}_3$ so that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$. Call x and y the components of $\vec{F}_3$.

EVALUATE: Writing all three forces in terms of their components, we get

$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

$(2.00\hat{i} + 2.00\hat{j})\text{ N} + (-2.00\hat{i} - 3.00\hat{j})\text{ N} + x\hat{i} + y\hat{j} = \vec{0}.$

Using the x-components gives

$2.00\text{ N} - 2.00\text{ N} + x = 0\text{N} \quad \rightarrow \quad x = 0\text{ N}$

Using the y-components gives

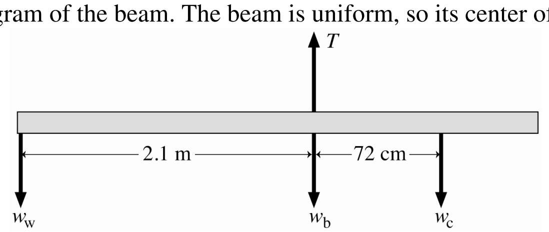
$2.00\text{ N k} - 3.00\text{ N} + y = 0\text{ N} \quad \rightarrow \quad y = 1.00\text{ N}$

Therefore the force is $\vec{F}_3 = 1.00\hat{j}\text{ N}$.

ASSESS: Even though the locations at which the forces act were given, we did not need to use them. The forces must add to zero no matter where they act. Only the torques make use of where the forces act.

69. **INTERPRET:** A uniform beam is supported by a cable at its center. A worker is at one end of the beam, and a bucket of concrete is on the other side of the cable, but not at the end of the beam. We want to find the mass of the bucket of concrete. The beam remains at rest, so it is in static equilibrium.

DEVELOP: The forces acting on the beam are the downward weight w_w of the worker, the downward weight w_c of the bucket of concrete, the tension T in the cable, and gravity w_b acting on the beam. The figure below shows these forces on a free-body diagram of the beam. The beam is uniform, so its center of mass is at its center.



We know that the vertical forces must balance, but we don't know T, or the weight w_c of the concrete, or the weight w_b of the beam. So it is best to start by balancing torques. For the pivot point, we choose the point where the cable is attached to the beam because this will eliminate T and w_b from the torque equation, leaving the weight of the concrete $w_c = m_c g$ as the only known.

EVALUATE: The lever arm for the worker is 2.1 m, and the lever arm for the concrete is 72 cm = 0.72 m. Writing out the torques gives

$m_w g(2.1\text{ m}) = m_c g(0.72\text{ m})$

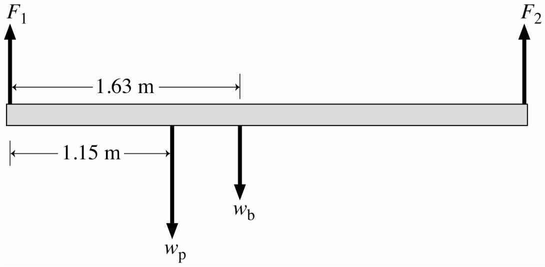
$(65\text{ kg})(2.1\text{ m}) = m_c(0.72\text{ m})$

$m_c = 190\text{ kg}.$

ASSESS: Note that g canceled out, so there was no need to put in its numerical value. Note also that it was not necessary to know the mass of the beam in this case.

70. **INTERPRET:** This problem involves forces and torques. The board is at rest, so it is in static equilibrium. Both ends rest on scales, and we want to know the reading in each scale. We know the mass and length of the board and that its center of gravity is at its middle since it is uniform, and we also know the mass and location of the person standing on it.

DEVELOP: The forces on the board are the upward forces due to the two scales, having magnitudes F_1 and F_2, the downward force of gravity w_b at its center, and the downward force due to the weight of the person w_p. The figure below shows these forces on a free-body diagram.



The forces and torques must both balance. If we balance forces, we have two unknowns (F_1 and F_2). So we start with torques and choose the pivot point to be the end of the board closest to the person. This will eliminate F_3 from the torque equation. The lever arms are 1.15 m for the weight of the person, 1.63 from the weight of the board, and 3.26 m for F_2. Once we have found F_2, we can balance vertical forces to find F_1, giving $F_1 + F_2 - w_b - w_p = 0$.

EVALUATE: Using the given numbers, balancing torques gives

$F_2(3.26\text{ m}) - (72.3\text{ kg})(9.81\text{ m/s}^2)(1.15\text{ m}) - (3.48\text{ kg})(9.81\text{ m/s}^2)(1.63\text{ m}) = 0\text{ N}$

$F_2 = 267\text{ N}.$

Now balance forces to find F_1.

$F_1 + 267\text{ kg} - (3.48\text{ kg})(9.81\text{ m/s}^2) - (72.3\text{ kg})(9.81\text{ m/s}^2) = 0$

$F_1 = 476\text{ N}.$

ASSESS: As a check, we calculate torques about the other end of the board to see if they balance. This gives

$-(765\text{ N})(3.26\text{ m}) + (72.3\text{ kg})(9.81\text{ m/s}^2)(1.63\text{ m}) + (3.48\text{ kg})(9.81\text{ m/s}^2)(1.63\text{ m}) = 0.43\ \text{N}\cdot\text{m}.$

It may appear that the torques do not balance. However the torque involved $(0.43\ \text{N}\cdot\text{m})$ is extremely small compared to the other torques in the problem, so the very small non-zero torque is just from rounding numbers during the calculations. Therefore our result does check.

71. **INTERPRET:** The height of a roller-coaster varies with its horizontal position x. We know the equation for the height h x as a function of x, and we want to find out where an equilibrium point occurs and whether it is stable or unstable.

DEVELOP: We know that the height h of the roller-coaster is given by $h = ax - bx^2$, so its gravitational potential energy U is $U = mgh = mg(ax - bx^2)$. At an equilibrium point, $dU/dx = 0$. If $d^2U/dx^2 > 0$, the point is one of stable equilibrium, but if $d^2U/dx^2 < 0$, it is a point of unstable equilibrium.

EVALUATE: Using $U = mg(ax - bx^2)$, we have

$dU/dx = mg(a - 2bx) = 0$

$x = a/2b.$

Now test for the type of equilibrium at this point.

$$\frac{d^2U}{dx^2} = \frac{d}{dx}\big[mg(a-2bx)\big] = mg(-2b) = -2bmg.$$

Since b, m, g are all positive, the second derivative is negative, so this is a point of unstable equilibrium.

ASSESS: If b were negative, the point would be one of stable equilibrium.

72. **INTERPRET:** We know the potential energy function for a particle and want to use it to investigate the equilibrium points for this particle.

DEVELOP: We know that $U = ax^3 - 2x^2 - 7x + 10$ in SI units. Since U is in joules and x is in meters, a has units of J/m^3, the 2 has units of J/m^2, the 7 has units of J/m, and the 10 has units of J. At an equilibrium point, $dU/dx = 0$. If $d^2U/dx^2 > 0$, the point is one of stable equilibrium, but if $d^2U/dx^2 < 0$, it is a point of unstable equilibrium. We want find a so that there is an equilibrium point at $x = 1.464$.

EVALUATE: (a) Calculate dU/dx and then solve for a when $x = 1.464$ m.

$dU/dx = 3ax^2 - 4x - 7 = 0$

$$a = \frac{4x+7}{3x^2} = \frac{4(1.464)\ \text{J/m} + 7\ \text{J/m}}{3(1.464\ \text{m})^2} = 1.999\ \text{J/m3}.$$

(b) Now take the second derivative.

$d^2U/dx^2 = 6ax - 4$

At $x = 1.464$ m with $a = 1.999$ J/m^3, we get

$d^2U/dx^2 = 6(1.999\ \text{J/m}^3)(1.464\ \text{m}) - 4\ \text{J/m}^2 = 13.6\ \text{J/m}^2 > 0.$

Therefore the point $x = 1.464$ m is one of stable equilibrium.

(c) We have shown that for an equilibrium point, $dU/dx = 3ax^2 - 4x - 7 = 0$. Using $a = 1.999$ J/m^3 and solving for x using the quadratic formula, we get $x = -0.7972$ m for the unknown root. Now evaluate d^2U/dx^2 at that point to test for stability.

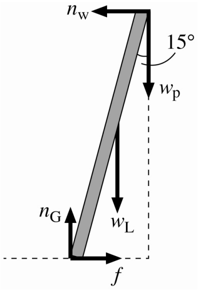
$d^2U/dx^2 = 6ax - 4 = 6(1.999\ \text{J/m}^3)(-0.7972\ \text{m}) - 4\ \text{J/m}^2.$

Since both terms are negative, d^2U/dx^2 is also negative, so the equilibrium is unstable at $x = -0.7972$ m.

ASSESS: The potential energy curve has a maximum at $x = -0.7972$ m since this is a point of unstable equilibrium, and a minimum at $x = 1.464$ m because this is a point of stable equilibrium.

73. **INTERPRET:** The ladder is in static equilibrium but is just ready to slip with the heaviest possible person at the top. Only friction at the ground prevents the ladder from slipping. We want the minimum coefficient of static friction between the ladder and the ground. The forces and the torques on the ladder must balance.

DEVELOP: Five external forces act on the ladder: friction f toward the wall at the ground, the upward normal force n_G due to the ground, the downward weight w_L of the ladder, the downward weight w_p of the person at the top of the ladder, and the normal force n_w away from the wall. The free-body diagram in the figure below shows these forces. The ladder is uniform, so its center of mass is at its center.



The ladder is just ready to slip because the person is the heaviest possible one who will not cause slipping. Therefore $f = \mu n_G$. For static equilibrium, the external forces and the external torques on the ladder both add to zero. Call the x-axis horizontal and the y-axis vertical. First balance vertical forces to find n_G.

$n_G - m_L g - m_p g = 0.$

Next take torques using the upper end of the ladder as the pivot point. Doing so will leave friction as the only unknown in the torque equation, so we can solve for f. Finally use $f = \mu n_G$ to find μ.

EVALUATE: First use $n_G - m_L g - m_p g = 0$ to find n_G.

$n_G = (5.0\ \text{kg})(9.81\ \text{m/s}^2) + (65\ \text{kg})(9.81\ \text{m/s}^2) = 686.7$ N.

Now take torques about the upper end of the ladder to find f. Call L the length of the ladder.

$-n_G L \sin 15° + m_L g(L/2) \sin 15° + fL \cos 15° = 0.$

Canceling L and putting in the known numbers gives

$-(686.7\ \text{N})(\sin 15°) + (5.0\ \text{kg})(9.81\ \text{m/s}^2)(1/2)(\sin 15°) + f(\cos 15°) = 0$

$f = 177.4$ N.

Now use $f = \mu n_G$ to find μ.

$177.4\ \text{N} = \mu(686.7\ \text{N})$

$\mu = 0.26.$

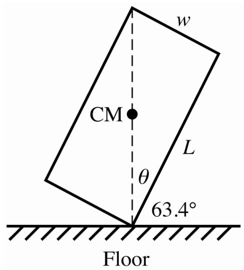
ASSESS: We check by calculating the net torque about the base of the ladder; it should be zero.

$\sum \tau = n_w L \cos 15° - m_p g L \sin 15° - m_L g(L/2) \sin 15°.$

Balancing horizontal forces tells us that $n_w = f$. Using this in the torque equation and factoring out L gives
$L[(177.4 \text{ N}) \cos 15° - (65 \text{ kg})(9.81 \text{ m/s}^2) \sin 15° - (5.0 \text{ kg})(9.81 \text{ m/s}^2)(1/2) = -0.03\text{N}L \approx 0.$
So our result checks.

74. **INTERPRET:** We know the angle at which a uniform rectangular block is balanced and want to find its relative dimensions.

DEVELOP: First draw a figure of the balanced block (see the figure below). The block is uniform, so its center of mass is at its middle.



For balance, the gravitational torque must be zero, so the center of mass of the block must be directly above the point of contact with the floor. In that case, the dashed line in the figure is vertical and therefore perpendicular to the floor. This means that $63.4° + \theta = 90°$, so $\theta = 26.6°$. A little trigonometry shows us that $\tan\theta = w/L$.

EVALUATE: $\tan 26.6° = w/L$

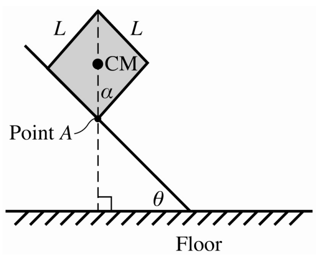
$L = w/(\tan 26.6°) = 2.00$

which tells us that $L = 2w$, so the block is twice as long as it is wide.

ASSESS: Our result is true only if the block is uniform with the center of mass at its middle. An irregular block would balance at a different angle, but the center of mass would still be over the point of contact with the floor.

75. **INTERPRET:** As the angle of an inclined surface is increased, a uniform cubical block on that surface tips over before it begins to slide. We want to find the minimum coefficient of friction between the block and the surface.

DEVELOP: First sketch the block when it is just ready to tip over. (See the figure below.) The block is uniform, so its center of mass is at its middle.



At the instant the block is just ready to tip, the gravitational torque about point A must be zero. In that case, the center of mass is directly over point A. It is tipped any more, gravity will rotate the block clockwise about A and tip it over. Thus $\alpha = \theta$ because the dashed line is vertical. Since a cube has sides of equal length, we know that $\tan \alpha = L/L = 1$, so $\alpha = 45°$ and therefore $\theta = 45°$. Now balance forces on the block. The forces parallel to the surface of the plane are f (up the plane) and $mg \sin\theta$ (down the plane). The forces perpendicular to the surface are n (away from the surface) and $mg \cos\theta$ (toward the surface). Balancing forces perpendicular to the surface gives $n = mg \cos\theta$.

Balancing forces parallel to the surface gives

$f = mg \sin\theta.$ (Eq. 1)

We want the minimum coefficient of friction to prevent slipping, so the block be just ready to slip, in which case

$f = \mu n = \mu mg \cos\theta.$ (Eq. 2)

EVALUATE: Combining equations (1) and (2) gives

$mg \sin\theta = \mu mg \cos\theta$

$\mu = \tan\theta = \tan 45° = 1.$

ASSESS: Our result is independent of the mass of the block, but holds true only if the block is uniform so its center of mass is at its center. A coefficient of friction of 1 is fairly large, but certainly not physically impossible.

76. **INTERPRET:** A nonuniform bar rests on two scales at its ends. We have some knowledge of how its density varies along the bar, and we also know its length and one of the scale readings. We want to find the complete density function of the bar and the reading of the other scale. The rod is not moving, so it is in static equilibrium.

DEVELOP: For static equilibrium, the forces and torques must balance. But to calculate these quantities, we need to know the mass of location of the center of mass of the bar. We know that the equation for the mass density λ of the bar is given by $\lambda = a + bx$, where b is known but we want to find a. Therefore we must express the mass M of the bar the location of its center of mass, x_{cm}, in terms of a. We use material described in Section 9.1 for these calculations. Calling L the length of the bar, its mass is

$$M = \int_0^L \lambda dx = \int_0^L (a+bx)dx = aL + \frac{bL^2}{2} = L\left(a + \frac{bL}{2}\right)$$

and its center of mass is located at

$$x_{cm} = \frac{\int_0^L xdm}{M} = \frac{\int_0^L x\lambda dx}{M} = \frac{\int_0^L x(a+bx)dx}{M} = \frac{\frac{aL^2}{2} + \frac{bL^3}{3}}{L\left(a+\frac{bL}{2}\right)} = \frac{L\left(\frac{a}{2}+\frac{bL}{3}\right)}{a+\frac{bL}{2}}.$$

We now use these quantities in balancing the torques and forces on the bar. The vertical forces acting on the bar are the force of the scale at the right end, the force of the scale at the left end (which is 16 N), and the gravitational force Mg at the center of mass. If we choose the right end as the pivot point, the only unknown in the equation will be the mass M, which contains the unknown a, so we can solve for a. The lever arm for Mg is $L - x_{cm}$, and the lever arm for the 16-N force is L. The torque equation is then $Mg(L - x_{cm}) = (16\text{ N})L$. Using the results that we just found for M and x_{cm}, we can solve for a. Then we can balance the vertical forces and solve for the force F_R due to the right-hand scale.

EVALUATE: Using our results for M and x_{cm}, the torque equation gives us

$Mg(L - x_{cm}) = (16\text{ N})L$

$$L\left(a+\frac{bL}{2}\right)\left[L - \frac{L\left(\frac{a}{2}+\frac{bL}{3}\right)}{a+\frac{bL}{2}}\right] = (16\text{ N})L.$$

Doing a bit of algebra to solve for a gives

$$a = 2\left(\frac{16\text{ N}}{gL} + \frac{bL}{3} - \frac{bL}{2}\right) = 2\left[\frac{16\text{ N}}{(9.81\text{ m/s}^2)(2.0\text{ m})} - \frac{(1.0\text{ kg/m}^2)(2.0\text{ m})}{6}\right] = 0.9643\text{ kg/m},$$ which rounds to $a =$ 0.96 kg/m.

Now balance vertical force4s to find the force F_R due to the right-hand scale.

$16\text{ N} + F_R = Mg.$

Using our earlier result for M, we have

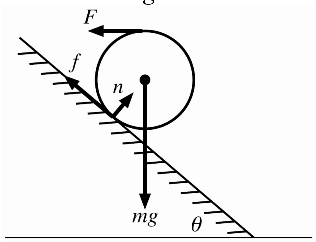
$$F_R = L\left(a+\frac{bL}{2}\right) - 16\text{ N} = (2.0\text{ kg})\left(0.9643\text{ kg/m} + \frac{(1.0\text{ kg/m}^2)(2.0\text{ m})}{2}\right) - 16\text{ N}$$

$F_R = 23\text{ N}.$

ASSESS: The center of mass is at $x_{cm} = \dfrac{L\left(\frac{a}{2}+\frac{bL}{3}\right)}{a+\frac{bL}{2}}$. Putting in our result for a and the known quantities, we find that $x_{cm} = 1.55$ m. This point is closer to the right end of the rod than the left end, which is reasonable since the density increases as we go from left to right in the rod. So more mass is in the right half than in the left half, making the center of mass to the right of the midpoint of the rod.

77. **INTERPRET:** A uniform wheel is supported at rest on a slope by a known horizontal force, so the wheel is in static equilibrium. We want to know the angle θ of the slope. Because the wheel is uniform, its center of mass is at its center.

DEVELOP: We know that the forces acting the wheel are the horizontal 38.8-N force F, its weight mg downward, and the normal force n perpendicular to the surface of the slope. The 38.8-N force must prevent the wheel from rolling down the slope, so it must point toward the slope instead of away from it. The statement of the problem says nothing about friction, but we cannot conclude that there is no friction. Gravity and the normal force act at (or through) the center of the wheel, so they exert no torque about the center of the wheel. There must be a torque about the center of the wheel to balance the torque due to the 38.8-N force, and that can only come from friction. The figure below shows a free-body diagram of the wheel, and we can see that the friction force must be up the slope to balance the torque due to the 38.8-N force. Balancing the torques about the center of the wheel tells us that $FR = fR$, where R is its radius, so $f = F$. Balancing forces parallel to the surface of the plane gives $f + R\cos\theta = mg\sin\theta$. Since $f = F$, this equation becomes $F + F\cos\theta = mg\sin\theta$.



EVALUATE: We want θ, so we solve the equation $F + F\cos\theta = mg\sin\theta$ for θ. To do this, we use the trigonometric identity $\sin^2\theta + \cos^2\theta = 1$ to put the equation only in terms of $\sin\theta$ (or $\cos\theta$). Solving for $\cos\theta$ and putting it into the force equation gives

$$F + F\sqrt{1-\sin^2\theta} = mg\sin\theta$$

$$\sqrt{1-\sin^2\theta} = (mg/F)\sin\theta - 1.$$

Squaring and rearranging gives

$[1 + (mg/F)^2]\sin^2\theta - 2(mg/F)\sin\theta = 0.$

This equation has two solutions, $\sin\theta = 0$, so $\theta = 0°$, and

$$\sin\theta = \frac{2mg/F}{1+(mg/F)^2} = \frac{2(28.5\text{ kg})(9.81\text{ m/s}^2)/(38.8\text{ N})}{1+\left[(28.5\text{ kg})(9.81\text{ m/s}^2)/(38.8\text{ N})\right]^2} = 0.2723,$$ which gives $\theta = 15.8°$. It may appear that this problem has two solutions, but the $\theta = 0°$ solution is not physically possible. Friction would have to oppose the torque due to F, but in that case F and f would be in the same direction, so the wheel could not be held at rest as specified in the problem. Therefore the only physically possible solution is $\theta = 15.8°$.

ASSESS: In this problem, we cannot use $f = \mu n$ because we do not know if the wheel is just ready to slip. We only know that it is being supported by the horizontal 38.8-N force.

Chapter 13: Alternate Problem Set in *Mastering Physics*

91. **INTERPRET:** This is a unit conversion problem.

DEVELOP: We know the heart rate in beats per second (Hz) and want to find it in beats per minute. 1.17 Hz = 1.17 s^{-1} = 1.17 beats/s.

EVALUATE: $\left(1.17\dfrac{\text{beats}}{\text{s}}\right)\left(\dfrac{60\text{ s}}{1\text{ min}}\right)$ = 70.2 beats/min.

ASSESS: 1.17 beats/s is around 1 beat/s, which is about 60 beats per minute. This is in the neighborhood of our answer, so our result seems reasonable.

92. **INTERPRET:** We know the period and want the frequency of an oscillation.

DEVELOP: The time for one cycle is the period T, and we know that $f = 1/T$. Use 11.6 fs = 11.6×10^{-15} s.

EVALUATE: Use $f = 1/T$.

$f = 1/(11.6\text{ fs}) = 1/(11.6\times10^{-15}\text{ s}) = 8.62\times10^{13}\text{ s}^{-1} = 86.2\times10^{12}\text{ s}^{-1} = 86.2\text{ THz}$.

ASSESS: The period is very short, so we expect a very high frequency, which is what we have found, so our answer seems reasonable.

93. **INTERPRET:** This problem involves simple harmonic motion (SHM). An object oscillates on a spring. We know the object's mass as well as the amplitude and frequency of the oscillations. We want to find the maximum speed the object achieves and the spring constant of the spring.

DEVELOP: For SHM we have $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ and $v(t) = -\omega A\sin\omega t$, where $\omega = 2\pi f$. We know f, m, and A, and we want k and the maximum speed v_{max} of the object.

EVALUATE: (a) The velocity at any time t is $v(t) = -\omega A\sin\omega t$. The maximum speed occurs when $\sin\omega t = 1$, and we disregard the minus sign since we only want the speed, not the direction of motion. Therefore

$v_{max} = \omega A = 2\pi fA = 2\pi(0.840\text{ Hz})(25.2\text{ cm}) = 133\text{ cm/s}$.

(b) Use $f = \dfrac{1}{2\pi}\sqrt{\dfrac{k}{m}}$ and solve for k, giving

$k = m(2\pi f)^2 = (0.225\text{ kg})[2\pi(0.840\text{ Hz})]^2 = 6.27\text{ N/m}$.

ASSESS: The units for k should be N/m, so we check to see if we have them correct. Just looking at the units of our calculation $m(2\pi f)^2$ gives

$\text{kg(Hz)}^2 = \text{kg}(1/\text{s})^2 = \dfrac{\text{kg}\cdot\text{m}}{\text{m}\cdot\text{s}^2} = \dfrac{\text{kg}\cdot\text{m/s}^2}{\text{m}} = \text{N/m}$. The units check, so we have some confidence in our answer.

94. **INTERPRET:** This problem involves simple harmonic motion (SHM). A quartz crystal oscillates at a known frequency. We want to know its maximum oscillation amplitude and its maximum acceleration.

DEVELOP: Model the crystal as a mass on a spring. For SHM, the acceleration as a function of time is $a(t) = -\omega^2 A\cos\omega t$. The largest that $\cos\omega t$ can be is 1, and we disregard the minus sign since we only are concerned with the maximum acceleration. We also use $\omega = 2\pi f$, so $a_{max} = \omega^2 A = (2\pi f)^2 A$. Solve for A when $a_{max} = 6.22\text{ km/s}^2 = 6220\text{ m/s}^2$.

EVALUATE: Using $a_{max} = \omega^2 A = (2\pi f)^2 A$ gives

$6220\text{ m/s}^2 = [2\pi(32{,}768\text{ Hz})]^2 A$

$A = 1.47\times10^{-7}\text{ m} = 147\times10^{-9}\text{ m} = 147\text{ nm}$.

ASSESS: For an oscillating crystal, we expect a very small amplitude, so an answer of A = 147 nm is plausible.

95. **INTERPRET:** This problem is about the oscillation of a simple pendulum. We know its frequency and want its length.

DEVELOP: For a simple pendulum, $T = 2\pi\sqrt{L/g}$. A tick occurs every time it reaches its maximum angle of swing, and this is twice per cycle (once on either side of the vertical). A tick occurs once each second, and 2 ticks make a cycle, so the period is 2.00 s.

EVALUATE: Solve $T = 2\pi\sqrt{L/g}$ for L and use the known numbers.

$$L = g(T/2\pi)^2 = (9.81\text{ m/s}^2)\left(\frac{2.00\text{ s}}{2\pi}\right)^2 = 0.994\text{ m} = 99.4\text{ cm}.$$

ASSESS: A length of about one meter is reasonable for the pendulum in a household grandfather clock.

96. **INTERPRET:** This problem is about a physical pendulum, in this case a baseball bat. We want to find its rotation inertia by knowing its period of oscillation.

DEVELOP: The angular frequency of a physical pendulum is $\omega = \sqrt{mgL/I}$, where L is the distance from the rotation axis to the center of mass and $\omega = 2\pi/T$.

EVALUATE: Solve $\omega = \sqrt{mgL/I} = 2\pi/T$ for I, giving $I = mgL(T/2\pi)^2$. In this case, L is ¾ the length of the bat, so $L = (3/4)(0.950\text{ m}) = 0.7125$ m. Therefore

$I = (0.942\text{ kg})(9.81\text{ m/s}^2)(0.7125\text{ m})[(1.85\text{ s})/2\pi]^2 = 0.571\ \text{kg}\cdot\text{m}^2$.

ASSESS: If the bat were treated as a simple pendulum of length 0.7125 m, its period would be $T = 2\pi\sqrt{L/g}$ = $2\pi\sqrt{\dfrac{0.7125\text{ m}}{9.81\text{ m/s}^2}}$ = 1.69 s, which is significantly different from 1.85 s.

97. **INTERPRET:** This problem is about resonance in driven oscillations.

DEVELOP: The car shakes violently because the frequency with which it his the bumps must be the same as (or very close to) the natural frequency of its suspension system. Therefore the natural period for the suspension system is the time for the car to go from one bump to the next bump at a constant 65 km/h. Use $x = vt$ for constant speed and $f = 1/T$.

EVALUATE: First find the time for the car to travel between adjacent bumps. Use $x = vt$, where $v = 65$ km/h = (65,000 km)/(3600 s) = 18.056 m/s. This gives

40 m = (18.056 m/s)t

t = 2.215 s.

The frequency is $f = 1/T = 1/(2.215\text{ s}) = 0.45$ Hz.

ASSESS: This type of resonance can be occur when driving on "washboard" gravel or dirt roads. It can be very dangerous and can easily cause the driver to lose control of the car.

98. **INTERPRET:** This problem involves an object oscillating vertically on a spring, so it deals with simple harmonic motion (SHM).

DEVELOP: When the mass is attached to the spring, it stretches the spring to a new equilibrium position. If L_0 was the original unstretched length of the spring, the mass stretches it by a distance x such that $kx = mg$, so the length of the spring is now $L_0 + x$ and the new equilibrium position is at $x = mg/k$. The system is now set into vertical oscillations of amplitude A. At the highest point in the object's oscillations, the spring again has a length L_0. At that point, we know that the object is at rest so it is a distance A from its new equilibrium point. We also know that the object has traveled from $L_0 + x$ to L_0, a distance of x. Therefore $A = x = mg/k$. For SHM, $\omega = \sqrt{k/m} = 2\pi f$, so $k/m = (2\pi f)^2$, which means that $m/k = 1/(2\pi f)^2$. We just saw that $A = mg/k$, so

$A = mg/k = (m/k)g = (1/2\pi f)^2 g = g/(2\pi f)^2$.

EVALUATE: $A = g/(2\pi f)^2 = (9.81\text{ m/s}^2)/[2\pi(3.24\text{ Hz})]^2 = 0.0237\text{ m} = 2.37$ cm.

ASSESS: The spring stretched 2.37 cm when the mass was attached to it.

99. **INTERPRET:** This is a problem about simple harmonic motion (SHM).

DEVELOP: The particle follows a frictionless path given by the equation $y = bx^2$. Its gravitational potential energy is $U = mgy = mgbx^2$. The particle oscillates back and forth about the lowest point ($y = 0$) in its path, so it behaves like a particle on a spring having a spring constant k. Therefore the potential energy using the equivalent spring constant is $U = mgbx^2 = \frac{1}{2}kx^2$, which tells us that $k = 2mgb$. The particle's angular frequency is $\omega = \sqrt{k/m}$, so $\omega = \sqrt{\dfrac{2mgb}{m}} = \sqrt{2gb}$. We know the period, so we use $\omega = 2\pi/T$, which gives $2\pi/T = \sqrt{2gb}$. We can now solve for b.

EVALUATE: Solving $2\pi/T = \sqrt{2gb}$ for b gives

$$b = \frac{(2\pi/T)^2}{2g} = \frac{\left(\dfrac{2\pi}{1.66\ \text{s}}\right)^2}{2(9.81\ \text{m/s}^2)} = 0.730\ \text{m}^{-1}.$$

ASSESS: Check units. Since $y = bx^2$ and x and y are both lengths, b should have units of 1/m, which agrees with our result.

100. **INTERPRET:** This problem involves a physical pendulum.

DEVELOP: We know that $\omega = \sqrt{mgL/I}$ and $\omega = 2\pi/T$. Meter sticks are normally uniform, so the center of mass of the stick is at its center. In the formula for ω, L is the distance from the hole to the center of mass of the meter stick. We will call this distance d so we don't confuse it with the length L of the meter stick. For this case, the I in the formula is the rotational inertia about the *hole*, so we need to use the parallel-axis theorem (from Section 10.3) to calculate it. Using this theorem gives

$I = I_{\text{cm}} + md^2 = \frac{1}{12}mL^2 + md^2 = I_{\text{hole}}$. The angular frequency is $\omega = \sqrt{\dfrac{mgd}{I_{\text{hole}}}} = \sqrt{\dfrac{mgd}{\frac{1}{12}mL^2 + md^2}}$. We know T, so we use $\omega = 2\pi/T$ to get

$\dfrac{2\pi}{T} = \sqrt{\dfrac{mgd}{\frac{1}{12}mL^2 + md^2}}$. We can now solve for d.

EVALUATE: Squaring $\dfrac{2\pi}{T} = \sqrt{\dfrac{mgd}{\frac{1}{12}mL^2 + md^2}}$ and solving for d gives

$$\left(\frac{2\pi}{T}\right)^2 = \frac{mgd}{\frac{1}{12}mL^2 + md^2}$$

$$d^2 - g\left(T/2\pi\right)^2 d + \tfrac{1}{12}L^2 = 0$$

$$d^2 - (9.81\ \text{m/s}^2)\left(\frac{2.53\ \text{s}}{2\pi}\right)^2 d + \tfrac{1}{12}(1.0\ \text{m})^2 = 0$$

$$d^2 - (1.5906\ \text{m})d + 0.08333\ \text{m}^2 = 0$$

There are two solutions to this quadratic equation: $d = 1.54$ m and $d = 0.0543$ m. The first solution is not physically possible because the hole would be outside the meter stick. For $d = 0.0543$ m $= 5.43$ cm, the distance of the hole from the upper end of the stick is 50.0 cm – 5.43 cm = 44.6 cm, which is at the 44.6 cm mark.

ASSESS: If the hole were at the upper end of the meter stick, d would be equal to $L/2$, so the angular frequency would be $\omega = \sqrt{\dfrac{mg(L/2)}{\frac{1}{12}mL^2 + m(L/2)^2}} = \sqrt{\dfrac{g/2}{L\left(\frac{1}{12}+\frac{1}{4}\right)}} = 3.84\ \text{rad/s}$, so the period would be $T = 2\pi/\omega = 1.64$ s, which is less than the given period of 2.53 s. This result is reasonable. As the hole gets closer to the center of mass, the period increases. If it were at the center of mass, the stick would not oscillate at all since it would be balanced, so ω would approach zero and T would approach infinity.

Chapter 14: Alternate Problem Set in *Mastering Physics*

85. **INTERPRET:** We need to relate the period and speed of a wave to its wavelength.

DEVELOP: We know the period T and the speed v, and we want the wavelength λ. We know that $f = 1/T$ and $v = \lambda f$.

EVALUATE: $v = \lambda f = \lambda (1/T)$

$5.35 \text{ m/s} = \lambda [1/(3.40 \text{ s})]$

$\lambda = 18.2$ m.

ASSESS: In one period, every point on the wave travels 18.2 m.

86. **INTERPRET:** We need to relate the frequency, wavelength, and speed for a wave.

DEVELOP: Use $v = \lambda f$. We know v and λ and want f.

EVALUATE: Solving for f gives

$$f = \frac{v}{\lambda} = \frac{1500 \text{ m/s}}{0.19\times 10^{-3} \text{ m}} = 7.9\times 10^{6} \text{ s}^{-1} = 7.9\times 10^{6} \text{ Hz} = 7.9 \text{ MHz}.$$

ASSESS: The maximum frequency of audible sound (for humans) is about 20 kHz, so this frequency is much higher than that. This is reasonable since this sound is called *ultrasound*.

87. **INTERPRET:** In this problem, we want to write a mathematical equation for a simple harmonic wave. We know its speed frequency, amplitude, and direction of travel.

DEVELOP: For a simple harmonic wave, the equation is a sinusoidal function, $y = A\cos(kx \pm \omega t)$. We know the frequency, we use $\omega = 2\pi f$. We know the speed v, so we use $v = \lambda f$ to get λ, and then we use $k = 2\pi/\lambda$ to find k. We know the amplitude is $A = 2.87$ cm.

EVALUATE: $\omega = 2\pi f = 2\pi(2.23 \text{ Hz}) = 14.0 \text{ s}^{-1}$.

Combining $v = \lambda f$ and $\lambda = 2\pi/k$ gives $v = \lambda f = (2\pi/k)f = 2\pi f/k$. Solving for k gives $k = 2\pi f/v = 2\pi(2.23 \text{ Hz})/(35.0 \text{ cm/s}) = 40.0 \text{ cm}^{-1}$. In the equation $y = A\cos(kx \pm \omega t)$, we use the + sign because the wave is traveling in the $-x$-direction. Therefore the equation is $y(x,t) = 2.87 \text{ cm} \cos(40.0 \text{ cm}^{-1}\, x + 14.0 \text{ s}^{-1}\, t)$.

ASSESS: The displacement is supposed to be a maximum at $x = 0$ when $t = 0$. For our equation, we have $y(0,0) = 2.87 \text{ cm} \cos(0 + 0) = 2.87$ cm, which is what we expect.

88. **INTERPRET:** This problem deals with a traveling wave on a string and the power carried by that wave.

DEVELOP: The speed of the wave is $v = \sqrt{\dfrac{F}{\mu}}$. We know the force F and the mass density μ, so we can find the string tension F. The average power carried by the wave is $\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v$. Using $\omega = 2\pi f$, we know all the quantities on the right-hand side of the equation, so we can find the power.

EVALUATE: (a) Solve $v = \sqrt{\dfrac{F}{\mu}}$ for F, giving

$F = \mu v^2 = (0.0156 \text{ kg/m})(37.2 \text{ m/s})^2 = 21.6$ N.

(b) Use $\bar{P} = \frac{1}{2}\mu\omega^2 A^2 v = \frac{1}{2}\mu(2\pi\omega)^2 A^2 v$ to find the power.

$\bar{P} = \frac{1}{2}(0.0156 \text{ kg/m})[2\pi(44.3 \text{ Hz})]^2(0.00125 \text{ m})^2(37.2 \text{ m/s}) = 0.0351 \text{ W} = 35.1 \text{ mW}$.

ASSESS: This is a very small power compared to a typical household 75-W lightbulb.

89. **INTERPRET:** This problem deals with the speed of sound in air.

DEVELOP: Use $v=\sqrt{\dfrac{\gamma P}{\rho}}$, where $\gamma = \frac{7}{5}$ for air. We know P and v and we want to find the density ρ.

EVALUATE: The pressure is $P = 101.3\ \text{kN/m}^2 = 101{,}300\ \text{N/m}^2$. Solve $v=\sqrt{\dfrac{\gamma P}{\rho}}$ for ρ, giving

$\rho = \gamma P/v^2 = \frac{7}{5}(101{,}300\ \text{N/m}^2)/(750\ \text{m/s})^2 = 0.25\ \text{kg/m}^3$.

ASSESS: The speed of sound in ordinary air is about 340 m/s, so the density we just found is not the density of air in an ordinary room. Since ρ is proportional to $1/v^2$, the air density in an ordinary room would be greater than 0.25 kg/m^3. In fact, it is approximately 1.3 kg/m^3, which is significantly different from what we just found.

90. **INTERPRET:** This problem is about beats. The sound waves from the two engines have slightly different frequencies, so they interfere to produce beats.
DEVELOP: Each revolution of the engine produces a sound wave, so the frequency of the sound from one engine is 560 Hz, and we want to find the possible frequencies of the other engine. The period of the beat is 6.0 s = $\frac{1}{10}$ min, so the frequency of the beat is $f_B = 1/(\frac{1}{10}\ \text{min}) = 10\ \text{min}^{-1}$. The frequency of the beat is equal to the difference between the two frequencies, so $f_B = |560\ \text{min}^{-1} - f_2|$.
EVALUATE: Since $f_B = |560\ \text{min}^{-1} - f_2|$, the possibilities for f_2 are
$560\ \text{min}^{-1} - 10\ \text{min}^{-1} = 550\ \text{min}^{-1}$ and $560\ \text{min}^{-1} + 10\ \text{min}^{-1} = 570\ \text{min}^{-1}$.
So the rotational speed of the other end could be either 550 rpm or 570 rpm.
ASSESS: A beat of frequency 10 min^{-1} could easily be detected just by listening.

91. **INTERPRET:** This problem is about the Doppler effect.

DEVELOP: The observer (firefighter) is moving toward the source (firehouse siren), so we use $f' = f\left(1 \pm \dfrac{u}{v}\right)$, where f' is the frequency the observer receives. The observer moves toward the source, so we use the + sign in the equation. We know the two frequencies and want to find the speed u of the observer.

EVALUATE: Solve $f' = f\left(1+\dfrac{u}{v}\right)$ for u, giving

$u = v\left(\dfrac{f'}{f}-1\right) = (343\ \text{m/s})\left(\dfrac{93.2\ \text{Hz}}{85.0\ \text{Hz}}-1\right) = 33.1\ \text{m/s}$. This is equivalent to 119 km/h.

ASSESS: The observer is moving toward the source and therefore should hear a higher frequency than the source is emitting. By choosing the + sign in the equation $f' = f\left(1 \pm \dfrac{u}{v}\right)$, we achieved this, so our choice was correct.

92. **INTERPRET:** This problem uses the definition of the intensity of a spherical sound wave.
DEVELOP: For a spherical wave, $I = P/4\pi r^2$. We solve for P at $r = 18$ m.
EVALUATE: We use $I = 1.4\ \text{mW/m}^2 = 0.0014\ \text{W/m}^2$. Solving for P gives
$P = 4\pi r^2 I = 4\pi(18\ \text{m})^2(0.0014\ \text{W/m}^2) = 5.7\ \text{W}$.
ASSESS: The power of this source is equivalent to 75/5.7 = 13 standard 75-W incandescent lightbulbs.

93. **INTERPRET:** This problem involves the speed of transverse waves on a stretched spring. Stretching the spring changes both the tension F in it and the mass density μ of the spring. We model the spring as an ordinary string.

DEVELOP: The speed of a wave on a string is $v=\sqrt{\dfrac{F}{\mu}}$, where F is the tension in the string and μ is its mass density. For a string, the tension is $F = kx$. Let L stand for the length of the spring.
Unstretched spring: $L = L_0$, $\mu_0 = m/L_0$.
At length L_1: $L_1 = L_0 + x_1$, so $x_1 = L - L_0$.
$\mu_1 = m/L_1$
$F_1 = kx_1 = k(L_1 - L_0)$

$$v_1 = \sqrt{\frac{F_1}{\mu_1}} = \sqrt{\frac{k(L_1 - L_0)}{m/L_1}}$$

At length L_2: $L_2 = L_1 + \frac{1}{4}L_1 = \frac{5}{4}L_1$

$\mu_2 = m/L_2 = m/(\frac{5}{4}L_1)$

$x_2 = L_2 - L_0 = \frac{5}{4}L_1 - L_0$

$F_2 = kx_2 = k(\frac{5}{4}L_1 - L_0)$

$$v_2 = \sqrt{\frac{F_2}{\mu_2}} = \sqrt{\frac{k\left(\frac{5}{4}L_1 - L_0\right)}{m/(\frac{5}{4}L_1)}}.$$

Using $v_2 = 2v_1$ gives

$$v_2 = \sqrt{\frac{k\left(\frac{5}{4}L_1 - L_0\right)}{m/(\frac{5}{4}L_1)}} = 2\sqrt{\frac{k(L_1 - L_0)}{m/L_1}}.$$

EVALUATE: Square both sides of the last equation, cancel k and m, and solve for L_0 in terms of L_1, which gives $L_0 = \frac{39}{44}L_1 = 0.89L_1$.

ASSESS: We have assumed that stretching the spring did not affect its spring constant k. We also assumed that the stretching was uniform over the length of the spring so that μ would be constant over the length of the spring.

94. **INTERPRET:** This problem is about the relationship between the intensity of a sound wave, its intensity level, and the distance from the source of sound.

DEVELOP: The intensity is $I = P/4\pi r^2$ and the intensity level is $\beta = 10\log\left(\frac{I}{I_0}\right)$. At 2.0 m from the source, $\beta = 75$ dB. We want to know the intensity level (β) at 8.0 from the source. We can use $I = P/4\pi r^2$ to find the intensity (I) at 8.0 m in terms of the intensity at 2.0 m. Then we can use that result to calculate the intensity level (β) at 8.0 m.

EVALUATE: Using is $I = P/4\pi r^2$, we have

$I_8 = P/[4\pi(8.0\text{ m})]^2$

$I_2 = P/[4\pi(2.0\text{ m})]^2$

Taking the ratio of I_8 to I_2 gives

$$\frac{I_8}{I_2} = \frac{P/\left[4\pi(8.0\text{ m})^2\right]}{P/\left[4\pi(2.0\text{ m})^2\right]} = \left(\frac{2.0\text{ m}}{8.0\text{ m}}\right)^2 = \frac{1}{16}$$

Therefore $I_8 = \frac{1}{16}I_2$.

Now use $\beta = 10\log\left(\frac{I}{I_0}\right)$ to find the intensity level at 8.0 m.

$$\beta_8 = 10\log\left(\frac{I_8}{I_0}\right) \quad\text{and}\quad \beta_2 = 10\log\left(\frac{I_2}{I_0}\right)$$

$$\beta_8 - \beta_2 = 10\log\left(\frac{I_8}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right) = 10\left[\log\left(\frac{I_8}{I_0}\right)\left(\frac{I_0}{I_2}\right)\right] = 10\log\left(\frac{I_8}{I_2}\right).$$

Now use $I_8 = \frac{1}{16}I_2$, which gives

$$\beta_8 - \beta_2 = 10\log\left(\frac{I_2/16}{I_2}\right) = 10\log(1/16) = -12\text{ dB}$$

$\beta_8 = \beta_2 - 12\text{ dB} = 75\text{ dB} - 12\text{ dB} = 63\text{ dB}$.

ASSESS: The intensity level at 8.0 m should be less than it is at 2.0 m because the intensity decreases with distance, and this agrees with our result. Note that the intensity I obeys the inverse-square distance law, but the sound level β does *not*.

Chapter 15: Alternate Problem Set in *Mastering Physics*

84. **INTERPRET:** This problem involves density.

DEVELOP: Density is $\rho = m/V$, and the volume of a sphere is $V = \frac{4}{3}\pi R^3$. Divide the densities to compare them, realizing that both stars have the same mass *m*.

$$\frac{\rho_{\text{sun}}}{\rho_{\text{ns}}} = \frac{m/V_{\text{sun}}}{m/V_{\text{ns}}} = \frac{V_{\text{ns}}}{V_{\text{sun}}} = \frac{\frac{4}{3}\pi R_{\text{ns}}^3}{\frac{4}{3}\pi R_{\text{sun}}^3} = \frac{R_{\text{ns}}^3}{R_{\text{sun}}^3}$$

$$\frac{R_{\text{ns}}}{R_{\text{sun}}} = \left(\frac{\rho_{\text{sun}}}{\rho_{\text{ns}}}\right)^{1/3}.$$

EVALUATE: Putting in the given numbers, we get

$$\frac{R_{\text{ns}}}{R_{\text{sun}}} = \left(\frac{1400\ \text{kg/m}^3}{4.0\times10^{17}\ \text{kg/m}^3}\right)^{1/3} = 1.5\times10^{-5}$$

Therefore $R_{\text{ns}} = 1.5\times10^{-5} R_{\text{sun}}$.

ASSESS: Since both stars have the same mass, the denser star should have the smaller radius, which agrees with our answer.

85. **INTERPRET:** We must use the definition of pressure.

DEVELOP: The pressure is $p = F/A$. We know F and p, so solve for A.

EVALUATE: $A = F/p = (12{,}000\ \text{N})/(360\times10^9\ \text{Pa}) = 3.3\times10^{-8}\ \text{m}^2$.

ASSESS: The area is circular, so we can use $A = \pi r^2$ to find its diameter. Its radius would be

$r = \sqrt{A/\pi} = \sqrt{\dfrac{3.3\times10^{-8}\ \text{m}^2}{\pi}} = 1.0\times10^{-4}\ \text{m}$ = 0.10 mm, so its diameter would be 0.20 mm. This is small, but not microscopic.

86. **INTERPRET:** This problem deals with the pressure in a fluid.

DEVELOP: The lake water rises up because the pressure at the water surface under the tornado funnel is less than the atmospheric pressure on the rest of the lake surface. It is this pressure difference, Δp, that raises the water. The magnitude of this pressure difference is $\Delta p = p_{\text{atm}} - p_{\text{f}}$, where p_{f} is the pressure at the water surface under the funnel. So $\Delta p = \rho gh$ due to 1.9 m of water. Therefore $\rho gh = p_{\text{atm}} - p_{\text{f}}$.

EVALUATE: Solving $\rho gh = p_{\text{atm}} - p_{\text{f}}$ for p_{f} gives

$p_{\text{f}} = p_{\text{atm}} - \rho gh = 101.3\times10^3\ \text{Pa} - (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(1.9\ \text{m}) = 8.266\times10^4\ \text{Pa}$.

Comparing this to atmospheric pressure we get

$$\frac{p_{\text{f}}}{p_{\text{atm}}} = \frac{8.266\times10^4\,\text{Pa}}{101.3\times10^5\ \text{Pa}} = 0.82 = 82\%.$$

The pressure under the funnel is 82% of atmospheric pressure.

ASSESS: An 18% reduction in pressure is *much* greater than pressure variations due to high-pressure and low-pressure weather systems.

87. **INTERPRET:** This problem deals with buoyancy and Archimedes' principle. The rock seems lighter in the water because some of its weight is balanced by the buoyancy force from the fluid.

DEVELOP: We apply Archimedes' principle: the buoyancy force F_{b} is equal to the weight of the fluid displaced by the rock. The weight of the rock is $F_{\text{g}} = m_{\text{r}}g$. In water, the maximum force you exert to lift the rock is

$F_{\max} = m_{\text{r}}g - F_{\text{b}} = m_{\text{r}}g - \rho_{\text{w}} Vg$

where ρ_w is the density of water and V is the volume of the rock. On dry land, the maximum force you can exert is the same as it is in water, but now that force can only lift a lighter rock because there is no buoyancy force to help. Calling m_L the greatest mass you can lift on land, we have $F_{max} = m_L g$. Equating our two expressions for F_{max} gives $m_L g = m_r g - \rho_w Vg$, which simplifies to $m_L = m_r - \rho_w V$. To find the volume V of the rock, we use $\rho_r = m_r/V$, so $V = m_r/\rho_r$. Using this in our equation for m_L gives $m_L = m_r - \rho_w\left(\frac{m_r}{\rho_r}\right) = m_r\left(1 - \frac{\rho_w}{\rho_r}\right)$.

EVALUATE: Put in the numbers:

$$m_L = m_r\left(1 - \frac{\rho_w}{\rho_r}\right) = (65\ \text{kg})\left(1 - \frac{1.0\ \text{g/cm}^3}{2.7\ \text{g/cm}^3}\right) = 41\ \text{kg}.$$

ASSESS: The buoyancy force under water is a substantial fraction of the weight of the rock, so it is not inconsequential. Note that in our final calculation, we used the densities in g/cm^3 without converting to kg/m^3. We could do this because the units canceled out, making it unnecessary to convert. Always be alert for such short cuts when solving problems!

88. **INTERPRET:** This is a buoyancy problem. If an object is floating, the buoyancy force is always equal to the weight of the object. Burt if the object just on the verge of sinking, it is fully submerged (but not under the surface of the liquid), so in this case the average density of the object is equal to the density of the fluid.

DEVELOP: For the drum to just barely float, the average density $\bar{\rho}$ of the drum with the water inside of it must be equal to the density of water ρ_w, so $\bar{\rho} = \rho_w$. Calling m_w the mass of water inside the drum, m_d the mass of the drum, and V_d the volume of the drum, the average density is $\bar{\rho} = \frac{m_d + m_w}{V_d}$, which must be equal to ρ_w, so

$$\frac{m_d + m_w}{V_d} = \rho_w.$$

EVALUATE: Using $\frac{m_d + m_w}{V_d} = \rho_w$ and solving for m_w, we have

$m_w = \rho_w V_d - m_d = (1000\ \text{kg/m}^3)(0.21\ \text{m}^3) - 19\ \text{kg} = 190\ \text{kg}.$

ASSESS: The drum cannot be totally full of water and still float since then its average density would be greater than the density of water due to the heavy steel. The volume of water in the drum is $V = m_w/\rho_w = (190\ \text{kg})/(1000\ \text{kg/m}^3) = 0.19\ \text{m}^3$, which is less than the 0.21 m^3 volume of the drum. Therefore our answer is reasonable.

89. **INTERPRET:** This problem involves fluid flow. We want to find the volume flow rate and the flow speed of the water in a hose.

DEVELOP: We know flow speed and diameter of the nozzle, plus the diameter of the hose. We want use this in formation to find the volume flow rate and the flow speed in the hose. The flow rate is given by vA and the mass flow rate is ρvA. The flow rate is constant along a flow tube, so the continuity equation is $v_1A_1 = v_2A_2$, where 1 and 2 are any two points along a flow tube. We know (or can find) v and A at the nozzle. We can use these to find v at the hose. Then we can use this v to find the volume flow rate.

EVALUATE: First find the speed of the fluid in the hose, v_h. We know the speed in the nozzle ($v_n = 31$ m/s) and the area of the nozzle (πr_n^2) and the area of the hose (πr_h^2). The continuity equation gives us

$$v_1A_1 = v_2A_2$$
$$v_hA_h = v_nA_n$$
$$v_h(\pi r_h^2) = v_n(\pi r_n^2)$$
$$v_h = v_n\left(\frac{r_n}{r_h}\right)^2 = v_n\left(\frac{D_n}{D_h}\right)^2, \text{ where } D \text{ is diameter.}$$
$$v_h = v_n\left(\frac{D_n}{D_h}\right)^2 = (31\ \text{m/s})\left(\frac{2.5\ \text{cm}}{15\ \text{cm}}\right)^2 = 0.8611\ \text{m/s}.$$

(a) The volume flow rate in the hose is

$v_h A_h = v_h(\pi r_h^2) = (0.8611 \text{ m/s})\pi(0.075 \text{ m})^2 = 0.015 \text{ m}^3/\text{s}$.

(b) As show above, $v_h = 0.86$ m/s.

ASSESS: The speeds are different in the hose and nozzle, but the volume flow rates are the same in both. The speed is greater in a narrow section than in a wide section, so we found that $v_h < v_n$ because $r_h > r_n$. We have also treated water as an incompressible fluid, which is true unless the pressure is extremely large.

90. **INTERPRET:** This problem involves buoyancy and Archimedes' principle.

DEVELOP: The range of the number of marbles in the beaker is determined by the following extreme cases.

Case (i): With 7 marbles inside of it, the beaker could be just at the point of sinking, in which case the buoyancy force on it would be equal to the weight of the beaker plus 7 marbles, and the volume of water displaced would be equal to the volume of the beaker.

Case (ii): With 8 marbles inside of it, the beaker could be just at the point of sinking, in which case the buoyancy force on it would be equal to the weight of the beaker plus 8 marbles, and the volume of water displaced would be equal to the volume of the beaker.

First find the mass m_B of the empty beaker. It floats with 1/3 of its volume submerged, so the buoyancy force is equal to the weight of the beaker. If we call V the volume of the beaker, then the submerged volume is $V/3$. Using ρ_w for the density of water and ρ_B for the average density of the empty beaker, Archimedes' principle gives

$F_b = m_B g$

$\rho_w g(V/3) = \rho_B gV$

$\rho_B = \rho_w/3$,

so the average density of the empty beaker is equal to 1/3 the density of water. This is the density of the glass plus the space inside, not just the glass.

EVALUATE: We can calculate the volume V of the beaker, so its mass is

$m_B = \rho_B V = \rho_B(\pi r^2)h = \frac{1}{3}\rho_w \pi r^2 h = \frac{1}{3}(1.00 \text{ g/cm}^3)\pi(2.00 \text{ cm})^2(12.0 \text{ cm}) = 50.27$ g.

Now look at the two cases.

Case (i): The average density of the beaker plus 7 marbles inside is equal to the density ρ_w of water. Calling m the mass of each marble and use $\rho = m/V$, we get

$$\frac{m_B + 7m}{V} = \rho_w$$

$m_B + 7m = \rho_w V$

$50.27 \text{ g} + 7m = \rho_w \pi r^2 h = (1.00 \text{ g/cm}^3)\pi(2.00 \text{ cm})^2(12.5 \text{ cm})$

$m = 14.4$ g.

Case (ii): The average density of the beaker plus 8 marbles inside is equal to the density ρ_w of water. Calling m the mass of each marble and use $\rho = m/V$, we get

$$\frac{m_B + 8m}{V} = \rho_w$$

$m_B + 8m = \rho_w V$

$50.27 \text{ g} + 8m = \rho_w \pi r^2 h = (1.00 \text{ g/cm}^3)\pi(2.00 \text{ cm})^2(12.5 \text{ cm})$

$m = 12.6$ g.

So the range of possible masses for the marbles is $12.6 \text{ g} \le m \le 14.4$ g.

ASSESS: To check for possible errors, use $\rho = m/V$ to calculate the average density of the beaker plus marbles in each case. It should be equal to the density of water. The volume of the empty beaker is $V = \pi r^2 h = \pi(2.00 \text{ cm})^2(12.5 \text{ cm}) = 150.8 \text{ cm}^3$.

Case (i): $\rho = \dfrac{7(14.4 \text{ g}) + 50.27 \text{ g}}{150.8 \text{ cm}^3} = 1.00 \text{ g/cm}^3$.

Case (ii): $\rho = \dfrac{8(12.6 \text{ g}) + 50.27 \text{ g}}{150.8 \text{ cm}^3} = 1.00 \text{ g/cm}^3$.

Both cases check, so our answer appears to be correct.

91. **INTERPRET:** This problem deals with buoyancy and Archimedes' principle.

DEVELOP: The buoyancy force is equal to the weight $m_s g$ of the swimmer plus the weight $m_f g$ of the float. Since the top of the float is just at the water surface, the volume of the water displaced is equal to the volume V of the float. Applying Archimedes' principle gives $F_b = m_s g + m_f g$. The buoyancy force is $F_b = \rho_w gV$, where ρ_w is the density of water. Therefore

$\rho_w gV = m_s g + m_f g$

$\rho_w V = m_s + m_f$

Use $\rho = m/V$ to find the volume V of the float.

$V = m_f/\rho_f = (11.6\text{ kg})/(158\text{ kg/m}^3) = 0.07342\text{ m}^3$.

EVALUATE: Archimedes' principle gives

$\rho_w V = m_s + m_f$

$(1000\text{ kg/m}^3)(0.07342\text{ m}^3) = m_s + 11.6\text{ kg}$

$m_s = 61.8\text{ kg}$.

ASSESS: Notice that we did not need (or know) the volume of the swimmer since he is on the float and thus not displacing water. A mass of 61.8 kg (≈136 lb) is quite reasonable for either a female or male swimmer.

92. **INTERPRET:** This problem involves fluid flow, so Bernoulli's equation applies.

DEVELOP: Bernoulli's equation is $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$. Let the subscript 1 refer to the water in the hose and the subscript 2 refer to the water in the sprinkler hole. The hose and sprinkler are normally at the same level, so $y_1 = y_2$. We assume that the water in the hose is traveling much slower than the water as it exits the sprinkler holes, since the holes are tiny compared to the width of a typical hose, so $v_1 \approx 0$ m/s. Under these condition s, Bernoulli's equation now becomes

$p_1 - p_2 = \frac{1}{2}\rho v_2^2$. The pressure p_2 is at the sprinkler hole, so it is equal to atmospheric pressure, $p_2 = p_{atm}$. Therefore $p_1 - p_2 = p_1 - p_{atm} = p_{gauge}$, which means that $p_{gauge} = \frac{1}{2}\rho v_2^2$. We can find v_2 using conservation of mechanical energy once the water has left the sprinkler hole. This gives $mgh = \frac{1}{2}mv_2^2$, so $gh = \frac{1}{2}v_2^2$. Therefore

$p_{gauge} = \frac{1}{2}\rho v_2^2 = \rho \frac{1}{2}v_2^2 = \rho g h.$

EVALUATE: Using $p_{gauge} = \rho g h$ gives

$p_{gauge} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)(9.45\text{ m}) = 9.27\times 10^4\text{ Pa} = 92.7\text{ kPa}$.

ASSESS: The gauge pressure is $\dfrac{92.7\text{ kPa}}{101.3\text{ kPa}} = 0.915 = 91.5\%$ of atmospheric pressure. This is why the water goes so high!

93. **INTERPRET:** This problem is about fluid dynamics. It involves Bernoulli's equation and the continuity equation.

DEVELOP: Bernoulli's equation is $p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$. Let the subscript 1 refer to the top of the water in the tank and 2 refer to the water just as it comes out of the hole at the bottom of the tank. Both points are at atmospheric pressure, so $p_1 = p_2 = p_{atm}$ and they cancel out. Call the bottom of the tank the origin, so $y_1 = 0$ m and $y_2 = y$. Bernoulli's equation then becomes $\frac{1}{2}v_1^2 + gy = \frac{1}{2}v_2^2$. The continuity equation $A_1v_1 = A_2v_2$ tells us that

$v_2 = v_1\dfrac{A_1}{A_2}$, so Bernoulli's equation becomes $\frac{1}{2}v_1^2 + gy = \frac{1}{2}\left(v_1\dfrac{A_1}{A_2}\right)^2 = \frac{1}{2}v_1^2\left(\dfrac{A_1}{A_2}\right)^2$. Solving for v_1^2 gives

$v_1^2 = \dfrac{2gy}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}$. To make the calculations less cumbersome, we can write this in a simplified form as $v_1^2 = K^2 y$,

where $K^2 = \dfrac{2g}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}$. The speed at the top v_1 is the rate at which y is changing, so $v_1 = dy/dt$. Thus the equation

$v_1^2 = K^2 y$ becomes $\left(\dfrac{dy}{dt}\right)^2 = K^2 y.$

EVALUATE: We now must solve the differential equation $\left(\frac{dy}{dt}\right)^2 = K^2 y$. Taking the square root gives $\frac{dy}{dt} = \pm K\sqrt{y} = \pm K y^{1/2}$. Since $A_1 > A_2$, K is positive. We use the negative square root because y is decreasing as t is increasing. Thus $\frac{dy}{dt} = -K y^{1/2}$. We solve this differential equation using separation of variables. Divide both sides by $y^{1/2}$ and multiply by dt, giving $\frac{dy}{y^{1/2}} = -K dt$. Integrating gives $\int_h^0 \frac{dy}{y^{1/2}} = -\int_0^T K dt$, where h is the initial height of the water and T is the time we want to find. The integration gives $-\frac{\sqrt{h}}{1/2} = -KT$, so

$$T = \frac{2\sqrt{h}}{K} = \frac{2\sqrt{h}}{\sqrt{\dfrac{2g}{\left(\dfrac{A_1}{A_2}\right)^2 - 1}}}.$$

$$\left(\frac{A_1}{A_2}\right)^2 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right)^2 = \left(\frac{D_1^2}{D_2^2}\right)^2 = \left[\left(\frac{100 \text{ cm}}{1.00 \text{ cm}}\right)^2\right]^2 = 1.00\times 10^8.$$

This ratio is much greater than 1, so we can neglect the –1 inside the square root in the denominator. Putting in the appropriate numbers gives

$$T = \frac{2\sqrt{2.00 \text{ m}}}{\sqrt{\dfrac{2(9.81 \text{ m/s}^2)}{1.00\times 10^8}}} = 6.39\times 10^3 \text{ s} = 106 \text{ min} = 1.77 \text{ h}.$$

ASSESS: Check the units in our calculation for T. Looking only at the units, we have $\frac{\sqrt{\text{m}}}{\sqrt{\text{m/s}^2}} = \frac{\sqrt{\text{m}}}{\sqrt{\text{m}}}\sqrt{\text{s}^2} = \text{s}$, which are the proper units of time.

Chapter 16: Alternate Problem Set in *Mastering Physics*

87. **INTERPRET:** This is a unit conversion problem.

DEVELOP: We want to convert a temperature of 11°F to Celsius units because those are the temperature units in use in Canada. Start with $T_F = \frac{9}{5} T_C + 32$ and solve for T_C, giving $T_C = \frac{5}{9} (T_F - 32)$.

EVALUATE: $T_C = \frac{5}{9} (T_F - 32) = T_C = \frac{5}{9} (11°F - 32) = -12°C$.

ASSESS: The temperature on the Celsius scale is negative because it is below the zero of that scale, which is 0°C = 32°F.

88. **INTERPRET:** This is a specific heat problem.

DEVELOP: We want to raise the temperature of the water and the pot from 20°C to 100°C. The formula for the heat required is $Q = mc\Delta T$. We can find the total heat required for the pot and water and then find the time this will take since we know the rate at which heat is supplied. The total heat needed is $Q_t = Q_{water} + Q_{pot}$, so $Q_t = m_w c_w \Delta T + m_p c_{al} \Delta T = (m_w c_w + m_p c_{al}) \Delta T$. Once we have found Q_t, we use $P = Q_t/t$ to find the time t. Express all quantities in SI units. The density of water is 1000 kg/m^3, which is equal to 1.0 kg/L, so the mass of 6.0 L is 6.0 kg. From Table 16.1, we have $c_w = 4184\ \text{J/kg}\cdot\text{K} = 4184\ \text{J/kg}\cdot°\text{C}$ and $c_{al} = 900\ \text{J/kg}\cdot°\text{C}$.

EVALUATE: Using $Q_t = (m_w c_w + m_p c_{al})\ \Delta T$, we get

$Q_t = [(6.0\text{ kg})(4184\ \text{J/kg}\cdot°\text{C}) + (2.2\text{ kg})(900\ \text{J/kg}\cdot°\text{C})](100°\text{C} - 20°\text{C})$

$Q_t = 2.167 \times 10^6$ J.

Now use $P = Q_t/t$ to find the time t. Solving for t gives

$t = Q_t/P = (2.167 \times 10^6\text{ J})/(1700\text{ W}) = 1275\text{ s} = 21\text{ min}$.

ASSESS: 6.0 L is about 1.6 gal of water, so 21 min is a reasonable time to bring that much water to a boil starting at room temperature.

89. **INTERPRET:** This is a problem in thermal conduction.

DEVELOP: The magnitude of the rate of heat flow is $H = kA\dfrac{\Delta T}{\Delta x}$. We want to find the thickness Δx of the concrete slab. We know (or can find) H, A, k, and ΔT. Assume that the temperature of the top of the slab is the same as the temperature of the house interior and that the bottom of the slab is at the same temperature as the ground. Also assume that $k = 1\ \text{W/m}\cdot\text{K}$ as shown in Table 16.2.

EVALUATE: Use $H = kA\dfrac{\Delta T}{\Delta x}$ and solve for Δx, giving

$3900\text{ W} = (1\ \text{W/m}\cdot\text{K})(8.0\text{ k})(12\text{ m})(19°\text{C} - 11°\text{C})/\Delta x$

$\Delta x = 0.20\text{ m} = 20\text{ cm}$.

ASSESS: A slab thickness of 20 cm is about 7.75 in., which is reasonable for a house. The answer will vary somewhat depending on the type of concrete used for the slab.

90. **INTERPRET:** This problem is about the radiation of energy, so the Stefan-Boltzmann law applies.

DEVELOP: The radiated power is $P = e\sigma AT^4$. For an ideal blackbody, $e = 1$. We know the power P and the area A and want to find the temperature T.

EVALUATE: Using $P = e\sigma AT^4$ gives

$P = e\sigma AT^4 = (1)(5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.8\text{ m}^2)T^4$

$T = 610$ K.

ASSESS: The temperature is 610 K = 337°C = 640°F. This is a very hot stove, so touching it would not be advisable. But it is much below the melting point of iron, which is over 2,000°F, depending on the type of iron.

91. **INTERPRET:** This problem is about the conduction of heat out of an oven.

DEVELOP: The rate of heat lost by the over (7.3 W per °C) is equal to the rate of heat supplied to the over (930 W). If we call ΔT the temperature difference between the inside of the oven (at temperature *T*) and the 20°C kitchen, then

ΔT = *T* – 20°C. Balancing the rates gives (7.3 W/°C)(ΔT) = 930 W.

EVALUATE: Solve (7.3 W/°C)(ΔT) = 930 W for ΔT.

ΔT = (930 W)/(7.3 W/°C) = 127°C.

T = ΔT + 20°C = 127°C + 20°C = 147°C.

ASSESS: An oven temperature of 147°C is about 300°F, which is typical for household oven cooking.

92. **INTERPRET:** This problem is about the radiation of energy by a lightbulb filament.

DEVELOP: We want to find the radiated power, so we use $P = e\sigma AT^4$. For a blackbody, *e* = 1.

EVALUATE: Using $P = e\sigma AT^4$ gives

$P = e\sigma AT^4 = (1)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(1.3\times10^{-5}\ \text{m}^2)(3100\ \text{K})^4 = 68\ \text{W}$.

This power is closest to that of a 60-W bulb.

ASSESS: The 60 W is the total power radiated by the bulb, but not all of it is visible light.

93. **INTERPRET:** This problem is about specific heat.

DEVELOP: The heat to change the temperature of an object is $Q = mc\Delta T$. From Table 16.1 we have *c* = 447 $\text{J/kg}\cdot\text{K}$ (iron) and *c* = 4184 $\text{J/kg}\cdot\text{K}$ (water). We know the heat and the temperature change, and we want the mass of the pan and water inside it.

EVALUATE: (a) For the empty pan, we have only iron, so

$Q = mc\Delta T$

37,500 J = *m*(447 $\text{J/kg}\cdot\text{K}$)(85°C – 20°C)

m = 1.3 kg.

(b) The *additional* heat required to heat the pan with the water in it is

884 kJ – 37.5 kJ = 846.5 kJ = 846,500 J.

This is the heat needed to heat just the water to 85°C, so we have

$Q = mc\Delta T$

846,500 J = *m*(4184 $\text{J/kg}\cdot\text{K}$)(85°C – 20°C)

m = 3.1 kg.

ASSESS: A 1.3-kg pan weighs about 2.8 lb, which is reasonable for a cast iron pan. Even though the water is less dense than the iron of the pan, it has more mass than the pan because it occupies a greater volume in the pan than does the iron.

94. **INTERPRET:** This problem involves specific heat. The energy from the microwave oven goes into the water and brings it to a boil.

DEVELOP: The power *P* of the microwave oven is equal to the rate at which heat enters the water (assuming that none goes into other objects). The heat that enters the water is $Q = mc\Delta T$. The rate at which the water is heated is *Q*/*t*, and this is equal to the power *P*, so $P = \dfrac{Q}{t} = \dfrac{mc\Delta T}{t}$.

EVALUATE: Put in the numbers. 330 mL of water has a mass of 330 g = 0.33 kg, and from Table 16.1 we have *c* = 4184 $\text{J/kg}\cdot\text{K}$ for water.

$$P = \frac{mc\Delta T}{t} = \frac{(0.33\ \text{kg})(4184\ \text{J/kg}\cdot\text{K})(100°\text{C} - 10°\text{C})}{(2.4)(60\ \text{s})} = 860\ \text{W}.$$

ASSESS: Typical microwaves ovens consume around 700 W to 1000 W, so our result is reasonable.

95. **INTERPRET:** In this problem, mechanical kinetic energy of a car is transformed into heat and this heat increases the temperature of the steel brakes. Therefore we need to use specific heat.

DEVELOP: All the kinetic energy of the car goes into heat at the brakes. This heat raises the temperature of the brakes, so we have $K_{car} = Q_{brakes} = mc\Delta T$, where m is the total mass of the brakes. From Table 16.1, $c = 502$ $\text{J/kg}\cdot\text{K}$ for steel, and $m = 20.0$ kg since there are four brakes. We now have $\frac{1}{2}M_{car}v^2 = mc\Delta T$ and solve for v.

EVALUATE: Putting in the numbers gives

$$\tfrac{1}{2}(1450\ \text{kg})v^2 = (20.0\ \text{kg})(502\ \text{J/kg}\cdot\text{K})(18.3°\text{C})$$

$v = 15.9$ m/s $= 57.3$ km/h.

ASSESS: A speed of 57.3 km/h is about 36 mi/h, which is a reasonable speed for a car to suddenly brake and be able to come to a stop.

96. **INTERPRET:** This problem involves the radiation of energy, so the Stefan-Boltzmann law applies. It also uses the fact that the radiation intensity decreases with distance from the source.

DEVELOP: For the asteroid's surface to remain at a constant temperature of 184 K, the power its surface radiates must be equal to the power the asteroid receives from the sun. The power radiated by the asteroid is $P_{rad} = e\sigma AT^4$. The asteroid behaves like a blackbody, so $e = 1$. If we model it as a sphere of radius R, its surface area is $A = 4\pi R^2$. Therefore the power it radiates is $P_{rad} = \sigma(4\pi R^2)T^4$.

To find the power the asteroid absorbs from the sun, we first realize that it absorbs only on the side facing the sun. In addition, only solar rays that are perpendicular to the asteroid's surface are absorbed. This means that the absorbing surface is equivalent to a disk of radius R having area πR^2. At a distance r from the sun, the intensity I_s of sunlight is $I_s = P_s/4\pi r^2$ (as we saw in Section 14.3 on wave intensity). The power absorbed by the asteroid is the intensity of sunlight times the absorbing area. Therefore $P_{abs} = I_s(\pi R^2)$, which we can write as $P_{abs} = (P_s/4\pi r^2)(\pi R^2) = P_sR^2/4r^2$.

For the asteroid to have a constant temperature, the absorbed power is equal to the radiated power. Therefore, equating our expressions for P_{abs} and P_{rad}, we have $P_sR^2/4r^2 = \sigma(4\pi R^2)T^4$. We want the distance of the asteroid from the sun, so we solve for r, giving $r = \sqrt{\dfrac{P_s}{16\pi\sigma T^4}}$. From the inside back over of the textbook, we see that the power radiated by the sun is $P_s = 3.85\times10^{26}$ W.

EVALUATE: (a) Putting in the numbers into $r = \sqrt{\dfrac{P_s}{16\pi\sigma T^4}}$ gives

$$r = \sqrt{\frac{3.85\times10^{26}\ \text{W}}{16\pi(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(184\ \text{K})^4}} = 3.43\times10^{11}\ \text{m} = 3.43\times10^{8}\ \text{km}.$$

(b) From Appendix E, we see that Mars is 2.28×10^8 km from the Sun and Jupiter is 7.79×10^8 km from the Sun. Therefore the asteroid is between the orbit of Mars and the orbit of Jupiter.

ASSESS: Our result is reasonable because the asteroid belt lies between Mars and Jupiter, so it is not surprising that this asteroid will be in that belt.

Chapter 17: Alternate Problem Set in *Mastering Physics*

82. **INTERPRET:** This problem involves the ideal-gas law.

DEVELOP: We use $pV = nRT$, with T in Kelvin units and $R = 8.314\ \text{J/K}\cdot\text{mol}$. We want to find the volume V.

EVALUATE: $T = -165°\text{C} = 108\ \text{K}$. Using $pV = nRT$ gives

$(877{,}000\ \text{Pa})V = (3.50\ \text{mol})(8.314\ \text{J/K}\cdot\text{mol})(108\ \text{K})$

$V = 3.58\times10^{-3}\ \text{m}^3 = 3.58\ \text{L}.$

ASSESS: A volume of 3.58 L is approximately one gallon.

83. **INTERPRET:** We have a change of phase from solid to liquid.

DEVELOP: The heat of transformation is $Q = Lm$. For melting, we use L_f, which for copper is $L_f = 205$ kJ/kg (from Table 17.1). We want the mass m of copper that melted.

EVALUATE: Using $Q = L_f m$ gives

$1.46\ \text{kJ} = (205\ \text{kJ/kg})m$

$m = 7.12\times10^{-3}\ \text{kg} = 7.12\ \text{g}.$

ASSESS: By using both Q and L_f in units having kJ, we avoided having to convert units.

84. **INTERPRET:** This problem involves thermal expansion.

DEVELOP: We treat this problem as linear expansion because the diameter expands, so we use $\Delta L = \alpha L\Delta T$. Table 17.2 gives $\alpha = 3.2\times10^{-6}\ \text{K}^{-1}$ for Pyrex glass. We know that $\Delta L = 1.000075\ \text{cm} - 1.000000\ \text{cm} = 7.5\times10^{-5}\ \text{cm}$. We want the final temperature T_f, so we write $\Delta T = T_f - 20°\text{C}$.

EVALUATE: Putting these quantities into $\Delta L = \alpha L\Delta T$ gives

$7.5\times10^{-5}\ \text{cm} = (3.2\times10^{-6}\ \text{K}^{-1})(1.000000\ \text{cm})\ \Delta T$

$\Delta T = 23°\text{C}$

$T_f = \Delta T + 20°\text{C} = 43°\text{C}.$

ASSESS: The percent change in the diameter is $[(0.000075\ \text{cm})/(1.000000\ \text{cm})]\times100 = 7.5\times10^{-3}\%$, which is very small. This is typical of thermal expansion in most cases.

85. **INTERPRET:** In this problem we need to use the ideal-gas law.

DEVELOP: Use $pV = nRT$, where $T = 20°\text{C} = 293\ \text{K}$. The volume V of a cylinder of radius r and height h is $V = \pi r^2 h$. In this case, $r = \frac{1}{2}(0.219\ \text{m})$.

EVALUATE: Using $pV = nRT$ with $V = \pi r^2 h$ gives

$p[\pi(\frac{1}{2})(0.219\ \text{m})^2(1.16\ \text{m})] = (278\ \text{mol})(8.314\ \text{J/K}\cdot\text{mol})(293\ \text{K})$

$p = 1.55\times10^{7}\ \text{Pa} = 15.5\times10^{6}\ \text{Pa} = 15.5\ \text{MPa}.$

ASSESS: Atmospheric pressure is about 10^5 Pa, so this gas is under a pressure 155 times as great, which is very high.

86. **INTERPRET:** We need to apply the ideal-gas law.

DEVELOP: Use $pV = nRT$ for the 3 situations described in the problem. For each situation, we summarize the information given. Call V the volume of the flask, which is the same in all 3 situations.

<u>Situation 1</u>: The flask is open to the room.

$p_1 = 101.3\ \text{kPa}$

$T_1 = 20°\text{C} = 293\ \text{K}$

$n = n_1$ (unknown)

V is unknown

Situation 2: The flask is open but is at 100°C.

$p_2 = 101.3$ kPa

$T_2 = 100°\text{C} = 373$ K

$n = n_2 = n_1 - 0.0268$ mol

Situation 3: The flask is closed.

p_3 is unknown

$T_3 = 20°\text{C} = 293$ K

$n = n_2$

Now apply $pV = nRT$ to each situation and solve for the unknowns.

EVALUATE: First find n_1 and n_2. For situation 1, we have $p_1V = n_1RT_1$, and for situation 2 we have $p_2V = n_2RT_2$. Taking the ratio of pV gives $\frac{p_2V}{p_1V} = \frac{n_2RT_2}{n_1RT_1} = \frac{n_2T_2}{n_1T_1}$. Since $p_1 = p_2$, the ratio is 1. Using $n_2 = n_1 - 0.0268$ mol gives $1 = \frac{(n_1 - 0.0268\text{ mol})(373\text{ K})}{n_1(293\text{ K})}$. Solving for n_1 gives $n_1 = 0.1250$ mol. Therefore $n_2 = 0.1250$ mol $- 0.0268$ mol $=$ 0.0982 mol.

(a) For situation 1, we have $p_1V = n_1RT_1$, so $V = n_1RT_1/p_1$. This gives

$V = (0.1250\text{ mol})(8.314\text{ J/K}\cdot\text{mol})(293\text{ K})/(1.013\times10^5\text{ Pa}) = 0.00300\text{ m}^3 = 3.00\text{ L}$.

(b) For situation 3, solve $pV = nRT$ for p_3, giving

$p_3 = n_2RT_3/V = (0.0982\text{ mol})(8.314\text{ J/K}\cdot\text{mol})(293\text{ K})/(0.00300\text{ m}^3)$

$p_3 = 7.96\times10^4\text{ Pa} = 79.6\text{ kPa}$.

ASSESS: We find that $n_2 < n_1$ because gas escaped the open flask as its temperature increased from 20°C to 100°C. Once the flask was closed (situation 3), no more gas could escape.

87. **INTERPRET:** This problem involves a phase change of water from liquid to gas.

DEVELOP: The minimum power would occur if it took the full half hour for the water to vaporize. We use $P = Q/t$ for the power and $Q = L_v m$ during the phase change. Since the phase changes from liquid to gas, we use L_v. Table 17.1 gives $L_v = 2257$ kJ/kg for water.

EVALUATE: $P = Q/t = mL_v/t = (0.96\text{ kg})(2257\text{ kJ/kg})/[(30)(60)\text{ s}] = 1.2\text{ kW}$.

ASSESS: 960 kg of water is around 2 lb. It is reasonable that we could vaporize that amount of water in a half hour, so our result for the power is reasonable.

88. **INTERPRET:** This problem involves a phase change of water from liquid to solid.

DEVELOP: The power is $P = Q/t$ and $Q = L_f m$. We used L_f because the phase change is from liquid to solid. For water, Table 17.1 gives $L_f = 334$ kJ/kg.

EVALUATE: Using $P = Q/t = L_f m/t$, we get

$P = (0.75\text{ kg})(334\text{ kJ/kg})/[(45)(60)\text{ s}] = 0.093\text{ kW} = 93\text{ W}$.

ASSESS: The 93 W is nearly the power consumed by an ordinary 100-W lightbulb.

89. **INTERPRET:** This problem deals with specific heat and a phase change of water from liquid to gas.

DEVELOP: The energy Q_h required to raise the temperature of the water to the boiling point of 100°C is given by $Q_h = mc\Delta T$. We use $Q_v = L_v m$ to calculate the energy Q_v to vaporize the water at its boiling point. Since heat is supplied at a steady rate, the percent of *time* to bring the water to the boiling point is the same as the percent of the *heat* supplied to do this. Thus the desired ratio r is

$r = \frac{Q_h}{Q_h + Q_v} = \frac{mc\Delta T}{mc\Delta T + L_v m} = \frac{c\Delta T}{c\Delta T + L_v}$. Table 16.1 gives $c = 4184\text{ J/kg}\cdot\text{K}$ for water and Table 17.1 gives $L_v = 2257$ kJ/kg for water.

EVALUATE: Putting the numbers into $r = \frac{c\Delta T}{c\Delta T + L_v}$ gives

$$r = \frac{(4184\text{ J/kg}\cdot\text{K})(100°\text{C} - 10°\text{C})}{(4184\text{ J/kg}\cdot\text{K})(100°\text{C} - 10°\text{C}) + 2{,}257{,}000\text{ J/kg}} = 0.143 = 14\%.$$

ASSESS: Most of the time is spent vaporizing the water because of the large heat of vaporization for water. It takes 2,257,000 J to vaporize a kilogram of water but only 4184 J to raise the temperature of that kilogram by 1°C.

90. **INTERPRET:** This problem deals with specific heat and a phase change from liquid to gas for water.

DEVELOP: The heat the reactor gives to the water is the heat to increase its temperature from 20°C to 100°C plus the heat to vaporize it. The heat Q_h to increase its temperature is $Q_h = mc\Delta T$. The heat Q_v to vaporize it is $Q_v = L_v m$. The total heat is $Q_{tot} = Q_h + Q_v$. Therefore the power P is

$P = Q_{tot}/t = (mc\Delta T + L_v m) = m(c\Delta T + L_v)/t.$

Table 16.1 gives $c = 4184\ \text{J/kg}\cdot\text{K}$ for water and Table 17.1 gives $L_v = 2257$ kJ/kg for water. The density of water is 1000 kg/m^3, so the mass of 420 m^3 of water is 420,000 kg. The temperature change is $\Delta T = 100°\text{C} - 20°\text{C} = 80°\text{C}$.

EVALUATE: Using $P = m(c\Delta T + L_v)/t$ gives

$P = (420{,}000\ \text{kg})[(4184\ \text{J/kg}\cdot\text{K})(80°\text{C}) + 2{,}257{,}000\ \text{J/kg}]/[(75)(60)\ \text{s}]$

$P = 2.4\times 10^8\ \text{W} = 240\ \text{MW}.$

ASSESS: This power is reasonable for a large power station.

91. **INTERPRET:** In this problem we deal with thermal expansion and the vibration of a simple pendulum. Changing the temperature of the pendulum causes the wire by which it swings to expand (or contract), which then affects the period of the pendulum.

DEVELOP: This pendulum clock is accurate at 21.0°C, but loses time at your home. By "losing time," we mean that the pendulum clock is running slow. For example, when the true time is 9:00 pm, the clock might read 8:50 pm. For it to lose time, the pendulum must make fewer swings than it should. Thus its period must be *longer* at your house than it was at 21.0°C. (We will us t for the period of a pendulum so we don't confuse it with T for temperature.) The period t of a pendulum is $t = 2\pi\sqrt{\frac{L}{g}}$. So since its period t has increased, its length L must have increased due to thermal expansion of the brass wire of the pendulum. Thus the temperature at your house must be *higher* than 21.0°C.

<u>At 21.0°C</u>: $t_0 = 2\pi\sqrt{\frac{L_0}{g}}$

<u>At your house at temperature T</u>: $t = 2\pi\sqrt{\frac{L}{g}}$

The temperature at your house is higher than 21.0°C, so $t = t_0 + \Delta T$.

The pendulum loses 1 min in 4 weeks, so the *fractional* loss r in the time for 4 weeks is

$r = \frac{1\ \text{min}}{4\ \text{weeks}} = \frac{1\ \text{min}}{4(7)(24)(60)\ \text{min}} = 2.4802\times 10^{-5}$. (We have used the fact that 1 week = 7 days, 1 day = 24 h, and 1 h = 60 min.) The fractional change in the period is the same as the fractional change in the pendulum over 4 weeks, so $r = \frac{\Delta t_0}{t_0}$, which we can express as $r = \frac{\Delta t_0}{t_0} = \frac{t - t_0}{t_0} = \frac{t}{t_0} - 1$. Using $t = 2\pi\sqrt{\frac{L}{g}}$ to express the periods in terms of the length of the pendulum gives

$$\frac{t}{t_0} - 1 = r$$

$$\frac{2\pi\sqrt{\frac{L}{g}}}{2\pi\sqrt{\frac{L_0}{g}}} - 1 = r.$$

L is the length of the pendulum at your house. Due to thermal expansion, it is longer than L_0 by an amount ΔL, where $\Delta L = \alpha L\Delta T$. Therefore its length at your house is $L = L_0 + \Delta L = L_0 + \alpha L\Delta T$. Putting this into the previous equation for r, canceling, and solving for ΔT gives

$$\sqrt{\frac{L_0 + \alpha L_0\Delta T}{L_0}} - 1 = r$$

$$\sqrt{1+\alpha\Delta T}-1=r$$
$$1+\alpha\Delta T=(r+1)^2$$
$$\Delta T=\frac{(1+r)^2-1}{\alpha}.$$

EVALUATE: Table 17.2 gives $\alpha = 19\times10^{-6}$ for brass. Thus $\Delta T=\dfrac{(1+r)^2-1}{\alpha}$ gives

$$\Delta T=\frac{(1+2.4802\times10^{-5})^2-1}{19\times10^{-6}\ \text{K}^{-1}}=2.61\ \text{K}=2.61°\text{C}.$$

The temperature at your house is 21.0°C + 2.61°C = 23.6°C.

ASSESS: If the pendulum clock were taken to a colder place, its length would decrease so it would swing with a shorter period, which would cause it to gain time. The fractional change in the period is very small because the fractional change in the length is very small, as is typical for thermal expansion.

Chapter 18: Alternate Problem Set in *Mastering Physics*

84. **INTERPRET:** This problem involves specific heat and the first law of thermodynamics.

DEVELOP: The first law of thermodynamics $\Delta E_{\text{int}} = Q + W$ applies here. In this case, Q = –2.7 kJ because this heat flows out of the water. We use $\Delta E_{\text{int}} = mc\Delta T$ to calculate the change in the internal energy of the water. From Table 16.1, we have $c = 4184\ \text{J/kg}\cdot\text{K}$.

EVALUATE: Combining our equations above gives $\Delta E_{\text{int}} = mc\Delta T = Q + W$.

$(0.550\ \text{g})(4184\ \text{J/kg}\cdot\text{K})(3.1°\text{C}) = -2700\ \text{J} + W$

$W = 9800\ \text{J} = 9.8\ \text{kJ}$.

ASSESS: W is positive, so work is done *on* the system.

85. **INTERPRET:** In this problem, we deal with isothermal expansion and the ideal-gas law.

DEVELOP: We use $pV = nRT$. Since T and n are constant, so we can express the ideal-gas law as $p_1V_1 = p_2V_2$. We know p_1 = 102.1 kPa and $V_2 = 2.50V_1$, and we want to find p_2. For an isotheral process, the work done on the gas is $W = -nRT\ln\left(\dfrac{V_2}{V_1}\right)$, and in this case T = 15.0°C = 288 K.

EVALUATE: **(a)** Using as $p_1V_1 = p_2V_2$ with $V_2 = 2.50V_1$, we have

$(102.1\ \text{kPa})V_1 = p_2(2.50V_1)$

$p_2 = 40.8$ kPa.

(b) Using $W = -nRT\ln\left(\dfrac{V_2}{V_1}\right)$, we get

$W = -(0.350\ \text{mol})(8.314\ \text{J/K}\cdot\text{mol})(288\ \text{K})\ \ln[(2.50V_1)/V_1] = -768\ \text{J}$.

This is the work done *on* the helium gas. The work done *by* the helium is +768 J.

ASSESS: The helium gas did positive work on the balloon because it increased the volume of the balloon.

86. **INTERPRET:** This problem deals with the first law of thermodynamics and the molar specific heat of an ideal gas.

DEVELOP: The first law of thermodynamics is $\Delta E_{\text{int}} = Q + W$. In this case, $Q = 0$ and W = +2.53 kJ. The internal energy change can be expressed as $\Delta E_{\text{int}} = nC_V\Delta T$, so $nC_V\Delta T = W$. We want to know whether this gas is monatomic or diatomic. For a monatomic gas, $C_V = \frac{3}{2}R = \frac{3}{2}(8.314\ \text{J/K}\cdot\text{mol}) = 12.5\ \text{J/K}\cdot\text{mol}$. For a diatomic gas, $C_V = \frac{5}{2}R = \frac{5}{2}(8.314\ \text{J/K}\cdot\text{mol}) = 20.8\ \text{J/K}\cdot\text{mol}$. We calculate C_V and compare it to the two values above to find what type of gas we have.

EVALUATE: Calculate C_V using $nC_V\Delta T = W$.

$(1.00\ \text{mol})C_V(122\ \text{K}) = 2530\ \text{J}$

$C_V = 20.7\ \text{J/K}\cdot\text{mol}$.

This value is much closer to $\frac{5}{2}R$ than to $\frac{3}{2}R$, so the gas is *diatomic*.

ASSESS: The gas is not perfectly ideal, because C_V is not exactly $\frac{5}{2}R$.

87. **INTERPRET:** In this case, we are dealing with the isothermal compression of an ideal gas.

DEVELOP: The work done on a gas during an isothermal process is given by $W = -nRT\ln\left(\dfrac{V_2}{V_1}\right)$. In this case, we know that W = 61.2 J, and we want to find n.

EVALUATE: Using $W = -nRT\ln\left(\frac{V_2}{V_1}\right)$ gives

61.2 J = $-n$(8.314 J/K·mol)(195 K) ln[(3.00 L)/(3.50 L)]

n = 0.245 mol.

ASSESS: The gas is compressed, so work is done on it, which makes W positive. There is no need to convert L to m^3 because the units cancel in the ration within the logarithm.

88. **INTERPRET:** We are dealing with the isothermal compression of an ideal gas.

DEVELOP: The work done on a gas during an isothermal process is given by $W = -nRT\ln\left(\frac{V_2}{V_1}\right)$. We know that 1.75 kJ of work on the gas will compress it so that $V_2 = \frac{1}{2}V_1$. We want the ratio of V_1/V_2 if we do 2.77 kJ of work on this gas. Using the first set of information, we find nRT. Then we use W = 2.77 kJ to get the desired volume ratio.

EVALUATE: Using $W = -nRT\ln\left(\frac{V_2}{V_1}\right)$ gives

$$1.75\text{ kJ} = nRT\ln\left(\frac{V_1/2}{V_1}\right) = +nRT\ln 2$$

$$nRT = \frac{1.75\text{ kJ}}{\ln 2}.$$

Now use this result and W = 2.77 kJ in the equation $W = -nRT\ln\left(\frac{V_2}{V_1}\right)$ and solve for V_1/V_2.

$$2.77\text{ kJ} = -\left(\frac{1.75\text{ kJ}}{\ln 2}\right)\ln\left(\frac{V_2}{V_1}\right)$$

$-1.0972 = \ln(V_2/V_1)$

$V_2/V_1 = e^{-1.0972}$

$V_1/V_2 = e^{+1.0972} = 3.00$

$V_1 = 3.00V_2$.

ASSESS: 2.77 kJ of work should compress the gas more than 1.75 kJ of work, which agrees with our result. With 1.75 kJ of work, we have

$\Delta V = V_2 - V_1 = \frac{1}{2}V_1 - V_1 = -\frac{1}{2}V_1$.

With 2.77 kJ of work, we have

$\Delta V = V_2 - V_1 = \frac{1}{3}V_1 - V_1 = -\frac{2}{3}V_1$.

89. **INTERPRET:** We are dealing with an adiabatic compression of a gas.

DEVELOP: For an adiabatic process, pV^γ = constant, which we can also express as $p_1V_1^\gamma = p_2V_2^\gamma$. In this case, the pressure triples, so $p_2 = 3p_1$, and γ = 1.53 for this gas. We want the factor by which the volume changes, so we want V_2/V_1.

EVALUATE: Using $p_1V_1^\gamma = p_2V_2^\gamma$ gives

$p_1V_1^\gamma = (3p_1)V_2^\gamma$

$$\left(\frac{V_2}{V_1}\right)^\gamma = \frac{1}{3}$$

$(V_2/V_1)^{1.53} = 1/3$

Using the y^x key on a calculator, we take the 1/1.53 power of both sides of the equation, which gives

$[(V_2/V_1)^{1.53}]^{1/1.53} = V_2/V_1 = (1/3)^{1/1.53} = 0.488$

$V_2 = 0.488V_1$.

Therefore the volume of the gas drops to 0.488 of the initial volume.

ASSESS: The pressure tripled, but the volume dropped approximately in half, not to 1/3 of its original value. So note that tripling the pressure does not reduce the volume to 1/3 of its value in an adiabatic process.

90. **INTERPRET:** A gas undergoes an adiabatic compression.

DEVELOP: For an adiabatic process, $TV^{\gamma-1}$ = constant, which we can also express as $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$. In this case, we know that T_1 = 30.0°C = 303 K, T_2 = 445°C = 718 K, and γ = 1.40 for the gas. We want the compression ratio, which is V_1/V_2.

EVALUATE: Using $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$ gives

$$(303\text{ K})V_1^{\gamma-1} = (718\text{ K})V_2^{\gamma-1}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \frac{718}{303}$$

$$\left(\frac{V_1}{V_2}\right)^{1.40-1} = \left(\frac{V_1}{V_2}\right)^{0.40} = \frac{718}{303}.$$

Using the y^x key on a calculator, we take the 1/0.40 power of both sides of the equation, which gives

$[(V_1/V_2)^{0.40}]^{1/0.40} = V_2/V_1 = (718/303)^{1/0.40} = 8.64$

Therefore $V_2/V_1 = 8.64$ is the compression ratio.

ASSESS: The temperature increased by a factor 718/303 = 2.37 as the volume decreased by a much greater factor of 8.64.

91. **INTERPRET:** In this problem, we are comparing adiabatic and isothermal expansions of a gas.

DEVELOP: We want to know if the gas is monatomic or diatomic. For a monatomic ideal gas $\gamma = \frac{5}{3}$, and for a diatomic gas $\gamma = \frac{7}{5}$. So we can find γ for the gas and use it to determine what type of gas it is. On a pV diagram for a gas, the slope of the curve is dp/dV.

Isothermal process: $pV = nRT$ = constant, so $p = nRT/V$. The slope is then

$dp/dV = -nRTV^{-2} = -(pV)V^{-2} = -p/V$.

Adiabatic process: pV^{γ} = constant, so $d(pV^{\gamma})/dV = 0$. Using the derivative of a product, we have

$$V^{\gamma}\frac{dp}{dV} + p\frac{dV^{\gamma}}{dV} = 0$$

$$V^{\gamma}\frac{dp}{dV} + p\gamma V^{\gamma-1} = 0$$

$$\frac{dp}{dV} = -\frac{p\gamma V^{\gamma-1}}{V^{\gamma}} = -p\gamma V^{-1} = -\frac{p}{V}\gamma.$$

EVALUATE: At the common point, we are given that the slope of the isothermal curve is 3/5 that of the adiabatic curve. Therefore at that point we know that

$$\left.\frac{dp}{dV}\right|_{\text{isothermal}} = \frac{3}{5}\left.\frac{dp}{dV}\right|_{\text{adiabatic}}.$$

Using the expressions for the slopes that we just found, we get

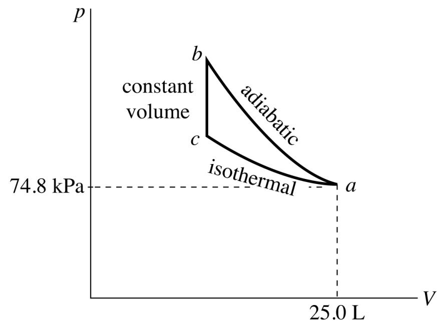
$$-\frac{p}{V} = \frac{3}{5}\left(-\frac{p}{V}\gamma\right)$$

$\gamma = \frac{5}{3}$, so the gas is *monatomic*.

ASSESS: The slopes of the two graphs must be compared at the point where the graphs intersect.

92. **INTERPRET:** This problem uses the ideal-gas law and involves adiabatic, isothermal, and constant-volume processes that make a complete cycle.

DEVELOP: Start by making a pV diagram showing this process (see the figure below). From this figure, we immediately see that the maximum pressure occurs at the end of the adiabatic expansion (point b on the figure). Therefore we want to find the pressure at b, p_b.



The total work done by the gas during this cycle is $W_{\text{tot}} = W_{ab} + W_{bc} + W_{ca}$. During segment bc no work is done because the volume remains constant. Segment ab is adiabatic, so that work is $W_{ab} = \dfrac{p_b V_b - p_a V_a}{\gamma - 1}$. Segment ca is isothermal, so that work is $W_{ca} = -nRT \ln\left(\dfrac{V_a}{V_c}\right)$. On ca we do not know the temperature T or n, but we do know that both are constant. So using the ideal-gas law, we have $p_a V_a = nRT$. We also know that $V_b = V_c$. Therefore we can express the work during ca as $W_{ca} = -p_a V_a \ln\left(\dfrac{V_a}{V_b}\right)$. The total work is then $W_{\text{tot}} = \dfrac{p_b V_b - p_a V_a}{\gamma - 1} - p_a V_a \ln\left(\dfrac{V_a}{V_b}\right)$.
Since segment ab is adiabatic, we can express V_b in terms of V_a using the fact that pV^γ is constant. Therefore $p_a V_a^\gamma = p_b V_b^\gamma$, so $V_b = V_a \left(\dfrac{p_a}{p_b}\right)^{1/\gamma}$. So the total work is

$$W_{\text{tot}} = \frac{p_b \left(\dfrac{p_a}{p_b}\right)^{1/\gamma} V_a - p_a V_a}{\gamma - 1} - p_a V_a \ln\left[\frac{V_a}{V_a \left(\dfrac{p_a}{p_b}\right)^{1/\gamma}}\right]$$

$$W_{\text{tot}} = \frac{p_b^{1-1/\gamma} p_a^{1/\gamma} V_a - p_a V_a}{\gamma - 1} - p_a V_a \ln\left[\left(\frac{p_b}{p_a}\right)^{1/\gamma}\right].$$

EVALUATE: We know that $W_{\text{tot}} = 365$ J, $\gamma = 1.67$, $p_a = 74.8$ kPa, and $V_a = 25.0\text{ L} = 0.0250\text{ m}^3$. Therefore we can solve the last equation for p_b. We cannot do this algebraically, but we can do it using numerical methods or mathematical software such as Mathematica. The result is $p_b = 242$ kPa.
ASSESS: To check, substitute $p_b = 242$ kPa into the final equation for W_{tot}. In addition, we found that $p_b > p_a$, which it should be.

93. **INTERPRET:** In this problem, a gas undergoes an isothermal compression followed by an adiabatic compression.
DEVELOP: For an isothermal process, T is constant. For an adiabatic process, $TV^{\gamma-1}$, which we can also express as $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$. Let A be the starting point, B the end of the isothermal compression, and C the end of the adiabatic compression. We know that $V_{\text{A}} = 17.5$ L, $V_{\text{B}} = \frac{1}{3}(17.5\text{ L})$, $T_{\text{A}} = T_{\text{B}} = 273$ K, $T_{\text{C}} = 435$ K, and $\gamma = 1.40$ for this gas. We want to find the final volume, which is V_{C}. Applying $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$ between B and C gives $T_{\text{B}} V_{\text{B}}^{\gamma-1} = T_{\text{C}} V_{\text{C}}^{\gamma-1}$. Solving for V_{C} gives $V_{\text{C}} = V_{\text{B}} \left(\dfrac{T_{\text{B}}}{T_{\text{C}}}\right)^{1/(\gamma-1)}$, so we can find V_{C}.

EVALUATE: Using $V_{\text{C}} = V_{\text{B}} \left(\dfrac{T_{\text{B}}}{T_{\text{C}}}\right)^{1/(\gamma-1)}$ with $V_{\text{B}} = \frac{1}{3}(17.5\text{ L})$, we have

$$V_{\text{C}} = \tfrac{1}{3}(17.5\text{ L})\left(\frac{273\text{ K}}{435\text{ K}}\right)^{1/(1.40-1)} = 1.82\text{ L}.$$

ASSESS: Our result gives $V_{\text{C}} < V_{\text{B}}$, which it should be since the gas is being compressed, so our result is reasonable.

Chapter 19: Alternate Problem Set in *Mastering Physics*

74. **INTERPRET:** This problem is about the efficiency of a Carnot heat engine.

DEVELOP: The Carnot efficiency is $e_{\text{Carnot}} = 1 - \dfrac{T_c}{T_h}$.

EVALUATE: Using $e_{\text{Carnot}} = 1 - \dfrac{T_c}{T_h}$, we have

$$e_{\text{Carnot}} = 1 - \frac{54.8\ \text{K}}{90.2\ \text{K}} = 0.392 = 39.2\%.$$

ASSESS: This engine converts 39.2% of the heat input into work.

75. **INTERPRET:** This problem deals with the coefficient of performance of a refrigerator and involves a phase change for water from liquid to solid.

DEVELOP: The coefficient of performance is $\text{COP} = \dfrac{Q_c}{W}$. W is the energy consumed to run the refrigerator and is 57.3 kJ in this case. Q_c is the heat removed at the cold reservoir. This is the heat to freeze the water at 0°C. For the phase change of the water, we have $Q_c = L_f m$. From Table 17.1, we know that L_f = 334 kJ/kg for water. We want to know the COP.

EVALUATE: $Q_c = L_f m$ = (334 kJ/kg)(0.674 kg) = 225.1 kJ.

COP = Q_c/W = (225.1 kJ)/(57.3 kJ) = 3.93.

ASSESS: We get a COP that is greater than 1, which is physically correct.

76. **INTERPRET:** This problem deals with the entropy change as ice melts, and it also involves a change of phase.

DEVELOP: During melting the temperature is constant, so the entropy change is $\Delta S = \dfrac{\Delta Q}{T}$. ΔQ is the heat given up by the iceberg as it melts, so $\Delta Q = L_f m$, which makes the entropy change $\Delta S = \dfrac{L_f m}{T}$. We know that T = 0°C = 273 K, and from Table 17.1 L_f = 334 kJ/kg.

EVALUATE: Using $\Delta S = \dfrac{L_f m}{T}$ and solving for m gives

$$m = \frac{T\Delta S}{L_f} = (273\ \text{K})(250 \times 10^9\ \text{J/K})/(334{,}000\ \text{J/kg}) = 2.0 \times 10^8\ \text{kg}.$$

ASSESS: As it melts, the iceberg gains entropy, but the ocean loses entropy. However the ocean loses less entropy than the iceberg gains because the temperature of the ocean is higher than that of the iceberg.

77. **INTERPRET:** We are working with a Carnot heat engine.

DEVELOP: The Carnot efficiency is $e_{\text{Carnot}} = 1 - \dfrac{T_c}{T_h}$. The efficiency of any engine is $e = \dfrac{W}{Q_h}$, where W is the work output of the engine and the power output is $P = W/t$. We know the heat input per cycle is Q_h = 895 J at 558 K. We want to find the power output, which is $P = W/t$.

EVALUATE: First use the temperature extremes to calculate the efficiency of the engine.

$$e_{\text{Carnot}} = 1 - \frac{T_c}{T_h} = 1 - \frac{288\ \text{K}}{558\ \text{K}} = 0.48387.$$

Now find the work output per cycle. An efficiency of 0.48387 means that 48.387% of the heat input at the hot reservoir is converted to work each cycle, so the work is

$W = (0.48387)Q_h = (0.48387)(895\text{ J}) = 433.06\text{ J}$.

At 221 cycles/second the power is

$P = W/t = (221)(433.06\text{ J})/(1.00\text{ s}) = 95{,}700\text{ J/s} = 95.7\text{ kW}$.

ASSESS: The waste heat per cycle is 895 J – 433 J = 462 J.

78. **INTERPRET:** This problem deals with efficiency of heat engines.

DEVELOP: The maximum possible efficiency is that of a Carnot engine $e_{\text{Carnot}} = 1 - \dfrac{T_c}{T_h}$.

EVALUATE: **(a)** $e_{\text{Carnot}} = 1 - \dfrac{T_c}{T_h} = 1 - \dfrac{318\text{ K}}{1750\text{ K}} = 0.818 = 81.8\%$.

(b) For the gas turbine stage, T_h = 1750 K and T_c is the intermediate temperature T_{int}. Thus we use $e_{\text{gas turbine}} = 1 - \dfrac{T_c}{T_h}$.

$$0.620 = 1 - \frac{T_{\text{int}}}{1750\text{ K}}$$

$T_{\text{int}} = 665$ K.

(c) The final stage is the steam stage, which operates between 318 K and 665 K, so

$$e_{\text{steam}} = 1 - \frac{318\text{ K}}{665\text{ K}} = 0.522 = 52.2\%.$$

ASSESS: The combined efficiency is greater than the efficiency of either of the stages.

79. **INTERPRET:** This problem is about a heat pump.

DEVELOP: For a heat pump, the coefficient of performance is COP = $\dfrac{Q_h}{W}$, and for this one its COP = 4.2. The rate at which heat is supplied to the building is Q_h/t = 23 kW. So we can express the COP as COP = $\dfrac{Q_h/t}{W/t}$, where W/t is the electric power consumption, which we want to find. Therefore we have $W/t = \dfrac{Q_h/t}{\text{COP}}$.

EVALUATE: $W/t = \dfrac{Q_h/t}{\text{COP}}$ = (23 kW)/4.2 = 5.5 kW.

ASSESS: This heat pump requires only 5.5 kW of electrical input to deliver 23 kW of heat to the building.

80. **INTERPRET:** In this problem, we deal with the entropy change due to the melting of ice and also the entropy change due to the temperature increase of the water from the melted ice.

DEVELOP: The pond gains entropy as the ice melts at 0°C and then gains more entropy as the melted water heats to its summer temperature. During melting, the temperature remains constant at 0°C = 273 K, so the entropy change is $\Delta S = \dfrac{\Delta Q}{T}$, where $\Delta Q = L_f m$. After the ice has all melted, the temperature of the water increase to its final temperature T_f. The entropy change during that part of the process is $\Delta S = \int \dfrac{dQ}{T}$, where $dQ = mcdT$.

Therefore $\Delta S = \int_{273\text{ K}}^{T_f} \dfrac{mcdT}{T} = mc\ln\left(\dfrac{T_f}{273\text{ K}}\right)$. The total entropy change of the ice and water during this entire process is $\Delta S_{\text{tot}} = \dfrac{L_f m}{273\text{ K}} + mc\ln\left(\dfrac{T_f}{273\text{ K}}\right)$. We know that m = 94 Mg = 94,000 kg, ΔS_{tot} = 150 MJ/K = 150×10^6 J/K, L_f = 334 kJ/kg (from Table 17.1), and $c = 4184\ \text{J/kg}\cdot\text{K}$ (from Table 16.1). We want to find T_f.

EVALUATE: Put the appropriate numbers into $\Delta S_{\text{tot}} = \dfrac{L_f m}{273\text{ K}} + mc\ln\left(\dfrac{T_f}{273\text{ K}}\right)$.

150×10^6 J/K = (334,000 J/kg)(94,000 kg)/(373 K)

+ (94,000 kg)(4184 J/kg·K) ln(T_f/273 K)

0.08898 = ln(T_f/273 K)

$T_f/273\text{ K} = e^{0.08898}$

$T_f = 298\text{ K} = 25°\text{C}$.

ASSESS: A pond temperature of 25°C is quite reasonable for summer. Therefore the entropy change of 150 MJ/K must also be reasonable.

81. **INTERPRET:** We are looking at entropy changes for three different processes.

DEVELOP: Use $\Delta S = \int \frac{dQ}{T}$, where $dQ = nCdT$. This gives $\Delta S = \int_{T_1}^{T_2} \frac{nCdT}{T} = nC\ln\left(\frac{T_2}{T_1}\right)$. The largest value of C will have the largest entropy change since the temperature extremes are the same in each case. At constant pressure, $C_P = \frac{7}{2}R$, and $C_V = \frac{5}{2}R$ for a diatomic gas. We want to identify the thermodynamic process in each case.

EVALUATE: **(a)** ΔS is smaller for this process than for the process in **(b)**, so this one may be carried out at constant volume. If so, ΔS should be

$\Delta S = nC_V \ln(T_2/T_1) = n(\frac{5}{2})(8.314\ \text{J/K}\cdot\text{mol}) \ln(520\text{ K}/310\text{ K}) = 54\text{ J/K}$.

Therefore this process is at constant volume.

(b) Now try $C_P = \frac{7}{2}R$. All the other numbers are the same as in part **(a)**, so our result is $\Delta S = 75\text{ J/K}$, which means that this process is at constant pressure.

(c) For an adiabatic process, $Q = 0$, so $\Delta S = 0$, which means that this process is adiabatic.

ASSESS: The entropy change is greater at constant pressure than at constant volume because the gas absorbs more heat than at constant volume to achieve the same temperature change.

82. **INTERPRET:** This problem involves entropy and specific heat.

DEVELOP: The aluminum pan loses heat as the water gains heat until both of them reach a final equilibrium temperature T_f. We first need to find that temperature in terms of the original temperature of the water, T_0, and then use it in the calculation of the total entropy change. It is T_0 that we ultimately want to find.

The quantity of heat lost by the aluminum pan is equal to the quantity of heat gained by the water. We use $Q = mc\Delta T$ for each one. The initial temperature of the aluminum pan is $155°\text{C} = 428\text{ K}$. Using $Q_{\text{water}} = Q_{\text{aluminum}}$, we have

$m_w c_w (T_f - T_0) = m_a c_a (428\text{ K} - T_f)$.

The total entropy change ΔS_{tot} is $\Delta S_{\text{tot}} = \Delta S_{\text{water}} + \Delta S_{\text{aluminum}}$. Since the temperature is changing during both of these entropy changes, we use $\Delta S = \int_{T_1}^{T_2} \frac{mcdT}{T} = mc\ln\left(\frac{T_2}{T_1}\right)$ for both of them. This gives

$$\Delta S_{\text{tot}} = m_w c_w \ln\left(\frac{T_f}{T_0}\right) + m_a c_a \ln\left(\frac{T_f}{428\text{ K}}\right).$$

EVALUATE: Now put in the appropriate numbers. First relate the equilibrium temperature T_f to the initial temperature of the water T_0.

$m_w c_w (T_f - T_0) = m_a c_a (428\text{ K} - T_f)$

$(3.58\text{ kg})(4184\ \text{J/kg}\cdot\text{K})(T_f - T_0) = (2.45\text{ kg})(900\ \text{J/kg}\cdot\text{K})(428\text{ K} - T_f)$

$T_f = 54.92\text{ K} + 0.87168T_0$

Now use this result for T_f in the entropy equation.

$$\Delta S_{\text{tot}} = m_w c_w \ln\left(\frac{T_f}{T_0}\right) + m_a c_a \ln\left(\frac{T_f}{428\text{ K}}\right)$$

$$162\text{ J/K} = (3.58\text{ kg})(4184\ \text{J/kg}\cdot\text{K})\ln\left(\frac{54.92\text{ K} + 0.87168T_0}{T_0}\right) + (2.45\text{ kg})(900\ \text{J/kg}\cdot\text{K})\ln\left(\frac{54.92\text{ K} + 0.87168T_0}{428\text{ K}}\right)$$

To solve this equation for T_0, it is necessary to use numerical methods. The result is

$T_0 = 289.4\text{ K} = 16.4°\text{C}$.

ASSESS: In our work above, we found that the final temperature of the water with the aluminum pan in it is given by $T_f = 54.92\text{ K} + 0.87168T_0$. Using our result for T_0, this gives us $T_f = 54.92\text{ K} + (0.87168)(289.4\text{ K}) = 307.2$

K = 34°C. This temperature is closer to that of the water than the aluminum since the specific heat of water is much larger than that of aluminum.

83. **INTERPRET:** This problem involves the efficiency of a heat engine and specific heat.
DEVELOP: The heat Q_{out} that is ejected by the power plant at the cold reservoir goes into the river and increases its temperature. Heat Q_{in} goes into the gas at the hot reservoir, so the efficiency of the power plant is $e = W/Q_{in} = 0.34$, and it produces 750 MW of power. This means that W/t = 750 MW. Thus we can write the efficiency as $e = \dfrac{W/t}{Q_{in}/t}$.

This gives $Q_{in}/t = \dfrac{W/t}{e}$. The ejected heat is $Q_{out} = Q_{in} - W$, so we have $Q_{out}/t = Q_{in}/t - W/t$, which gives $Q_{out}/t = \dfrac{W/t}{e} - W/t = (W/t)\left(\dfrac{1}{e} - 1\right)$. To find the temperature increase of the river, we use $Q = mc\Delta T$, where Q = Q_{out}. Dividing by t gives $Q_{out}/t = (m/t)c\Delta T$. Using our previous result, we have $(W/t)\left(\dfrac{1}{e} - 1\right) = (m/t)c\Delta T$.

We know that W/t = 750 MW = 750×10^6 W, c = 4184 J/kg·K, e = 0.34, and m/t = (110 m^3/s)(1000 kg/m^3) = 1.1×10^5 kg/s.
EVALUATE: Putting in the appropriate numbers and solving for ΔT gives

$$(W/t)\left(\frac{1}{e} - 1\right) = (m/t)c\Delta T$$

$(750\times10^6$ W)(1/0.34 – 1) = $(1.1\times10^5$ kg/s)(4184 J/kg·K) ΔT

ΔT = 3.2 K = 3.2°C.
ASSESS: A temperature increase of 3.2°C would be very harmful to wildlife in the river, such as fish and amphibians. A higher thermodynamic efficiency would help.

Chapter 20: Alternate Problem Set in *Mastering Physics*

86. **INTERPRET:** In this problem, we want to find out how many coulombs of charge are contained in 10^{20} electrons.

DEVELOP: Each electron has a charge of 1.6×10^{-19} C.

EVALUATE: 10^{20} electrons $\times$ 1.6×10^{-19} C per electron = 16 C.

ASSESS: 16 C may sound small, but it is a huge amount of charge compared to the charges involved in ordinary laboratory work.

87. **INTERPRET:** This problem involves the use of Coulomb's law for the force between two charged particles.

DEVELOP: Coulomb's law gives the magnitude of the force as $F_{12}=\dfrac{kq_1q_2}{r^2}$. We know the force and the charges, and we want to find the distance r between them. The proton and electron each have charge of magnitude $e = 1.60\times10^{-19}$ C.

EVALUATE: We use $F_{12}=\dfrac{kq_1q_2}{r^2}$ with $q_1 = q_2 = e = 1.60\times10^{-19}$ C.

$82.4\times10^{-9}\ \text{N} = (9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2/r^2$

$r = 5.29\times10^{-11}\ \text{m} = 52.9\times10^{-12}\ \text{m} = 52.9\ \text{pm}.$

ASSESS: A separation of 52.9 pm is the distance of the electron from the proton in the Bohr model of the hydrogen atom.

88. **INTERPRET:** This problem makes use of Coulomb's law for the force between two charged particles.

DEVELOP: The magnitude of the force between two very small charged objects (point charges) is $F_{12}=\dfrac{kq_1q_2}{r^2}$. We know that the force on the second object having charge q_2 is given by $\vec{F}=\left(-1.16\hat{i}-1.02\hat{j}\right)\text{nN}$. We also know that the magnitude of this charge is $F=\dfrac{kq_1q_2}{r^2}$. In terms of its components, the magnitude of this force is $F=\sqrt{F_x^2+F_y^2}$. In this case, $q_1 = e$, and we want to find q_2.

EVALUATE: Use $F=\sqrt{F_x^2+F_y^2}$ to find the magnitude of F from its components.

$F=\sqrt{F_x^2+F_y^2} = \sqrt{(-1.16\ \text{nN})^2+(-1.02\ \text{nN})^2} = 1.545\ \text{nN}.$

Now use Coulomb's law to find q_2. The square of the distance between the two charges is $r^2 = (0.410\times10^{-9}\ \text{m})^2 + (0.360\times10^{-9}\ \text{m})^2$, and we just found that the magnitude of the force is 1.545 nN = 1.545×10^{-9} N. Therefore

$$F=\frac{kq_1q_2}{r^2}$$

$$1.545\times10^{-9}\ \text{N} = \frac{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})q_2}{(0.410\times10^{-9}\ \text{m})^2+(0.360\times10^{-9}\ \text{m})^2}$$

$q_2 = 3.194\times10^{-19}$ C, which is extremely close to $2e$.

The direction of the force on q_2 is toward the proton (which is positive), so the proton is attracting q_2. Thus q_2 must be negative. So $q_2 = -2e$, which is choice **(d)**.

ASSESS: If q_2 were $+2e$, the force on it would be directed away from the proton, so the force on q_2 would be $\vec{F}=\left(1.16\hat{i}+1.02\hat{j}\right)\text{nN}$.

89. **INTERPRET:** This problem deals with the force that an electric field exerts on a charge.

DEVELOP: The magnitude of the force on a charge q in an electric field E is $F = qE$. For part **(a)**, we use this to find E. Then use it again in **(b)** to find the magnitude of the second charge.

EVALUATE: **(a)** Using $F = qE$ gives

155×10^{-3} N = $(68.0\times 10^{-9}$ C$)E$

$E = 2.28\times 10^{6}$ N/C = 2.28 MN/C.

(b) Using $F = qE$ again gives

96.2×10^{-3} N = $q(2.28\times 10^{6}$ N/C)

$q = 4.22\times 10^{-8}$ C = 42.2 nC.

ASSESS: The force is weaker on the second charge than on the first charge (96.2 mN compared to 155 mN), so the second charge must be smaller, as we found (42.2 nC compared to 68.0 nC).

90. **INTERPRET:** This problem deals with the electric field due to a long charged wire and the force this field exerts on a proton.

DEVELOP: The magnitude of the electric field E_w due to a very long wire is $E_w = \dfrac{2k\lambda}{y}$. The force on the proton due to this field is $F = qE = eE_w = e\left(\dfrac{2k\lambda}{y}\right) = \dfrac{2ek\lambda}{y}$. We want the magnitude of the force F.

EVALUATE: Using $F = \dfrac{2ek\lambda}{y}$ gives

$F = 2(1.60\times 10^{-19}\text{ C})(9.0\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(1950\text{ N/C})/(0.223\text{ m})$

$F = 2.52\times 10^{-5}$ N = 25.2×10^{-6} N = 25.2 μN.

ASSESS: We treated the wire as being infinite.

91. **INTERPRET:** This problem deals with the force on a proton in an electric field. We also make use of constant-acceleration kinematics formulas and Newton's second law of motion.

DEVELOP: The force on the proton is $F = qE = eE$ to the left, so it slows down. We know its initial speed and stopping distance, so we can find its acceleration using $v^2 = v_0^2 + 2a(x - x_0)$. Next we use Newton's second law to find the force on the proton, and finally we use $F = eE$ to find the electric field E.

EVALUATE: First find the acceleration of the proton. Calling to the right positive, we have

$v^2 = v_0^2 + 2a(x - x_0)$

$(0\text{ m/s})^2 = (386{,}000\text{ m/s})^2 + 2a(0.0147\text{ m})$

$a = -5.068\times 10^{12}$ m/s^2 to the left.

From Newton's second law we know $F = ma$ and we also know that $F = eE$. Combining these gives

$eE = ma$

$(1.60\times 10^{-19}\text{ C})E = (1.67\times 10^{-27}\text{ kg})(5.068\times 10^{12}\text{ m/s}^2)$

E = 52,900 N/C = 52.9 kN/C.

ASSESS: The enormous acceleration of the proton is much, much greater than we encounter with ordinary matter, but it is not unusual when dealing with subatomic particles such as protons and electrons.

92. **INTERPRET:** This problem makes use of Coulomb's law. In this case, we add the forces due to two charges.

DEVELOP: We apply $F_{12} = \dfrac{kq_1q_2}{r^2}$ to find the force that each charge exerts on the third charge q_3. This charge is at $x = \left(3+\sqrt{6}\right)a$. For q_3 to experience zero electric force, q and $3Q$ must have opposite signs so their forces on q_3 will be in opposite directions and cancel. The magnitudes of the two forces on q_3 must be equal, so $F_{\text{due to }3Q} = F_{\text{due to }q}$. The distance between q and q_3 is $\left(3+\sqrt{6}\right)a - a = \left(2+\sqrt{6}\right)a$. Applying Coulomb's law gives

$$\frac{k(3Q)q_3}{\left[\left(3+\sqrt{6}\right)a\right]^2} = \frac{kqq_3}{\left[\left(2+\sqrt{6}\right)a\right]^2}.$$ Solve for q in terms of Q.

EVALUATE: Canceling out all common factors in our previous equation gives
$\dfrac{3Q}{\left(3+\sqrt{6}\right)^2}=\dfrac{q}{\left(2+\sqrt{6}\right)^2}$. Now solve for q, which includes rationalizing the denominator.

$$q = 3Q\left(\frac{2+\sqrt{6}}{3+\sqrt{6}}\right)^2 = 3Q\left(2+\sqrt{6}\right)^2\left[\frac{3-\sqrt{6}}{\left(3+\sqrt{6}\right)\left(3-\sqrt{6}\right)}\right]^2 = 3Q\frac{\left(2+\sqrt{6}\right)^2\left(3-\sqrt{6}\right)^2}{9} = 2Q$$

Since q and $3Q$ have opposite signs, the final answer is $q = -2Q$.

ASSESS: The magnitude of q is $2Q$, which is less than the magnitude of $3Q$. This is reasonable because, for the two forces on q_3 to cancel, the charge q that is closer to q_3 must have a smaller magnitude than the charge $3Q$ that is farther away. Note also that the answer does not depend on q_3, and it does not depend on whether q_3 is positive or negative.

93. **INTERPRET:** This problem is about the electric field of an electric dipole.
DEVELOP: The electric field due to a dipole depends on the *cube* of the distance from the dipole, which we can express as $E \propto \dfrac{1}{r^3}$, which we can write as $E = \dfrac{K}{r^3}$, where K is a constant. In this case, we know the electric field is 288 N/C at 6.36 cm from the dipole, and we want to know what it would be at a distance of 8.49 cm.

EVALUATE: **(a)** At 6.36 cm, $E_{6.36} = \dfrac{K}{(6.36\text{ cm})^3}$, and at 8.49 cm, $E_{8.49} = \dfrac{K}{(8.49\text{ cm})^3}$. Taking the ratio of the two fields gives

$$\frac{E_{8.49}}{E_{6.36}} = \frac{\dfrac{K}{(8.49\text{ cm})^3}}{\dfrac{K}{(6.36\text{ cm})^3}} = \left(\frac{6.36}{8.49}\right)^3 = 0.420.$$

$E_{8.49}$ = (0.420)(288 N/C) = 121 N/C.

(b) We do not know the direction we moved relative to the dipole, so we cannot find the dipole moment.

(c) If, at 6.36 cm, we are along the perpendicular bisector of the dipole, then we know that $E = kp/y^3$. In this case the dipole moment p is $p = y^3E/k$. So

$p = (0.0636\text{ m})^3(288\text{ N/C})/(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2) = 8.23\times10^{-12}\text{ C}\cdot\text{m} = 8.23\text{ pC}\cdot\text{m}$.

If, at 6.36 m, we are along a line along the axis of the dipole, $E = 2kp/x^3$. In this case, p would be $p = x^3E/2k$, which is half the value we just found, or $4.12\text{ pC}\cdot\text{m}$. So we only know that p is between $4.12\text{ pC}\cdot\text{m}$ and $8.23\text{ pC}\cdot\text{m}$.

ASSESS: Even though we don't know the exact value of p, we do know its value within a fairly narrow range.

94. **INTERPRET:** In this problem, we deal with the electric field due to a long charged wire, the force on a charge in orbit in that field, and Newton's second law for circular motion.
DEVELOP: The charged particle makes a circular orbit because it is attracted to the charged wire. Newton's second law tells us that $F_{net} = ma$, and for circular motion the acceleration is $a = v^2/r$. Therefore $F_{net} = mv^2/r$. The net force is due to the electric attraction of the wire, so $F_{net} = qE$. For a long charged wire, $E = 2k\lambda/r$. Equating our two expressions for F_{net} and using the formula for the electric field gives $q\left(\dfrac{2k\lambda}{r}\right)=\dfrac{mv^2}{r}$.

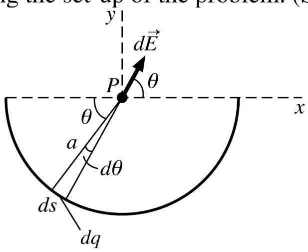
EVALUATE: Solving our previous equation for v, we get $v = \sqrt{\dfrac{2qk\lambda}{m}}$. Putting in the given numbers gives

$$v = \sqrt{\frac{2\left(2.15\times10^{-9}\text{ C}\right)\left(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(14.8\times10^{-6}\text{ C/m}\right)}{6.84\times10^{-9}\text{ kg}}} = 289\text{ m/s}.$$

ASSESS: It may be surprising that the speed does not depend on the distance from the wire. This is true because both the force on the charge and its acceleration are proportional to $1/r$, so r cancels from the equation in Newton's second law.

95. **INTERPRET:** In this problem, the charge density on a semicircular loop varies along the loop. To find the magnitude of the electric field at the center of the semicircle, we will need to integrate.

DEVELOP: First draw a figure showing the set-up of the problem. (See the figure below.)



In the figure, P is the center of the semicircle, $d\vec{E}$ is the electric field due to an infinitesimal amount of charge dq (assumed positive) on a tiny segment of loop of length ds that subtends an angle $d\theta$ at P. If $d\theta$ is in radians, then $ds = a\,d\theta$. The charge dq on ds is $dq = \lambda ds = \lambda a d\theta = \lambda_0 \sin\theta\, a d\theta$. To find the magnitude of $d\vec{E}$, we treat dq as a point-charge. Therefore $dE = \dfrac{kdq}{a^2} = \dfrac{k\lambda_0 \sin\theta\, a d\theta}{a^2} = \dfrac{k\lambda_0 \sin\theta d\theta}{a}$. We shall integrate the x- and y-components separately. The x-component of $d\vec{E}$ is $dE_x = \sin\theta dE$, and the y-component is $dE_y = \cos\theta dE$. For part **(a)**, we integrate dE_x and dE_y to find E_x and E_y at P. For **(b)** we use $Q = \int dq = \int \lambda ds = \int_0^\pi \lambda_0 \sin\theta\, a d\theta$ to find the total charge on the loop.

EVALUATE: **(a)** First find E_x at P.

$$E_x = \int dE_x = \int \cos\theta dE = \int_0^\pi \cos\theta \frac{k\lambda_0}{a} \sin\theta d\theta = \frac{k\lambda_0}{a}\int_0^\pi \sin\theta\cos\theta d\theta = \frac{k\lambda_0}{a}\frac{\sin^2\theta}{2}\bigg|_0^\pi = 0.$$

Now find E_y at P using the same procedure as for E_x.

$$E_y = \int dE_y = \int \sin\theta dE = \int_0^\pi \sin\theta \frac{k\lambda_0}{a}\sin\theta d\theta = \frac{k\lambda_0}{a}\int_0^\pi \sin^2\theta d\theta = \frac{k\lambda_0}{a}\left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^\pi \quad E_y = \frac{k\pi\lambda_0}{2a}.$$

Since $E_x = 0$ at P, the magnitude of E is just E_y, so $E = \dfrac{k\pi\lambda_0}{2a}$.

(b) We integrate the charge density to get

$$\int dq = \int \lambda ds = \int_0^\pi \lambda_0 \sin\theta\, a d\theta = \lambda_0 a(-\cos\theta)\Big|_0^\pi = 2a\lambda_0.$$

ASSESS: It is reasonable that $E_x = 0$ because the charge density $\lambda = \lambda_0 \sin\theta$ is symmetric about $\theta = \pi/2$. Therefore there is just as much charge on the right side of the loop as on the left side. Therefore the x-components of the electric field from the right-hand charge cancel the x-components of the field from left-hand charge. The y-components, however, all point upward in Figure 20-10, so they add instead of canceling.

Chapter 21: Alternate Problem Set in *Mastering Physics*

80. **INTERPRET:** This problem involves the calculation of electric flux through a surface.

DEVELOP: For a uniform electric field, the flux through a surface is $\Phi = \vec{E} \cdot \vec{A}$, which can be written as $\Phi = EA\cos\theta$. In this problem we know A and E, and we want to find the angle θ for several values of the flux.

EVALUATE: **(a)** The flux is zero, but E and A are not zero, so it must be true that $\cos\theta = 0$, which means that $\theta = 90°$.

(b) Using $\Phi = EA\cos\theta$ gives

$1610\ \text{N}\cdot\text{m}^2/\text{C} = (788\ \text{N/C})(2.14\ \text{m}^2)\cos\theta$

$\cos\theta = 0.9547$

$\theta = 17.3°$.

(c) Using the same procedure as in **(b)** gives

$1430\ \text{N}\cdot\text{m}^2/\text{C} = (788\ \text{N/C})(2.14\ \text{m}^2)\cos\theta$

$\cos\theta = 0.8480$

$\theta = 32.0°$.

ASSESS: As θ approaches 0°, the flux gets larger because the electric gets closer to being perpendicular to the surface.

81. **INTERPRET:** This problem involves Gauss's law.

DEVELOP: We know the flux through a closed surface, so use Gauss's law in the form $\Phi = \dfrac{q_{\text{enclosed}}}{\varepsilon_0}$ to get the enclosed charge. Now convert this charge to the number of electrons, using the fact that the magnitude of the charge on each electron is $e = 1.60\times 10^{-19}$ C.

EVALUATE: Solving Gauss's law for the enclosed charge gives

$q_{\text{enclosed}} = \varepsilon_0\Phi = (8.85\times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(-17{,}000\ \text{N}\cdot\text{m}^2/\text{C}) = -1.505\times 10^{-7}$ C.

Let N be the number of electrons. Therefore $Ne = |q_{\text{enclosed}}|$, so

$N = |q_{\text{enclosed}}|/e = (1.505\times 10^{-7}\ \text{C})/(1.60\times 10^{-19}\ \text{C}) = 9.4\times 10^{11}$ electrons.

ASSESS: The sock contains many more than 9.4×10^{11} electrons, but all the others are balanced by protons so that flux is zero.

82. **INTERPRET:** This problem deals with the electric field due to a uniformly charged sphere.

DEVELOP: Outside the sphere the electric field is the as for a point charge at its center, so $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$. Let R be the radius of the sphere and Q the charge on it. At the surface of the sphere, $E_{\text{surface}} = \dfrac{Q}{4\pi\varepsilon_0 R^2}$. At 10 cm above the surface, the field is $E_{\text{above}} = \dfrac{Q}{4\pi\varepsilon_0 (R + 0.10\ \text{m})^2}$. Using these two equations, we can solve for Q and R. We know that Q is positive because the field points away from the surface of the sphere.

EVALUATE: Putting the numbers into the two equations for the electric field gives

At the surface: $95.4\ \text{kN/C} = \dfrac{Q}{4\pi\varepsilon_0 R^2}$

At 10 cm above the surface: $14.7\ \text{kN/C} = \dfrac{Q}{4\pi\varepsilon_0 (R + 0.10\ \text{m})^2}$

Take the ratio of the fields: $\dfrac{95.4 \text{ kN/C}}{14.7 \text{ kN/C}} = \dfrac{\dfrac{Q}{4\pi\varepsilon_0 R^2}}{\dfrac{Q}{4\pi\varepsilon_0 (R+0.10 \text{ m})^2}}$

Dividing out common factors gives

$$6.490 = \left(\frac{R+0.10 \text{ m}}{R}\right)^2$$

$$\frac{R+0.10 \text{ m}}{R} = \pm\sqrt{6.490} = \pm 2.548$$

R + 0.10 m = ±2.548R

$$R = \frac{-0.10 \text{ m}}{1 \pm 2.548}.$$

Since R must be positive, we use the negative sign in the denominator. This gives R = 0.06462 m, which rounds to R = 0.0646 m = 6.46 cm.

Now find the charge using the electric field at the surface of the sphere.

$$14{,}700 \text{ N/C} = \frac{Q}{4\pi\varepsilon_0 R^2} = \frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)Q}{(0.06462 \text{ m})^2}$$

$Q = +4.43\times10^{-8}$ C = +44.3 nC.

Therefore our final answers are

(a) $Q = +4.43\times10^{-8}$ C = +44.3 nC

(b) R = 0.0646 m = 6.46 cm.

ASSESS: Note that the 10 cm given in the problem is the distance from the *surface* of the sphere, not from its center.

83. **INTERPRET:** This problem involves the electric field due to a thin charged wire.

DEVELOP: The wire is only 25.0 cm long, so at a distance of 28.7 m it appears essentially like a point-charge, so its electric field is $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$. We can use this equation to find q. If we are close to the wire but not near its ends, we can model it as an infinite line of charge, so its electric field is $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The charge density is $\lambda = q/L$. Use the q we just found to get λ and then use λ to find the field at 4.5 mm from the axis of the wire.

EVALUATE: **(a)** Using $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ gives

$$34.6 \text{ N/C} = \frac{(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)q}{(28.7 \text{ m})^2}$$

$q = 3.17\times10^{-6}$ C = 3.17 μC.

(b) At a distance of 4.5 mm, the field is given by $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$. The charge density is

$\lambda = q/L = (3.17\times10^{-6} \text{ C})/(0.250 \text{ m}) = 1.268\times10^{-5}$ C/m. This gives

$$E = \frac{2(9.0\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.268\times10^{-5} \text{ C/m})}{0.0045 \text{ m}} = 5.07\times10^7 \text{ N/C} = 50.7 \text{ MN/C}.$$

ASSESS: At a distance of 28.7 m, the formula $E = \dfrac{\lambda}{2\pi\varepsilon_0 r}$ would be completely inappropriate to calculate the strength of the electric field because it only applies for infinite (or at least very long) lines of charge.

84. **INTERPRET:** In this problem we use the electric field due to a charged metal plate.

DEVELOP: Metal is an electric conductor, so the field near the plate is $E = \sigma/\varepsilon_0$. Since $\sigma = Q/A$, the charge on the plate is $Q = \sigma A$. The area of a square having sides of length L is $A = L^2$, so the charge is $Q = \sigma L^2$. But $\sigma = \varepsilon_0 E$, so $Q = \varepsilon_0 E L^2$.

EVALUATE: Using $Q = \varepsilon_0 EL^2$ gives

$Q = (8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(1.68\times10^6\ \text{N/C})(0.750\ \text{m})^2 = 8.36\times10^{-6}\ \text{C} = 8.36\ \mu\text{C}.$

ASSESS: Half this charge is on each side of the plate.

85. **INTERPRET:** In this problem we use the electric field due to a uniform sphere of charge.

DEVELOP: For points outside the balloon, $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$. We know the field the balloon's surface and at 58.7 cm from its center. We want to find the charge q on the balloon and the radius R of the balloon.

EVALUATE: **(a)** We know the field at 58.7 cm from the center, so use $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$.

$$3760\ \text{N/C} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q}{(0.587\ \text{m})^2}$$

$Q = 1.44\times10^{-7}\ \text{C} = 144\ \text{nC}.$

(b) At the surface of the balloon, use $E = \dfrac{q}{4\pi\varepsilon_0 R^2}$, which gives

$$26{,}800\ \text{N/C} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.44\times10^{-7}\ \text{C})}{R^2}$$

$R = 0.220\ \text{m} = 22.0\ \text{cm}.$

ASSESS: For all points at or beyond the balloon's surface, the field is equivalent to a point-charge of 144 nC at the balloon's center. But this is not true for points inside the balloon. The balloon behaves like a spherical shell of charge, so the field inside of it is zero.

86. **INTERPRET:** In this problem, we deal with the electric field produced by a point-charge and by a uniform spherical shell. The point-charge is at the center of the shell.

DEVELOP: A uniform spherical shell of charge produces no electric field inside of the shell. Therefore any field within the shell must be due to the point-charge at its center. The field outside the shell is due to the shell and the point-charge. The field due to a point-charge is $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$, and this formula also applies for the shell for points outside the shell. Call Q the point-charge at the center of the shell and q the charge on the shell.

EVALUATE: **(a)** At 5.40 cm from the shell's center, the electric field is due only to the point-charge Q at the center, so $E = \dfrac{Q}{4\pi\varepsilon_0 r^2}$. Putting in the numbers gives

$$12.2\times10^6\ \text{N/C} = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)Q}{(0.0540\ \text{m})^2}$$

$Q = 3.95\times10^{-6}\ \text{C} = 3.95\ \mu\text{C}.$

This is only the *magnitude* of Q. The electric field at 5.40 cm is directed inward, so Q must be negative. Therefore $Q = -3.95\times10^{-6}\ \text{C} = -3.95\ \mu\text{C}.$

(b) Outside the shell the total electric field is due to the point-charge Q and the charge q on the shell. The shell is positive and the point-charge is negative, so the fields are in opposite directions. The charge on the shell is greater in magnitude than the point-charge, so the net field will point outward from the center. Its magnitude is $E = E_{\text{shell}} - E_{\text{pt-chge}} = E_q - E_Q$, which is

$$E = \frac{q}{4\pi\varepsilon_0 r^2} - \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{(q-Q)}{4\pi\varepsilon_0 r^2}$$

$$E = \frac{(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(87.2\times10^{-6}\ \text{C} - 3.95\times10^{-6}\ \text{C})}{(0.0500\ \text{m})^2} = 3.00\times10^6\ \text{N/C} = 3.00\ \text{MN/C}.$$

The direction is away from the center of the shell.

ASSESS: If the point-charge and sphere had the same sign charge, the field outside would have been the sum of their magnitudes.

87. **INTERPRET:** This problem involves the force on a charged particle in an electric field, the electric field due to a nonconducting sheet of charge, and static equilibrium.

DEVELOP: The bead is in equilibrium, so the forces on it should balance. The downward force of gravity must be balanced by an upward electric force due to the charged floor. Since the bead is negative, the floor must also be negative to exert an upward force on the bead. The field due to the nonconducting floor is $E = \sigma/2\varepsilon_0$, and the force on the bead is $qE = q\sigma/2\varepsilon_0$, where q is the charge on the bead. For equilibrium, $F_{\text{electric}} = F_{\text{gravity}}$, which tells us that $q\sigma/2\varepsilon_0 = mg$. We want the surface charge density on the floor, so we solve for σ.

EVALUATE: Solving $q\sigma/2\varepsilon_0 = mg$ for σ gives $\sigma = \dfrac{2mg\varepsilon_0}{q}$. Putting in the numbers, we get

$$\sigma = \frac{2(11.2\times10^{-6}\ \text{kg})(9.81\ \text{m/s}^2)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}{0.326\times10^{-6}\ \text{C}}$$

$\sigma = 5.97\times10^{-9}\ \text{C/m}^2 = 5.97\ \text{nC/m}^2$. But this gives only the magnitude of σ. We have seen that the floor is negatively charged, so

$\sigma = -5.97\times10^{-9}\ \text{C/m}^2 = -5.97\ \text{nC/m}^2$.

ASSESS: Notice that we used $q = 0.326\ \mu\text{C}$, and not $q = -0.326\ \mu\text{C}$, in the calculation for σ. This was because the formula $E = \sigma/2\varepsilon_0$ gives only the magnitude of E, so it cannot be negative. Notice also that we used $E = \sigma/2\varepsilon_0$ and not $E = \sigma/\varepsilon_0$ for the electric field due to the floor. But $E = \sigma/\varepsilon_0$ is for a *conducting* sheet, and the floor is nonconducting.

88. **INTERPRET:** In this problem we use the electric field due to a spherical shell and due to a point-charge.

DEVELOP: A uniform spherical shell of charge produces no electric field inside of the shell. Therefore any field within the shell must be due to the point-charge at the center. The field at the surface of the shell is due to the shell and the point-charge. The electric field due to a point-charge is $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$, and this formula also applies for the shell for points at the surface of the shell.

EVALUATE: Call the point-charge q. To find its magnitude, use $E = \dfrac{q}{4\pi\varepsilon_0 r^2}$ with the given value of E at $r = \frac{1}{2}R$.

$$\frac{2kQ}{R^2} = \frac{kq}{\left(\frac{1}{2}R\right)^2}$$

$q = \frac{1}{2}Q$.

This is just the magnitude of q. We know that the field at $r = \frac{1}{2}R$ points inward, so q must be negative. Therefore $q = -\frac{1}{2}Q$.

The field at the surface of the sphere is due to the point-charge q and the shell having positive charge Q. These fields point in opposite directions, and the field due to Q has a greater magnitude than the field due to q. Therefore the magnitude of the field is

$$E = \frac{kQ}{R^2} - \frac{kq}{R^2} = \frac{k(Q-q)}{R^2} = \frac{k\left(Q-\frac{1}{2}Q\right)}{R^2} = \frac{kQ}{2R^2}.$$

ASSESS: Notice that in the last equations we did not use $q = -\frac{1}{2}Q$. This was because we had already made use of the fact that q and Q have opposite signs by subtracting the magnitudes of the fields. If we had been careless, we might have said that $Q - q = Q - (-\frac{1}{2}Q) = \frac{3}{2}Q$, which would have been *wrong*!

89. **INTERPRET:** This problem deals with the electric field due to a sphere of charge.

DEVELOP: If the electric field outside the sphere is zero, the total charge q inside the sphere must be zero. The volume charge density inside the sphere depends on r, so the total charge inside the sphere is $q = \int \rho dV$. In this case, dV is a thin spherical shell of radius r and thickness dr. The area of this shell is $4\pi r^2$, so its volume is $dV = 4\pi r^2 dr$, and we are given that $\rho = \rho_0 - br^3$. The integration then becomes

$q = \int_0^R \rho 4\pi r^2 dr$, and this integral must equal zero.

EVALUATE: Using $\rho = \rho_0 - br^3$, the integral becomes

$$q = \int_0^R \rho 4\pi r^2 dr = \int_0^R \left(\rho_0 - br^3\right) 4\pi r^2 dr = 4\pi \int_0^R \left(\rho_0 r^2 - br^5\right) dr = \frac{4\pi R^3}{3}\left(\rho_0 - \frac{bR^3}{2}\right).$$

This charge must be zero, so $\rho_0 - \frac{bR^3}{2} = 0$, which gives $b = \frac{2\rho_0}{R^3}$.

ASSESS: Check units. Since $\rho = \rho_0 - br^3$, br^3 must have units of charge density, C/m^3, so b must have units of C/m^6. Our result has units of $(C/m^3)/m^3 = C/m^6$, which are the correct units.

Chapter 22: Alternate Problem Set in *Mastering Physics*

86. **INTERPRET:** In this problem we must relate the work done on a charge to the potential difference through which it moved.

DEVELOP: The work produces a change in potential energy ΔU_{AB}. The potential difference due to this potential energy difference is $\Delta V_{AB} = \dfrac{\Delta U_{AB}}{q}$. We know the work and the potential difference, so we can solve for the charge q of the particle.

EVALUATE: Solving for q gives

$$q = \frac{\Delta U_{AB}}{\Delta V_{AB}} = \frac{45.2\ \text{J}}{12.0\ \text{V}} = 3.77\ \text{C}.$$

ASSESS: Check the units of our answer: $\dfrac{\text{J}}{\text{V}} = \dfrac{\text{J}}{\text{J/C}} = \text{C}$, so the units are correct.

87. **INTERPRET:** For this problem, we relate the kinetic energy gained by particles to the potential through which they are accelerated.

DEVELOP: Use $\Delta V_{AB} = \dfrac{\Delta U_{AB}}{q}$. In this case, $\Delta U_{AB} = \Delta K_{AB}$, so $\Delta V_{AB} = \dfrac{\Delta K_{AB}}{q}$. Solving for q gives $q = \dfrac{\Delta K_{AB}}{\Delta V_{AB}}$.

We want the charge of each of the three particles.

EVALUATE: <u>Particle A</u>: $q_\text{A} = \dfrac{16\ \text{aJ}}{100\ \text{V}} = \dfrac{16\times 10^{-18}\ \text{J}}{100\ \text{V}} = 16\times 10^{-20}\ \text{C}.$

Using $e = 1.60\times 10^{-19}$ C, we convert the charge to units of e.

$q_\text{A} = (16\times 10^{-20}\ \text{C})(e/1.60\times 10^{-19}\ \text{C}) = e.$

<u>Particle B</u>: We use the same procedure as for particle A.

$$q_\text{B} = \frac{32\ \text{aJ}}{100\ \text{V}} = 2q_\text{A} = 2e.$$

<u>Particle C</u>: $q_\text{C} = \dfrac{144\ \text{aJ}}{100\ \text{V}} = 9q_\text{A} = 9e.$

ASSESS: The energy gain is proportional to the charge, so a particle with a large charge gains more kinetic energy than one with a smaller charge.

88. **INTERPRET:** We need to find the potential difference using the electric field.

DEVELOP: We just want the magnitude of the potential difference, so we use $\Delta V_{AB} = \int_A^B \vec{E}\cdot d\vec{r}$. In this case, A is the origin and B is the point at which $y = 3L$. We integrate along the y-axis, so $d\vec{r} = dy\hat{j}$. We are given that

$$\vec{E} = \frac{E_0 y}{L}\hat{j}.$$

EVALUATE: Carry out the integration for the quantities indicated above.

$$\Delta V_{AB} = \int_A^B \vec{E}\cdot d\vec{r} = \int_0^{3L} \frac{E_0 y}{L}\hat{j}\cdot dy\hat{j} = \frac{E_0}{L}\frac{y^2}{2}\bigg|_0^{3L} = \tfrac{9}{2}E_0 L.$$

ASSESS: Check units. The SI units of our answer are $\dfrac{\text{N}}{\text{C}}\cdot\text{m} = \dfrac{\text{N}\cdot\text{m}}{\text{C}} = \dfrac{\text{J}}{\text{C}} = \text{V}$, so the units check.

89. **INTERPRET:** We want to relate the spacing of equipotentials to the strength of the electric field.

DEVELOP: For a uniform electric field, $E_x = \dfrac{\Delta V}{\Delta x}$. In this case, Δx = 2.54 cm between equipotentials, and we want to know the potential difference ΔV between them.

EVALUATE: Solving $\dfrac{\Delta V}{\Delta x}$ for ΔV gives

$\Delta V = E_x \Delta x = (44.0\text{ N/C})(0.0254\text{ m}) = 1.12\text{ V}$.

ASSESS: If the field were stronger, the spacing would be smaller for the same potential difference.

90. **INTERPRET:** We know the potential as a function of *x*, *y*, and *z*, and we want to use this potential to find the electric field.

DEVELOP: Since we know $V(x,y,z)$, we use $E_x = -\dfrac{\partial V}{\partial x}$, $E_y = -\dfrac{\partial V}{\partial y}$, and $E_z = -\dfrac{\partial V}{\partial z}$ to find the components of the electric field. Then we evaluate *V* and *E* at the point (1.0 m, 1.0 m, 1.0 m), and finally we write the field in vector form.

EVALUATE: **(a)** Evaluate *V* at the point (1.00 m, 1.00 m, 1.00 m).

$$V(1.00\text{ m}, 1.00\text{ m}, 1.00\text{ m}) = (4.40\text{ V/m}^2)(1.00\text{ m})(1.00\text{ m}) - (3.85\text{ V/m}^2)(1.00\text{ m})(1.00\text{ m}) + (6.72\text{ V/m}^3)(1.00\text{ m})^3$$

$V = 7.37$ V.

(b) First get the components of $\vec{E}$ and evaluate them at (1.00 m, 1.00 m, 1.00 m).

$$E_x = -\frac{\partial V}{\partial x} = -(4.40\text{ V/m}^2)y - 3(6.72\text{ V/m}^3)x^2$$

$$E_x = -(4.40\text{ V/m}^2)(1.00\text{ m}) - 3(6.72\text{ V/m}^3)(1.00\text{ m})^2 = -24.6\text{ V/m}$$

$$E_y = -\frac{\partial V}{\partial y} = -(4.40\text{ V/m}^2)x + (3.85\text{ V/m}^2)z$$

$$E_y = -(4.40\text{ V/m}^2)(1.00\text{ m}) + (3.85\text{ V/m}^2)(1.00\text{ m}) = -0.55\text{ V/m}$$

$$E_z = -\frac{\partial V}{\partial z} = (3.85\text{ V/m}^2)y = (3.85\text{ V/m}^2)(1.00\text{ m}) = 3.85\text{ V/m}.$$

Now write the field in vector form.

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} = \left(-24.6\hat{i} - 0.55\hat{j} + 3.85\hat{k}\right)\text{V/m}.$$

ASSESS: The units of the numerical coefficients in the equation for *V* are not given in the problem, but each of them must have units that are not shown because each term in the sum must have units of volts. Since *x*, *y*, and *z* are measured in meters, the units of the numbers are clear. For example, the first term is 4.40*xy*. Since *x* and *y* both have units of meters, and 4.40*xy* must have units of volts, the units of the 4.40 coefficient must be V/m^2. Note also that *V* is a scalar but $\vec{E}$ is a vector.

91. **INTERPRET:** In this problem, we are dealing with energy conservation and the relationship between electric potential and potential energy.

DEVELOP: The proton's potential energy is converted to kinetic energy, and the potential energy change can be expressed in terms of the potential difference as $\Delta U = q\Delta V$. Therefore $q\Delta V = \Delta K$. The proton stops, so its final kinetic energy is zero, and its initial kinetic energy is $K = \frac{1}{2}mv^2$. So it must be true that $\frac{1}{2}mv^2 = q\Delta V$. We want *v*, so we solve for it, giving $v = \sqrt{\dfrac{2q\Delta V}{m}} = \sqrt{\dfrac{2e\Delta V}{m_{\text{proton}}}}$.

EVALUATE: Putting in the numbers gives

$$v = \sqrt{\frac{2(1.60\times10^{-19}\text{ C})(2450\text{ V})}{1.67\times10^{-27}\text{ kg}}} = 6.85\times10^5\text{ m/s} = 685\text{ km/s}.$$

ASSESS: Check units. The SI units of *v* in our calculation are $\sqrt{\dfrac{\text{CV}}{\text{kg}}} = \sqrt{\dfrac{\text{C}\left(\dfrac{\text{J}}{\text{C}}\right)}{\text{kg}}} = \sqrt{\dfrac{\text{J}}{\text{kg}}} = \sqrt{\dfrac{\text{kg}\cdot\text{m}^2/\text{s}^2}{\text{kg}}}$ = m/s, which are correct units for speed.

92. **INTERPRET:** We want the potential inside a uniformly charged spherical shell.

DEVELOP: The electric field inside the shell is zero, so the potential must be constant inside. Since the potential is V at the center, that must be its value everywhere inside, including at the surface. At the surface of a uniformly charged sphere having a charge Q, the potential is $V = kQ/R$.

EVALUATE: At the surface of our sphere, the potential at the surface is $V = kQ/R$. Solving for Q gives $Q = RV/k$.

ASSESS: Careful! The electric field is zero inside the shell, but the potential is constant but *not zero*.

93. **INTERPRET:** We know the electric field and want to use it to find the potential.

DEVELOP: The potential difference between two points is $\Delta V_{AB} = -\int_A^B \vec{E} \cdot d\vec{r}$. In this case, the electric field is $\vec{E} = b\sqrt{x}\,\hat{i}$, which is along the x-axis. So we are integrating along that axis, which makes $d\vec{r} = dx\hat{i}$.

EVALUATE: Using the above quantities, the potential is

$V(x) = -\int_0^x b\sqrt{x'}\,\hat{i} \cdot dx'\hat{i} = -\int_0^x b\sqrt{x'}\,dx' = -\tfrac{2}{3}bx^{3/2}$.

ASSESS: This result for $V(x)$ gives the potential at point x relative to the origin.

94. **INTERPRET:** In this problem we must calculate the potential due to two point-charges.

DEVELOP: The potential due to a single point-charge is $V = kq/r$. The total potential at any place is the scalar sum of the individual potentials due to all the charges. If we let q be the charge at $x = a$, the potential at $x = a/4$ is the sum of the potential due to Q and q, which is $V = V_Q + V_q = \dfrac{kQ}{a/4} + \dfrac{kq}{3a/4}$.

EVALUATE: Solve the previous equation using $V = 0$ at $x = a/4$.

$$\frac{kQ}{a/4} + \frac{kq}{3a/4} = 0$$

$$\frac{4kQ}{a} + \frac{4kq}{3a} = 0$$

$$Q + \frac{q}{3} = 0$$

$q = -3Q$.

ASSESS: The electric field at $x = a/4$ would *not* be zero since the fields at that point due to both charges point in the $+x$-direction. It does *not* follow that the electric field must be zero when the potential is zero.

95. **INTERPRET:** We need to use the relationship between the electric field and potential for a sphere of charge.

DEVELOP: The potential difference between the surface and center of the sphere is $\Delta V = \int_0^R \vec{E} \cdot d\vec{r}$, and we are given that $\Delta V = \dfrac{2E_0R}{3}$. We use the given electric field $\vec{E} = E_0\left(\dfrac{r}{R}\right)^\alpha \hat{r}$ for the integration, and we want to find the exponent α.

EVALUATE: $\Delta V = \dfrac{2E_0R}{3} = \int_0^R E_0\left(\dfrac{r}{R}\right)^\alpha \hat{r} \cdot d\vec{r} = \int_0^R \dfrac{E_0}{R^\alpha} r^\alpha dr = \dfrac{E_0}{R^\alpha} \dfrac{r^{\alpha+1}}{\alpha+1}\Bigg|_0^R$

Evaluating our integration result at the two limits gives

$\dfrac{E_0}{R^\alpha} \dfrac{r^{\alpha+1}}{\alpha+1}\Bigg|_0^R = \dfrac{E_0}{R^\alpha} \dfrac{R^{\alpha+1} - 0^{\alpha+1}}{\alpha+1} = \dfrac{E_0R}{\alpha+1}$, provided that $\alpha > -1$. Therefore we have

$\dfrac{2E_0R}{3} = \dfrac{E_0R}{\alpha+1}$. Solving for α gives $\alpha = \tfrac{1}{2}$.

ASSESS: Since $\alpha > -1$, our solution is valid, so the electric field is $\vec{E} = E_0\sqrt{\dfrac{r}{R}}\,\hat{r}$.

74. **INTERPRET:** This problem involves the work needed to assemble a set of four identical charges.

DEVELOP: The work to assemble two charges is $W = \dfrac{kq_1q_2}{r}$. In this case, the charges are all identical, and we assemble 4 of them in a line spaced at 2.50-cm intervals. Calling the charges q and the spacing d, the work is $W = \dfrac{kq^2}{d}$ for neighboring charges. The total work W_{tot} for all 4 charges is $W_{\text{tot}} = W_{12} + W_{13} + W_{14} + W_{23} + W_{24} + W_{34}$. We want to find the charge q.

EVALUATE: Applying the total work to the set of charges here gives

$$W_{\text{tot}} = W = \frac{kq^2}{d} + \frac{kq^2}{2d} + \frac{kq^2}{3d} + \frac{kq^2}{d} + \frac{kq^2}{2d} + \frac{kq^2}{d} = kq^2\left(\frac{3}{d} + \frac{2}{2d} + \frac{1}{3d}\right) = \frac{13kq^2}{d}.$$

Putting in the numbers and solving for q gives

$$4900\text{ J} = \frac{13(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)q^2}{3(0.0250\text{ m})}$$

$q = 5.60\times10^{-5}$ C = 56.0 μC.

ASSESS: The work does not depend on the path followed by the charges as they were assembled or the order in which it was done. We found that the magnitude of the charges is 56.0 μC, but we do not know if they were positive or negative charges. The same amount of work would be required in both cases, just so all the charges have the same charge.

75. **INTERPRET:** This problem involves the energy stored in a parallel-plate capacitor.

DEVELOP: The work that is done to transfer the charge between the plates is the energy stored in the capacitor. The work is $W = \frac{1}{2}CV^2$, but $V = Q/C$ so the work is $W = \frac{1}{2}C(Q/C)^2 = Q^2/2C$. We can use this result to find the capacitance C. For a parallel-plate capacitor, $C = \dfrac{\varepsilon_0 A}{d}$ with the area being $A = L^2$ for a square. We can use this result to find d using our value for C.

EVALUATE: Solving $W = Q^2/2C$ for C gives

$C = Q^2/2W = (7.28\times10^{-6}\text{ F})^2/[2(1.47\text{ J})] = 1.803\times10^{-11}$ F.

Now solve $C = \dfrac{\varepsilon_0 A}{d}$ for d, giving

$$d = \frac{\varepsilon_0 L^2}{C} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(0.0575\text{ m})^2}{1.803\times10^{-11}\text{ F}} = 1.62\times10^{-3}\text{ m} = 1.62\text{ mm}.$$

ASSESS: It seems reasonable that a square metal sheet 5.75 cm on a side could be machined to have a spacing of 1.62 mm.

76. **INTERPRET:** This problem deals with a parallel-plate capacitor.

DEVELOP: The capacitance is $C = \dfrac{\varepsilon_0 A}{d}$, and the area is $A = \pi r^2$, so $C = \dfrac{\varepsilon_0 \pi r^2}{d}$.

EVALUATE: Solving $C = \dfrac{\varepsilon_0 \pi r^2}{d}$ for d gives

$$d = \frac{\varepsilon_0 \pi r^2}{C} = \frac{(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)\pi(0.22\text{ m})^2}{740\times10^{-12}\text{ F}} = 1.8\times10^{-3}\text{ m} = 1.8\text{ mm}.$$

ASSESS: It seems reasonable that a round metal sheet 22 cm in radius could be machined to have a spacing of 1.8 mm.

77. **INTERPRET:** We are dealing with capacitors in series.

DEVELOP: Call C_1 the capacitor with the 80.0-V potential and C_2 the other capacitor. Since they are in series with 120 V across the combination, C_2 must have a voltage $V_2 = 40$ V across it, and they both have the same charge Q. We use $C_1 = Q/V_1$ to find Q, and then use $C_2 = Q/V_2$ to find C_2.

EVALUATE: First find Q. Solving $C_1 = Q/V_1$ for Q gives

$Q = C_1V_1 = (0.22\ \mu\text{F})(80.0\ \text{V}) = 17.6\ \mu\text{C}$.

Now use Q to find C_2.

$C_2 = Q/V_2 = (17.6\ \mu\text{C})/(40) = 0.44\ \mu\text{F}$.

ASSESS: Note that the larger capacitor has the smaller potential across it.

78. **INTERPRET:** This problem deals with capacitors in series.

DEVELOP: The equivalent series capacitance C_s of the 3 capacitors is given by $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. We know C_1, C_2, and C_s and want to find C_3.

EVALUATE: Using the formula for capacitors in series gives

$$\frac{1}{0.750\ \mu\text{F}} = \frac{1}{1.50\ \mu\text{F}} + \frac{1}{2.50\ \mu\text{F}} + \frac{1}{C_3}$$

$C_3 = 3.75\ \mu\text{F}$.

ASSESS: We found that C_3 is greater than the equivalent series capacitance, which must be true of all capacitors in series.

79. **INTERPRET:** In this problem we need to use the definition of capacitance.

DEVELOP: Capacitance C is defined as $C = Q/V$, so $Q = CV$. We want to compare the charge stored in each capacitor at its maximum rated voltage.

EVALUATE: 1.0-μF capacitor: $Q = CV = (1.0\ \mu\text{F})(250\ \text{V}) = 250\ \mu\text{C}$.

470-pF capacitor: $Q = CV = (470\ \text{pF})(3000\ \text{V}) = 1.4\times10^6\ \text{pC} = 1.4\ \mu\text{C}$.

We find that the 1.0-μF capacitor rated at 250 V can store more charge.

ASSESS: The charge storage ability of a capacitor depends on *both* its capacitance and its voltage rating.

80. **INTERPRET:** This problem involves the use of power.

DEVELOP: Power is $P = U/t$, so $t = U/P$. We know that an energy of $U = 2.5$ J is delivered by a 5.0-kW power source, and we want to find the time t during which the energy was delivered.

EVALUATE: Using $t = U/P$ gives

$t = (2.5\ \text{J})/(5000\ \text{J/s}) = 5.0\times10^{-4}\ \text{s} = 0.50\ \mu\text{s}$.

ASSESS: The larger the power, the shorter the time needed to deliver a given amount of energy.

81. **INTERPRET:** This problem involves a parallel-plate capacitor with a dielectric insulating material between its plates.

DEVELOP: The maximum safe voltage is the breakdown voltage V_b. The electric field between the plates is $E = V/d$, so the breakdown electric field is given by $E_b = V_bd$. We know the breakdown voltage and plate spacing d. From these we can find the breakdown field and compare it with those given in Table 23.1. The capacitance without the dielectric is $C_0 = \frac{\varepsilon_0 A}{d}$, and with the dielectric it is $C = \kappa C_0$, where κ is the dielectric constant of the material between the plates.

EVALUATE: **(a)** Find the breakdown field.

$E_b = V_b/d = (1500\ \text{V})/(25.0\times10^{-6}\ \text{m}) = 6.00\times10^7\ \text{V/m} = 60\ \text{MV/m}$.

From Table 23.1 we see that this is the breakdown field for Teflon, so that must be the insulating material.

(b) We use $C = \kappa C_0 = \frac{\kappa\varepsilon_0 A}{d}$. The area is $A = 50.0\ \text{cm}^2 = 5.0\times10^{-3}\ \text{m}^2$, and $\kappa = 2.1$ from Table 23.1.

$$C = \frac{\kappa\varepsilon_0 A}{d} = (2.1)(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(5.0\times10^{-3}\ \text{m}^2)/(25.0\times10^{-6}\ \text{m})$$

$C = 3.7\times 10^{-9}$ F = 3.7 nF.

ASSESS: Without the dielectric, the capacitance would be only C/κ = 1.77 nF.

82. **INTERPRET:** This problem deals with the energy density of an electric field.

DEVELOP: The energy density in an electric field is $u = \frac{1}{2}\varepsilon_0 E^2$. The total energy U within a volume is the product of the energy density times the volume. In this case, $\vec{E} = E_0\left(\frac{r}{R}\right)\hat{r}$, which is not uniform, so we must integrate the energy density over the volume of the sphere of radius R. Thus $U = \int u dV$. The field depends only on r, so we take dV to be a spherical shell of radius r and area $4\pi r^2$, so $dV = 4\pi r^2 dr$.

EVALUATE: The energy is $U = \int u dV = \int\left(\frac{1}{2}\varepsilon_0 E^2\right)dV = \int_0^R \frac{1}{2}\varepsilon_0\left[E_0\left(\frac{r}{R}\right)\right]^2 4\pi r^2 dr$

$$U = \frac{2\pi\varepsilon_0 E_0^2}{R^2}\int_0^R r^4 dr = \frac{2}{5}\pi\varepsilon_0 E_0^2 R^3.$$

ASSESS: Check the units of our answer by putting the SI units into our result, giving

$$\left(\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)\left(\frac{\mathrm{N}}{\mathrm{C}}\right)^2\left(\mathrm{m}^3\right) = \left(\frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)\left(\frac{\mathrm{N}^2}{\mathrm{C}^2}\right)\left(\mathrm{m}^3\right) = \mathrm{N\cdot m} = \mathrm{J}$$, which are the correct SI units for energy.

83. **INTERPRET:** This problem asks for the total energy stored in a series-parallel combination of capacitors.

DEVELOP: For capacitors in parallel, $C_\mathrm{p} = C_1 + C_2$, and for capacitors in series, $\frac{1}{C_\mathrm{s}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. The energy stored in a capacitor is $U = \frac{1}{2}CV^2$. We first calculate the equivalent capacitance of the combination, and then we use $U = \frac{1}{2}CV^2$ to calculate the stored energy U.

EVALUATE: For the parallel combination, we have

$C_\mathrm{p} = C_1 + C_2$ = 2.0 μF + 3.0 μF = 5.0 μF.

This combination is in series with capacitor C_3 = 9.5 μF, so

$$\frac{1}{C_\mathrm{s}} = \frac{1}{C_\mathrm{p}} + \frac{1}{C_3} = \frac{1}{5.0\ \mu\mathrm{F}} + \frac{1}{9.5\ \mu\mathrm{F}}$$

C_s = 3.276 μF.

The potential difference across the equivalent capacitance is 150 V, so the energy stored in it is

$U = \frac{1}{2}C_\mathrm{s}V^2 = \frac{1}{2}(3.276\ \mu\mathrm{F})(150\ \mathrm{V})^2 = 3.7\times 10^4\ \mu\mathrm{J}$ = 37 mJ.

ASSESS: If we connected only the 9.5-μF capacitor across the voltage source, it could store $U = \frac{1}{2}CV^2 = \frac{1}{2}(9.5\ \mu\mathrm{F})(150\ \mathrm{V})^2$ = 0.11 J, which is much more than the combination could store.

Chapter 24: Alternate Problem Set in *Mastering Physics*

76. **INTERPRET:** We use the definition of current.

DEVELOP: A steady current is given by $I = \dfrac{\Delta Q}{\Delta t}$. In this case, ΔQ is the charge of 100 trillion electrons, which is $10^{14}e$, and Δt = 1.0 s.

EVALUATE: First find ΔQ, giving ΔQ = $(10^{14})(1.6\times10^{-19}\text{ C}) = 1.6\times10^{-5}$ C. Now find *I*.

$I = \dfrac{\Delta Q}{\Delta t}$ = $(1.6\times10^{-5}\text{ C})/(1.0\text{ s}) = 1.6\times10^{-5}\text{ A} = 16\ \mu\text{A}$.

ASSESS: A current of 16 μA is typical of currents in many laboratory circuits.

77. **INTERPRET:** We use the definition of current density.

DEVELOP: The magnitude of the current density is $J = E/\rho$. In this case, $J = 3.25\text{ MA/m}^2$, and from Table 24.1 we find that $\rho = 2.65\times10^{-8}\ \Omega\cdot\text{m}$ for aluminum. We want to find the electric field *E*.

EVALUATE: Solving for *E* gives

$E = \rho J = (2.65\ \times10^{-8}\ \Omega\cdot\text{m})(3.25\times10^{6}\text{ A/m}^2) = 0.0861\text{ V/m}$.

ASSESS: This field is very small compared to those found in electronic equipment, such as capacitors.

78. **INTERPRET:** This problem requires the use of Ohm's law.

DEVELOP: Ohm's law is $V = V/R$. In this case, $V = 360$ V and $I = 0.30$ A, and we want to find *R*

EVALUATE: Solving Ohm's law for *R* gives

$R = V/I = (360\text{ V})/(0.30\text{ A}) = 1200\ \Omega = 1.2\text{ k}\Omega$.

ASSESS: This resistance is typical of many standard laboratory resistors.

79. **INTERPRET:** This problem requires the use of electrical power.

DEVELOP: The power is $P = IV$. We know $P = 2.0$ kW and $V = 240$ V, and we want to find the current *I*.

EVALUATE: Using $P = IV$ gives

2000 W = (250 V)*I*

$I = 8.3$ A.

ASSESS: This current is typical of many household currents.

80. **INTERPRET:** In this problem, we must use Ohm's law.

DEVELOP: Use Ohm's law ($I = V/R$) to find the current through the person's body. Then consult Table 24.3 to find the biological effects of this current.

EVALUATE: The current through the person is

$I = V/R = (120\text{ V})/(75{,}000\ \Omega) = 1.6\times10^{-3}\text{ A} = 1.6\text{ mA}$.

From Table 24.3, we see that a current between 0.5 mA and 2 mA will result in the threshold of sensation. Therefore you will feel only a slight tingle.

ASSESS: The person's $75\text{-k}\Omega$ resistance is for a dry body. If you are sweating or damp, your resistance is much lower, so you will receive a much larger current, which could be very dangerous.

81. **INTERPRET:** We must use current density.

DEVELOP: The current density is $J = I/A$, which tells us that $I = JA$. The current in the wire (I_w) and the current in the filament (I_f) must be the same, so $I_w = I_f$, which means that $J_wA_w = J_fA_f$. The cross-sectional area of the wire is

$A = \pi r^2 = \pi(d/2)^2$. We know that $J_w = 0.24\ \text{MA/m}^2 = 0.24\times10^6\ \text{A/m}^2$. We want to find the current density in the filament (J_f) and the total current in the circuit.

EVALUATE: **(a)** Using that $J_w A_w = J_f A_f$ and $A = \pi r^2 = \pi(d/2)^2$, we have

$J_w \pi(d_w/2)^2 = J_f \pi(d_f/2)^2$

$$J_f = J_w\left(\frac{d_w}{d_f}\right)^2 = (0.24\ \text{MA/m}^2)\left(\frac{2.1\ \text{mm}}{0.055\ \text{mm}}\right)^2 = 350\ \text{MA/m}^2 = 3.5\times10^8\ \text{A/m}^2.$$

(b) To find the current, look at the wire since we are given information about it.

$I = JA = J\pi r^2 = J\pi(d/2)^2 = (0.24\times10^6\ \text{A/m}^2)\ \pi[(2.1\times10^{-3}\ \text{m})/2]^2 = 0.83\ \text{A}$.

ASSESS: To check our answer in (b), calculate the current in the filament.

$I = JA = (3.5\times10^8\ \text{A/m}^2)\ \pi[(0.055\times10^{-3}\ \text{m})/2]^2 = 0.83\ \text{A}$, which agrees with our answer in **(b)**.

82. **INTERPRET:** In this problem, we need to use Ohm's law, resistivity, and current density.

DEVELOP: The resistance of the wire is $R = \rho L/A$. The voltage between the ends of the wire is $V = RI$. The current in the wire is $I = JA$. Combining these conditions gives $V = RI = (\rho L/A)(JA) = \rho LJ$. From Table 24.1 we find that $\rho = 9.71\times10^{-8}\ \Omega\cdot\text{m}$.

EVALUATE: Using $V = \rho LJ$ gives

$V = (9.71\times10^{-8}\ \Omega\cdot\text{m})(6.50\ \text{m})(4.379.71\times10^6\ \text{A/m}^2) = 2.76\ \text{V}$.

ASSESS: Check units. In standard SI units, our answer is $(\Omega\cdot\text{m})(\text{m})(\text{A/m}^2) = \Omega\cdot\text{A} = \text{V}$, which are the correct units of voltage.

83. **INTERPRET:** We need to use Ohm's law and resistivity.

DEVELOP: Ohm's law gives $V = RI$, and $R = \rho L/A$. Combining these expressions gives $V = \rho LI/A$. In this case, V, A, and I are the same for both wires, so we write the last equation as $VA/I = \rho L$, which tells us that ρL is constant. From Table 24.1, we find the values of ρ for copper and aluminum, which are $\rho_a = 2.65\times10^{-8}\ \Omega\cdot\text{m}$ and $\rho_c = 1.68\times10^{-8}\ \Omega\cdot\text{m}$. Using the fact that ρL is constant, we have $\rho_a L_a = \rho_c L_c$, so we can calculate L_a/L_c.

EVALUATE: Using $\rho_a L_a = \rho_c L_c$ gives

$$\frac{L_a}{L_c} = \frac{\rho_c}{\rho_a} = \frac{1.68\times10^{-8}\,\Omega\cdot\text{m}}{2.65\times10^{-8}\,\Omega\cdot\text{m}} = 0.634$$

$L_a = 0.634L_c$.

ASSESS: Since V and I are the same for both wires, they must have the same resistance by Ohm's law. Our result says that the aluminum wire is shorter than the copper wire. This is reasonable, since $\rho_a > \rho_c$, so we need less aluminum than copper for the two wires to have the same resistance.

84. **INTERPRET:** We need to use electrical power as well as mechanical power.

DEVELOP: The power delivered to the motor is $P = IV$. Since the motor is 100% efficient, all this power goes into the mechanical power $P = Fv$ of lifting the 15-N weight, where the force is its weight mg. Therefore $IV = Fv = mgv$. We want the speed v.

EVALUATE: Using $IV = mgv$ gives

$(0.63\ \text{A})(6.0\ \text{V}) = (15\ \text{N})v$

$v = 0.25\ \text{m/s} = 25\ \text{cm/s}$.

ASSESS: A 6.0-V battery probably could not maintain this speed for very long and would run down. If the motor was less than 100% efficient, the weight would rise more slowly but would still drain the battery at the same rate while the current was 0.63 A.

85. **INTERPRET:** This problem involves electrical power and specific heat.

DEVELOP: The power delivered as heat Q to the water is $P = V^2/R$, so the rate of heating is $Q/t = V^2/R$. The heat needed to raise the temperature of the water to the boiling point is $Q = mc\Delta T$. Combining these equations gives $\dfrac{mc\Delta T}{t} = \dfrac{V^2}{R}$. We know that $c = 4184\ \text{J/kg}\cdot\text{K}$ from Table 16.1, and m is the mass of 250 mL of water. Since $1\ \text{mL} = 1\ \text{cm}^3$, and $1\ \text{cm}^3$ of water has a mass of 1 g, the mass of 250 mL of water is 250 g, which is 0.25 kg. We want the time t.

EVALUATE: Solving $\dfrac{mc\Delta T}{t} = \dfrac{V^2}{R}$ for t gives

$t = mcR\,\Delta T\,/V^2 = (0.25\text{ kg})(4184\ \text{J/kg}\cdot\text{K})(13\ \Omega)(100°\text{C} - 10°\text{C})/(120\text{ V})^2 = 85\text{ s}.$

ASSESS: Check the units of t in our calculation. In SI units, these are

$$\frac{(\text{kg})\left(\dfrac{\text{J}}{\text{kg}\cdot\text{K}}\right)(\Omega)(°\text{C})}{\text{V}^2} = \frac{\text{J}\cdot\Omega}{\text{V}^2} = \frac{\text{J}\left(\dfrac{\text{V}}{\text{A}}\right)}{\text{V}^2} = \frac{\text{J}}{\text{A}\cdot\text{V}} = \frac{\text{J}}{\dfrac{\text{C}}{\text{s}}\cdot\dfrac{\text{J}}{\text{C}}} = \text{s}$$

, which is the correct SI unit for time.

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87. **INTERPRET:** This problem deals with power in an electric circuit.

DEVELOP: The power is $P = IV$, and in terms of energy, it is $P = dU/dt$. Equating the two expressions for P, we have $dU/dt = IV$. The current is $I = dQ/dt$, so we get $dU/dt = V\,dQ/dt$. The charge dQ flows in time dt, and the energy dU is delivered in the same time dt, so $dU = VdQ$, which gives $U = QV$.

EVALUATE: Using $U = QV$ gives us

$28.5\text{ J} = Q(12.0\text{ V})$

$Q = 2.38\text{ C}.$

ASSESS: Although this is a large amount of charge, it is balanced by an equal amount of charge of the opposite sign so the circuit remains neutral.

88. **INTERPRET:** This problem involves resistors in a series-parallel combination.

DEVELOP: For the two resistors in series, the equivalent series resistance R_s is $R_s = R_1 + R_2$. We use this to find the equivalent series resistance. For the parallel circuit, the equivalent parallel resistance R_p is given by $\frac{1}{R_p} = \frac{1}{R_s} + \frac{1}{R_3}$. We want to find the equivalent resistance of the combination of resistors.

EVALUATE: For the series resistors

$R_s = R_1 + R_2 = 47.0\text{ k}\Omega + 39.0\text{ k}\Omega = 86.0\text{ k}\Omega.$

For the parallel combination, R_s is in parallel with R_3, so

$$\frac{1}{R_p} = \frac{1}{R_s} + \frac{1}{R_3} = \frac{1}{86.0\text{ k}\Omega} + \frac{1}{22.0\text{ k}\Omega}$$

$R_p = 17.5\text{ k}\Omega$, which is the equivalent resistance of the combination.

ASSESS: The equivalent parallel resistance R_p is less than both of the resistors in the parallel combination, as it should be.

89. **INTERPRET:** This problem involves resistors in a series-parallel combination.

DEVELOP: First use $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$ to find the resistance of the parallel combination. Then use $R_s = R_p + R_3$ to find the resistance of R_3, knowing that $R_s = 43.0\text{ k}\Omega$.

EVALUATE: First find R_p.

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{47.0\text{ k}\Omega} + \frac{1}{39.0\text{ k}\Omega}$$

$R_p = 21.314\ \Omega.$

Now find R_3.

$R_s = R_p + R_3$

$43.0\text{ k}\Omega = 21.314\text{ k}\Omega + R_3$

$R_3 = 21.7\text{ k}\Omega.$

ASSESS: $R_3 < R_s$, which it must be since the equivalent series resistance is greater than each of the resistors in the series combination.

90. **INTERPRET:** This problem involves a battery having internal resistance and requires application of Ohm's law.
DEVELOP: The voltmeter reads the terminal voltage of the battery, which is given by $V_t = \mathcal{E} - R_{int}I$. We use Ohm's law to get the current in the circuit, which gives $\mathcal{E} = (R_{int} + R_{voltmeter})I$. We want to find the terminal voltage, and we know the other quantities.
EVALUATE: First find the current in the circuit.
$\mathcal{E} = (R_{int} + R_{voltmeter})I$
6.000 V = (343.6 Ω + 250,000 Ω)I
$I = 2.3967 \times 10^{-5}$ A.
Now use this current to find the terminal voltage of the battery.
$V_t = \mathcal{E} - R_{int}I$ = 6.000 V – (343.6 Ω)(2.3967×10^{-5} A) = 5.992 V.
ASSESS: The terminal voltage is nearly equal to the rated voltage of the battery because the current in the circuit is small, which is due to the large resistance of the voltmeter.

91. **INTERPRET:** This problem involves the time constant of a charging capacitor.
DEVELOP: The voltage across the capacitor is given by $V_C = \mathcal{E}(1 - e^{-t/RC})$, where RC is the time constant. We want to solve for t in terms of RC when V_C is 99.9% of $\mathcal{E}$.
EVALUATE: Using $V_C = \mathcal{E}(1 - e^{-t/RC})$ when $V_C = 0.999\mathcal{E}$ gives
$0.999\mathcal{E} = \mathcal{E}(1 - e^{-t/RC})$
$0.001 = e^{-t/RC}$
Taking natural logarithms of both sides gives
$\ln(0.001) = -t/RC$
$t = -RC \ln(0.001) = 0.691RC$.
To the nearest integer, $t = 7RC$ = 7 time constants.
ASSESS: In most laboratory circuits, the time constant is typically very small (milliseconds or even less), so 7 time constants is usually just a small fraction of a second.

92. **INTERPRET:** In this problem, we deal with the internal resistance of a battery and electrical power.
DEVELOP: The power is all dissipated in the internal resistance of the battery, so $P = \dfrac{V^2}{R_{int}}$. In this case, $V = \mathcal{E}$ = 6.00 V (the internal emf of the battery).
EVALUATE: Solving for R_{int} gives
$$R_{int} = \frac{\mathcal{E}^2}{P} = \frac{(6.00\text{ V})^2}{18.5\text{ W}} = 1.95\ \Omega.$$
ASSESS: The current is $I = \dfrac{\mathcal{E}}{R_{int}} = \dfrac{6.00\text{ V}}{1.95\ \Omega}$ = 3.08 A. This is quite a large current for a 6.00-V battery to produce and could not be maintained for very long.

93. **INTERPRET:** This problem deals with electrical power and resistors in parallel.
DEVELOP: The lamps are in parallel, so the potential is 120 V across each one. The total power (P_{tot}) is the sum of the power in each lamp (P_L). There are 75 identical lamps, so $P_{tot} = 75P_L$. The circuit breaker blows because the current I exceeded 15 A, so $P_{tot} = IV$, where I = 15 A and V = 120 V. Equating the two expressions for the total power gives $75P_L = IV$. We want to find the power rating P_L of each lamp.
EVALUATE: Using $75P_L = IV$ gives
$75P_L$ = (15 A)(120 V)
P_L = 24 W.
ASSESS: The wattage of 24 W of each bulb is the *minimum* wattage. The breaker could have been just ready to blow with 74 bulbs in the circuit if P_L were a bit greater than 24 W.

94. **INTERPRET:** This problem involves a charging capacitor.
DEVELOP: The voltage across a charging capacitor is given by $V_C = \mathcal{E}(1 - e^{-t/RC})$. We know R, and we also know that $V_C = \frac{1}{2}\mathcal{E}$ when t = 1.83 s. We want to find C, so we solve for it in the voltage equation.
EVALUATE: Solving $V_C = \mathcal{E}(1 - e^{-t/RC})$ for C when $V_C = \frac{1}{2}\mathcal{E}$ gives

$\frac{1}{2}\mathcal{E} = \mathcal{E}(1 - e^{-t/RC})$

$\frac{1}{2} = e^{-t/RC}$

$\ln(\frac{1}{2}) = \ln(e^{-t/RC}) = -t/RC$

$$C = -\frac{t}{R\ln(1/2)} = -\frac{1.83\ \text{s}}{(12{,}000\ \Omega)\ln(1/2)} = 2.20 \times 10^{-4}\ \text{F} = 220\ \mu\text{F}.$$

ASSESS: A capacitance of 220 μF is rather large for typical laboratory circuits. This is why it takes nearly two seconds for the capacitor to charge up to half of the battery voltage. In typical laboratory circuits, this usually happens in a small fraction of a second.

95. **INTERPRET:** This problem involves the internal resistance of a battery in a circuit as well as Ohm's law.

DEVELOP: The internal emf $\mathcal{E}$ of the battery is 4.500 V. The voltmeters both read less than 4.500 V because there is a potential drop across the internal resistance R_{int}. In both cases, the voltmeters read the terminal voltage of the battery. First use voltmeter A to find the internal resistance of the battery. Then use this in the circuit with voltmeter B connected.

EVALUATE: **(a)** With voltmeter A connected, the terminal voltage is 4.365 V, and this is also the voltage across the voltmeter. We can find the current by applying Ohm's law to voltmeter A.

$V_A = R_A I$

$4.365\ \text{V} = (15{,}000\ \Omega)I$

$I = 2.910 \times 10^{-4}\ \text{A}.$

The voltage drop across the internal resistance is $V_{\text{int}} = R_{\text{int}}I$. We know that this drop is 4.500 V – 4.365 V = 0.135 V. Therefore

$V_{\text{int}} = R_{\text{int}}I$

$0.135\ \text{V} = R_{\text{int}}\,(2.910 \times 10^{-4}\ \text{A})$

$R_{\text{int}} = 463.9\ \Omega$, which rounds to 464 Ω to 3 significant figures.

(b) Now consider the circuit with voltmeter B in it. First get the current by looking at the voltage drop across the internal resistance. This drop is 4.500 V – 4.412 V = 0.088 V. Applying Ohm's law to the internal resistance gives us

$R_{\text{int}}I = V_{\text{int}}$

$(463.9\ \Omega)I = 0.088\ \text{V}$

$I = 1.9 \times 10^{-4}\ \text{A}.$

Now apply Ohm's law to voltmeter B.

$V_B = R_B I$

$4.412\ \text{V} = R_B(1.9 \times 10^{-4}\ \text{A})$

$R_B = 23{,}000\ \Omega = 23\ \text{k}\Omega.$

ASSESS: Both voltmeters have typically large resistance, so in both cases the terminal voltage is very close to the voltage rating of the battery.

96. **INTERPRET:** In this problem, we deal with electrical power and energy.

DEVELOP: Average is $\overline{P} = \frac{U}{t}$, so the energy is $U = \overline{P}\,t$. In the electrical circuit, the average power is $\overline{P} = IV$.

Therefore $U = \overline{P}\,t = IVt$.

At 85 A, it takes 1.0 h (which is 3600 s) to run down the battery.

EVALUATE: $U = IVt = (85\ \text{A})(12\ \text{V})(3600\ \text{s}) = 3.7 \times 10^{6}\ \text{J} = 3.7\ \text{MJ}.$

ASSESS: We have assumed that the battery maintains a constant 12 V while discharging, but in fact that would not be the case. The voltage would decrease over time.

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92. **INTERPRET:** This problem involves the magnetic force on a moving charge.

DEVELOP: The magnitude of the force that the magnetic field exerts on the moving proton is $|F_B| = |q|vB\sin\theta$. We want the speed v and know the other quantities. The force is 1.60 fN = 1.60×10^{-15} N.

EVALUATE: Using $|F_B| = |q|vB\sin\theta$ gives

1.60×10^{-15} N = $(1.60\times10^{-19}$ C)v(55.2×10^{-3} T) sin(30.0)

$v = 3.62\times10^{5}$ m/s.

ASSESS: This proton is moving fast, but only about 1/800 the speed of light.

93. **INTERPRET:** This problem deals with the cyclotron frequency of electrons.

DEVELOP: The cyclotron frequency is $f = qB/2\pi m$. We know that B = 86 mT = 0.086 T and want to find the frequency f.

EVALUATE: $f = qB/2\pi m$ = $(1.60\times10^{-19}$ C)(0.086 T)/[$2\pi(9.11\times10^{-31}$ kg)]

$f = 2.4\times10^{9}$ Hz = 2.4 GHz.

ASSESS: The time for each cycle is $T = 1/f = 1/(2.4\times10^{9}\text{ Hz}) = 4.2\times10^{-10}$ s.

94. **INTERPRET:** This problem is about the magnetic force on a current-carrying wire in a magnetic field.

DEVELOP: The magnetic force is $F = IlB\sin\theta$. We know all the quantities except for θ. The field is B = 49.6 mT = 0.0496 T.

EVALUATE: Using $F = IlB\sin\theta$ gives

0.631 N = (15.0 A)(1.00 m)(0.0496 T) sin θ

θ = 58.0°.

ASSESS: The direction of the force is perpendicular to the wire and to the magnetic field.

95. **INTERPRET:** In this problem, we are working with the magnetic field due to a circular current-carrying wire loop.

DEVELOP: The magnetic field along the line perpendicular to the circular loop at its center is $B = \dfrac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}}$.

We know that $B = 41.2\ \mu$T at the center (x = 0 m) and B = 5.15 nT at 20.0 cm from the center of the loop (x = 0.200 m). We want to find the radius a of the loop and the current I in the loop.

EVALUATE: **(a)** If we take the ratio of the magnetic field at the center of the loop to the field at 20.0 cm from the center, the current I will cancel, leaving a as the only unknown. Using the given values, this ratio is

$\dfrac{B_\text{center}}{B_{20}} = \dfrac{41.2\times10^{-6}\text{ T}}{5.15\times10^{-9}\text{ T}} = 8000$. Using the formula for B, this ratio is

$$\frac{B_\text{center}}{B_{20}} = \frac{\dfrac{\mu_0 I a^2}{2\left(0^2+a^2\right)^{3/2}}}{\dfrac{\mu_0 I a^2}{2\left((0.200\text{ m})^2+a^2\right)^{3/2}}} = \frac{\left((0.200\text{ m})^2+a^2\right)^{3/2}}{\left(a^2\right)^{3/2}} = \left(\frac{(0.200\text{ m})^2+a^2}{a^2}\right)^{3/2}.$$

Equating the two expressions for the ratio of the fields gives

$$\left(\frac{(0.200\text{ m})^2+a^2}{a^2}\right)^{3/2} = 8000.$$

Taking the 2/3 root of both sides gives

$$\frac{(0.200\text{ m})^2 + a^2}{a^2} = 800^{2/3} = 400$$

$$(0.200\text{ m})^2 + a^2 = 400a^2$$

$$a = \frac{0.200\text{ m}}{\sqrt{399}} = 0.0100\text{ m} = 1.00\text{ cm}.$$

(b) Now use the field at the center of the loop to find the current *I*. Putting in $x = 0$ m gives $B = \mu_0 I/2a$. Solving for *I* we get

$$I = 2aB/\mu_0 = 2(0.0100\text{ m})(41.2\times10^{-6}\text{ T})/(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}) = 0.657\text{ A} = 657\text{ mA}.$$

ASSESS: To check, use the calculated current and radius to calculate *B* at 20.0 cm from the center.

$$B = \frac{\mu_0 I a^2}{2\left(x^2+a^2\right)^{3/2}} = \frac{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(0.657\text{ A})(0.0100\text{ m})^2}{2\left[(0.200\text{ m})^2 + (0.0100\text{ m})^2\right]^{3/2}} = 5.14\times10^{-9}\text{ T} = 5.14\text{ nT}.$$ The slight discrepancy in the third digit is due to rounding, so this result agrees with the given value of 5.15 nT.

96. **INTERPRET:** In this problem, we are dealing with the torque on a current-carrying loop in a magnetic field.

DEVELOP: The magnitude of the magnetic torque is $\tau = \mu B\sin\theta$, where $\mu = NIA$. Thus the torque is $\tau = NIAB\sin\theta$. For this loop, $N = 1$ and its area is L^2 since it is a square with sides of length $L = 5.00$ cm. We want to find the current *I* in the loop, and we know the torque and other quantities.

EVALUATE: Using $\tau = NIAB\sin\theta$, we have

$1.75\times10^{-3}\text{ N}\cdot\text{m} = (1)I(0.0500\text{ m})(1.46\text{ T})\sin(22.4°)$

$I = 1.26$ A.

ASSESS: The torque could be increased by adding more turns, which would increase *N*.

97. **INTERPRET:** This problem deals with the magnetic dipole moment.

DEVELOP: We model Jupiter as having a very small magnetic dipole at its center having a dipole moment $\mu = 2.3\times10^{27}\text{ A}\cdot\text{m}^2$. Since this dipole is very small, we can treat points on the "surface" of Jupiter as being very far away. Therefore the formula $B = \dfrac{\mu_0\mu}{2\pi x^3}$ applies to points on the axis of the dipole, which is the same as an axis through Jupiter's poles. For points on Jupiter's equator, we use the analogy with an electric dipole. In that situation, the field for a given distance along a line perpendicular to the dipole axis is one-half as strong as the field along the axis. Therefore the field at the equator will be one-half as strong as the field at the poles. From Appendix E we know that the radius *R* of Jupiter is $R = 69.9\times10^6$ m.

EVALUATE: **(a)** At the poles, $x = R$, so the field is

$$B = \frac{\mu_0\mu}{2\pi x^3} = \frac{\mu_0\mu}{2\pi R^3} = \frac{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(2.3\times10^{27}\text{ A}\cdot\text{m}^2)}{2\pi(69.9\times10^6\text{ m})^3}$$

$B = 1.3\times10^{-3}\text{ T} = 1.3\text{ mT} = 13\text{ G}$.

(b) As discussed above, $B_{\text{equator}} = \frac{1}{2}B_{\text{poles}} = \frac{1}{2}(1.3\text{ mT}) = 0.65\text{ mT} = 6.5\text{ G}$.

ASSESS: Earth's magnetic field is about 0.5 G near its surface, which is less than 1/10 the field at Jupiter's "surface."

98. **INTERPRET:** This problem deals with the circular motion of charges in a magnetic field.

DEVELOP: The magnetic field will deflect the electrons in the *z*-plane, where they will follow a semicircle. The radius of this circle is their maximum penetration into the magnetic field region. The radius is given by $r = mv/qB$. We know that $r = 3.45$ mm and we want to find *v*.

EVALUATE: Solving $r = mv/qB$ for *v* gives

$v = qBr/m = (1.60\times10^{-19}\text{ C})(188\times10^{-4}\text{ T})(0.00345\text{ m})/(9.11\times10^{-31}\text{ kg})$

$v = 1.14\times10^7\text{ m/s} = 11.4\text{ Mm/s}$.

ASSESS: This speed is nearly 4% the speed of light.

99. **INTERPRET:** This problem deals with the kinetic energy of a particle in circular motion in a magnetic field.
DEVELOP: The radius of the path is $R = mv/qB$ and the kinetic energy is $K = \frac{1}{2}mv^2$.

Solve the first problem for v and put that result into the kinetic energy equation to find K.
EVALUATE: Solving $R = mv/qB$ for v gives $v = qBR/m$. Putting this into the kinetic energy equation gives

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{qBR}{m}\right)^2 = \tfrac{1}{2}m\left(\frac{q^2B^2R^2}{m^2}\right) = \frac{q^2B^2R^2}{2m}.$$

ASSESS: Check units. Putting in the SI units for the result gives

$$\frac{\mathrm{C^2\cdot T^2\cdot m^2}}{\mathrm{kg}} = \frac{\mathrm{C^2}\left(\dfrac{\mathrm{N\cdot s}}{\mathrm{C\cdot m}}\right)^2\mathrm{m^2}}{\mathrm{kg}} = \frac{\mathrm{N^2\cdot s^2}}{\mathrm{kg}} = \frac{\dfrac{\mathrm{kg^2\cdot m^2}}{\mathrm{s^4}}\cdot\mathrm{s^2}}{\mathrm{kg}} = \mathrm{kg\cdot m^2/s^2} = \mathrm{J},$$

which is the correct SI unit of energy.

100. **INTERPRET:** In this problem, we use the magnetic field due to a long current-carrying wire.
DEVELOP: We model the lightning as a very long wire carrying a current I. Its magnetic is $B = \mu_0 I/2\pi r$.
We want I and know the other quantities.
EVALUATE: Solve $B = \mu_0 I/2\pi r$ for I, giving

$$I = \frac{2\pi rB}{\mu_0} = \frac{2\pi(500\ \mathrm{m})(40\times10^{-6}\ \mathrm{T})}{4\pi\times10^{-7}\,\mathrm{T\cdot m/A}} = 10^5\ \mathrm{A} = 100\ \mathrm{kA}.$$

ASSESS: Very large currents such as this one are indeed possible in lightning strikes. That is why they are so dangerous!

101. **INTERPRET:** This problem involves current density and the use of Ampere's law.
DEVELOP: Current density is $J = I/A$. In this case, J is not uniform over the cross-sectional area of the wire, so we need to break the area up into infinitesimal segments of area dA, so $dI = JdA$. The total current is then $I = \int JdA$. Since J depends only on r, we choose dA to be a thin ring of radius r and thickness dr, centered on the central axis of the wire. Its area is $dA = 2\pi r dr$, so $I = \int JdA = \int_0^R J2\pi r dr$. To find the magnetic field inside the wire, we will need to express I as a function of r and apply Ampere's law, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encircled}}$.

EVALUATE: **(a)** Using $I = \int JdA = \int_0^R J2\pi r dr$, we have

$$I = \int_0^R J_0\left(1-\frac{r^2}{R^2}\right)2\pi r dr = 2\pi J_0\int_0^R\left(r-\frac{r^3}{R^2}\right)dr = 2\pi J_0\left(\frac{R^2}{2}-\frac{R^4}{4R^2}\right) = \tfrac{1}{2}\pi R^2 J_0.$$

(b) First express I as a function of r using the same approach we used in part **(a)**.

$$I(r) = \int_0^r J_0\left(1-\frac{r'^2}{R^2}\right)2\pi r' dr' = 2\pi J_0\int_0^r\left(r'-\frac{r'^3}{R^2}\right)dr' = 2\pi J_0\left(\frac{r^2}{2}-\frac{r^4}{4R^2}\right)$$

$$I(r) = \pi r^2 J_0\left(1-\frac{r^2}{2R^2}\right).$$

Now apply Ampere's law, $\oint \vec{B}\cdot d\vec{l} = \mu_0 I_{\text{encircled}}$. For our path of integration, we choose a circle of radius r centered on the axis of the wire. On this path, B is constant in magnitude and is directed tangent to the circle, so we can take it out of the integral. This leaves just $\oint d\vec{l}$, which is just the circumference of the circle, $2\pi r$. Therefore the left-hand side of the equation is just $B2\pi r$. The right-hand side is μ_0 times the current within the circle of radius r, which we just found to be $I_{\text{enclosed}} = \pi r^2 J_0\left(1-\dfrac{r^2}{2R^2}\right)$. Equating the two sides of the equation gives

$$B2\pi r = \mu_0\pi r^2 J_0\left(1-\frac{r^2}{2R^2}\right)$$

$B = \dfrac{\mu_0 J_0}{2}\left(r-\dfrac{r^3}{2R^2}\right)$, which can also be expressed as $B = \dfrac{\mu_0 J_0}{4}\left(2r-\dfrac{r^3}{R^2}\right)$.

ASSESS: Check units: **(a)** The SI units of $\frac{1}{2}\pi R^2 J_0$ are $\text{m}^2(\text{A/m}^2) = \text{A}$, which is the unit of current. **(b)** The SI units of $\dfrac{\mu_0 J_0}{2}\left(r - \dfrac{r^3}{2R^2}\right)$ are $\left(\dfrac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\dfrac{\text{A}}{\text{m}^2}\right)(\text{m}) = \text{T},$ which is the unit for the magnetic field. We can also check B at the surface of the wire, where $r = R$. We have $B = \dfrac{\mu_0 J_0}{2}\left(R - \dfrac{R^3}{2R^2}\right) = \dfrac{\mu_0 J_0}{2}\left(R - \dfrac{R}{2}\right) = \dfrac{\mu_0 J_0 R}{4}$. Just at the surface, the wire is equivalent to a current of $I = \frac{1}{2}\pi R^2 J_0$, so the magnetic field it produces at the surface is given by $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{\mu_0\left(\frac{1}{2}\pi R^2 J_0\right)}{2\pi R} = \dfrac{\mu_0 R J_0}{4}$, which is the same as we just calculated from our result, so our answer checks at the surface of the wire.

Chapter 27: Alternate Problem Set in *Mastering Physics*

84. **INTERPRET:** This problem deals with magnetic flux.

DEVELOP: For a uniform magnetic field, the flux through a surface is $\Phi_B = BA\cos\theta$. We want to find B and know the other quantities. The area is $A = \pi r^2 = \pi(d/2)^2$.

EVALUATE: Using $\Phi_B = BA\cos\theta$ gives

$1.48\times10^{-4}\ \mathrm{T\cdot m^2} = B\pi(0.0275\ \mathrm{m})^2\cos(30.0°)$

$B = 7.19\times10^{-2}\ \mathrm{T} = 71.9\ \mathrm{mT}.$

ASSESS: This is a reasonable magnitude for such a magnetic field.

85. **INTERPRET:** This problem involves the self-inductance of a solenoid.

DEVELOP: The self-inductance of a solenoid is $L = \mu_0 n^2 Al$. If N is the number of turns, $n = N/l$, so we can express L as $L = \mu_0(N/l)^2 Al = \mu_0 N^2 A/l$, where $A = \pi r^2$. We want to find the number of turns N.

EVALUATE: Using $L = \mu_0 N^2 A/l$ gives

$0.0032\ \mathrm{H} = (2\pi\times10^{-7}\ \mathrm{T\cdot m/A})N^2\pi(0.020\ \mathrm{m})^2/(0.45\ \mathrm{m})$

$N = 955$ turns.

ASSESS: The density of turns is $n = 955/(45\ \mathrm{cm}) = 21$ turns/cm. This is a reasonable density if we use very thin wire.

86. **INTERPRET:** This problem involves the energy stored in an inductor.

DEVELOP: The stored energy is $U = \frac{1}{2}LI^2$. Call U_1 the initial energy and U_2 the energy after increasing the current. Using what is given in the problem, we have $U_2 = U_1 + 126$ mJ. The currents are $I_1 = 345$ mA and $I_2 = 1.00$ A. Writing the energy in terms of L gives $\frac{1}{2}LI_1^2 = \frac{1}{2}LI_2^2 + 126$ mJ. We want to solve for L.

EVALUATE: Solving $\frac{1}{2}LI_1^2 = \frac{1}{2}LI_2^2 + 126$ mJ for L, we get

$L\frac{1}{2}\left(I_2^2 - I_1^2\right) = 0.126\ \mathrm{J}$

$L\frac{1}{2}[(1.00\ \mathrm{A})^2 - (0.345\ \mathrm{A})^2] = 0.126\ \mathrm{J}$

$L = 0.286\ \mathrm{H} = 286\ \mathrm{mH}.$

ASSESS: This is rather large for an ordinary laboratory inductor, but not impossibly large.

87. **INTERPRET:** In this problem, we are dealing with the energy density in a magnetic field.

DEVELOP: The magnetic energy density is $u_B = \dfrac{B^2}{2\mu_0}$, so the total energy U_B is given by $U_B = u_B$(volume). We know B and U_B and we want the volume occupied by the field. (Note that we are *not* using V for volume so we don't confuse it with the voltage V.)

EVALUATE: $U_B = u_B(\text{volume}) = \dfrac{B^2}{2\mu_0}(\text{volume})$

$$41\times10^9\ \mathrm{J} = \frac{(11.8\ \mathrm{T})^2}{2(4\pi\times10^{-7}\ \mathrm{T\cdot m/A})}(\text{volume})$$

volume = 740 m^3.

ASSESS: The energy density is $(41\times10^9\ \mathrm{J})/(740\ \mathrm{m^3}) = 5.5\times10^7\ \mathrm{J/m^3} = 55\ \mathrm{J/cm^3}$. Therefore every cubic centimeter of this space contains 55 J of magnetic energy.

88. **INTERPRET:** In this problem, we apply Faraday's law to find the induced electric field.

DEVELOP: Faraday's law is $\oint \vec{E}\cdot d\vec{l} = -\dfrac{d\Phi_B}{dt}$. We can disregard the minus sign since we only want the magnitude of the induced electric field. Call R the radius of the solenoid, and take a path of integration to be a circle of radius r centered on the axis of the solenoid. On this path, E is constant and tangent to the path, so we take it out of the integral. This leaves only $\oint dl$, which is the circumference of the path, which is $2\pi r$. The rate of change of the flux is $\dfrac{d\Phi_B}{dt} = \dfrac{d(BA)}{dt} = A\dfrac{dB}{dt}$, where A is the area of the solenoid since that is where the magnetic field occurs. Thus $A = \pi R^2$. We want to find the induced electric field E at a distance $r = 12.8$ cm from the axis of the solenoid.

EVALUATE: Applying the above conditions to Faraday's law gives

$$\oint \vec{E}\cdot d\vec{l} = \frac{d\Phi_B}{dt}$$

$$E2\pi r = \pi R^2\, dB/dt$$

$$E(2\pi)(0.128 \text{ m}) = \pi(0.100 \text{ m})^2(1140 \text{ T/s})$$

$$E = 44.5 \text{ V/m}.$$

ASSESS: This is a rather weak electric field compared to the fields found in items such as capacitors and oscilloscopes.

89. **INTERPRET:** In this problem, we must make use of magnetic induction, Faraday's law, and Ohm's law.

DEVELOP: Faraday's law is $\mathcal{E} = -\dfrac{d\Phi_B}{dt}$. We can disregard the minus sign since were interested only in the magnitude of the magnetic field. The field is perpendicular to the loop, so $\Phi_B = BA$. Therefore $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \dfrac{d(BA)}{dt} = A\dfrac{dB}{dt}$. Applying Ohm's law to the loop gives $\mathcal{E} = RI$. We know that $I = b\sqrt{t}$, so $\mathcal{E} = A\dfrac{dB}{dt} = RI = Rb\sqrt{t}$. We then integrate to find $B(t)$.

EVALUATE: Separating variables in $A\dfrac{dB}{dt} = Rb\sqrt{t}$ and integrating gives

$$dB = \frac{Rb}{A}\sqrt{t}\,dt$$

$$B = \int \frac{Rb}{A}\sqrt{t}\,dt = \frac{Rb}{A}\frac{t^{3/2}}{3/2} + \text{constant}.$$

When $t = 0$, $B = 0$, so the constant is zero. Therefore the field is

$$B = \frac{2Rb}{3A}t^{3/2}.$$

ASSESS: Check units. The SI units of our result are

$$\frac{\Omega\cdot(\text{A}\cdot\text{s}^{-1/2})}{\text{m}^2}\cdot\text{s}^{3/2} = \frac{\Omega\cdot\text{A}\cdot\text{s}}{\text{m}^2} = \frac{\left(\frac{\text{V}}{\text{A}}\right)(\text{A}\cdot\text{s})}{\text{m}^2} = \frac{\text{V}\cdot\text{s}}{\text{m}^2} = \frac{\frac{\text{J}}{\text{C}}\cdot\text{s}}{\text{m}^2} = \frac{\text{N}\cdot\text{m}\cdot\text{s}}{\text{C}\cdot\text{m}^2} = \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}} = \text{T},$$

which is the correct SI unit for the magnetic field.

90. **INTERPRET:** This is a problem involving induction.

DEVELOP: As the coil turns, the magnetic flux through it changes due to the varying angle between the field and the area. This change induces a voltage in the coil. The flux through the coil is $\Phi_B = BAN\cos\theta$, and the magnitude of the induced voltage is $\mathcal{E} = \dfrac{d\Phi_B}{dt}$. Therefore the magnitude of $\mathcal{E}$ is $\mathcal{E} = \dfrac{d\Phi_B}{dt} = \dfrac{d(BAN\cos\theta)}{dt}$ $= BAN\sin\theta\dfrac{d\theta}{dt}$. The angle is steadily changing, so $\theta = \omega t$ and $d\theta/dt = \omega$. Therefore we have $\mathcal{E} =$ $BAN\omega\sin\omega t$. The peak voltage is $\mathcal{E}_{\max} = BAN\omega$. We know that $\omega = 2\pi f$, $A = \pi r^2$, and we want to find $\mathcal{E}_{\max}$.

EVALUATE: Using $\mathcal{E}_{\max} = BAN\omega$, $\omega = 2\pi f$, and $A = \pi r^2$ gives

$\mathcal{E}_{\max}$ = (0.288 T)π(0.0640 m)^2(50)2π(25.0 s^{-1}) = 29.1 V.

ASSESS: The induced voltage $BAN\omega\sin\omega t$ varies sinusoidally between ±29.1 V.

91. **INTERPRET:** In this problem, we are dealing with induction, Ohm's law, and a solenoid.

DEVELOP: A current is induced in the coil, so the magnetic flux through it must be changing. This flux is due to the magnetic field in the solenoid, which itself is due to the current in the solenoid, so the current in the solenoid must be changing with time. The magnetic field inside the solenoid is $B = \mu_0 I_{\text{sol}} n$. The magnetic flux through the coil of radius r is $\Phi_B = BA\cos\theta = BA\pi r^2\cos 0° = B\pi r^2$. Using the field inside the solenoid, the flux becomes $\Phi_B = \mu_0 I_{\text{sol}} n\pi r^2$. The induced voltage $\mathcal{E}$ in the coil is $\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d\left(\mu_0 I_{\text{sol}} n\pi r^2\right)}{dt} = \mu_0 n\pi r^2 \frac{dI_{\text{sol}}}{dt}$. Ohm's law for the coil is $\mathcal{E} = RI_{\text{coil}}$. Equating our two expressions for $\mathcal{E}$ gives $\mu_0 n\pi r^2 \frac{dI_{\text{sol}}}{dt} = RI_{\text{coil}}$. We also know that $n = N/l$ = (2000 turns)/(2.00 m) = 1000 m^{-1} and r = 6.10 cm = 0.0610 m. We want to find $\frac{dI_{\text{sol}}}{dt}$.

EVALUATE: Using $\mu_0 n\pi r^2 \frac{dI_{\text{sol}}}{dt} = RI_{\text{coil}}$ and putting in the numbers gives

$$(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})\,(1000\ \text{m}^{-1})\pi(0.0610\ \text{m})^2\frac{dI_{\text{sol}}}{dt} = (5.35\ \Omega)(0.0187\ \text{A})$$

$$\frac{dI_{\text{sol}}}{dt} = 6810\ \text{A/s} = 6.81\ \text{kA/s}.$$

ASSESS: We did not need to use the diameter of the solenoid since it is the flux change through the *coil* that induces the voltage in the coil. A rate of change of 6.81 kA/s in the solenoid does not mean that the current in the solenoid ever reaches 6.81 kA; it simply means that the current is changing rapidly.

92. **INTERPRET:** In this problem, we are dealing with induction by an alternating-current generator.

DEVELOP: As the coil turns, the magnetic flux through it changes due to the varying angle between the field and the area. This change induces a voltage in the coil. The flux through the coil is $\Phi_B = BAN\cos\theta$, and the magnitude of the induced voltage is $\mathcal{E} = \frac{d\Phi_B}{dt}$. Therefore the magnitude of $\mathcal{E}$ is $\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d(BAN\cos\theta)}{dt} = BAN\sin\theta\frac{d\theta}{dt}$. The angle is steadily changing, so $\theta = \omega t$ and $d\theta/dt = \omega$. Therefore we get $\mathcal{E} = BAN\omega\sin\omega t$. The peak voltage is $\mathcal{E}_{\max} = BAN\omega$. We know that $\omega = 2\pi f$, A is the area of a rectangle, and we want to find $\mathcal{E}_{\max}$.

EVALUATE: Using $\mathcal{E}_{\max} = BAN\omega$ and $\omega = 2\pi f$ gives

$\mathcal{E}_{\max}$ = (0.144 T)(0.754 m)(1.30 m)(125)2π(60.0 Hz) = 6650 V = 6.65 kV.

ASSESS: This peak voltage is *much* greater than the voltage in household circuits.

93. **INTERPRET:** This problem involves the current in an *RL* circuit.

DEVELOP: The current in an *RL* circuit is given by $I = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$. We want the time for the current to reach half of its maximum value. The maximum value is $\frac{\mathcal{E}_0}{R}$, so we must solve the equation $\frac{1}{2}\frac{\mathcal{E}_0}{R} = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$ for t.

EVALUATE: Solving $\frac{1}{2}\frac{\mathcal{E}_0}{R} = \frac{\mathcal{E}_0}{R}\left(1 - e^{-Rt/L}\right)$ for t, we get $\frac{1}{2} = 1 - e^{-Rt/L}$. Taking natural logarithms of both sides of the equation gives

$\ln(½) = -Rt/L$

$t = -(L/R)\ln(½) = -[(377\ \text{mH})/(151\ \Omega)]\ln(½) = 1.73$ ms.

ASSESS: By leaving L in mH, we could calculate t directly in ms.

Chapter 28: Alternate Problem Set in *Mastering Physics*

77. **INTERPRET:** This problem relates angular frequency and peak voltage to frequency and rms voltage.

DEVELOP: Angular frequency is $\omega = 2\pi f$, so $f = 2\pi/\omega$. The rms voltage is $V_{\text{rms}} = \dfrac{V_{\text{p}}}{\sqrt{2}}$.

EVALUATE: **(a)** $f = 2\pi/\omega = (2512\ \text{s}^{-1})/2\pi = 399.8\ \text{Hz} = 0.3998\ \text{kHz}$.

(b) $V_{\text{rms}} = \dfrac{V_{\text{p}}}{\sqrt{2}} = \dfrac{294\ \text{V}}{\sqrt{2}} = 208\ \text{V}$.

ASSESS: We find that $V_{\text{rms}} < V_{\text{p}}$, as it should be.

78. **INTERPRET:** This problem deals with capacitive reactance.

DEVELOP: Capacitive reactance is $X_C = \dfrac{1}{\omega C}$, where $\omega = 2\pi f$. We know the reactance at 60.0 Hz is 803 Ω, and we want to find C and the reactance at 1.00 kHz.

EVALUATE: **(a)** Solve $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$ for C, giving

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(60.0\ \text{Hz})(803\ \Omega)} = 3.30\times10^{-6}\ \text{F} = 3.30\ \mu\text{F}.$$

(b) At 1.00 kHz, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi(1000\ \text{Hz})(3.30\times10^{-6}\ \text{F})} = 48.2\ \Omega$.

ASSESS: Increasing the frequency decreased the capacitive reactance, but the capacitance itself remained the same.

79. **INTERPRET:** This problem involves the current through a resistor and a capacitor with an AC power source, as well as capacitive reactance.

DEVELOP: The current through the resistor is $I_R = V/R$. The current through the capacitor is $I_C = V/X_C$. The currents are the same, so $V/R = V/X_C$, which tells us that $R = X_C$. The reactance is $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$, so

$$R = \frac{1}{2\pi f C}.$$

EVALUATE: Using $R = \dfrac{1}{2\pi f C}$, we have

$$R = \frac{1}{2\pi(60.0\ \text{Hz})(1.50\times10^{-6}\ \text{F})} = 1770\ \Omega = 1.77\ \text{k}\Omega.$$

ASSESS: The resistance is equal to the capacitive reactance, but it is *not* correct to say that the resistance is equal to the capacitance.

80. **INTERPRET:** In this problem, we deal with the oscillation of an LC circuit and the energy in the capacitor and inductor.

DEVELOP: The angular frequency is $\omega = 1/\sqrt{LC}$ and $\omega = 2\pi f = 2\pi/T$. Use this to find C since we know T and L. The peak current occurs when the capacitance is completely discharged since at that time all the energy is in the inductor. The energy in a inductor is $U_L = \frac{1}{2}LI^2$ and the energy in a capacitor is $U_C = \frac{1}{2}CV^2$. Use this to find the peak current.

EVALUATE: **(a)** Combine $\omega = 1/\sqrt{LC}$ and $\omega = 2\pi f = 2\pi/T$, giving $\frac{1}{\sqrt{LC}} = \frac{2\pi}{T}$. Squaring and solving for C gives

$$\left(\frac{2\pi}{T}\right)^2 = \frac{1}{LC}$$

$$C = \frac{1}{L}\left(\frac{T}{2\pi}\right)^2 = \left(\frac{1}{0.322\text{ H}}\right)\left(\frac{0.00675\text{ s}}{2\pi}\right)^2 = 3.58\times10^{-6}\text{ F} = 3.58\ \mu\text{F}.$$

(b) Use the peak voltage to find the maximum energy stored in the capacitor. $U_C = \frac{1}{2}CV^2$, so the peak energy in the capacitor is

$$\text{U}_{\text{Cp}} = \tfrac{1}{2}CV_\text{p}^2 = \tfrac{1}{2}(3.58\times10^{-6}\text{ F})(1.22\text{ V})^2 = 2.67\times10^{-6}\text{ J}.$$

The peak current occurs when all of this energy has been transferred to the inductor, in which case the peak energy in the inductor is $U_{L_\text{p}} = 2.67\times10^{-6}$ J. The peak current occurs when the inductor energy is at its peak, so $\frac{1}{2}LI_\text{p}^2 = 2.67\times10^{-6}$ J. This gives

$$\tfrac{1}{2}(0.322\text{ H})I_\text{p}^2 = 2.67\times10^{-6}\text{ J}$$

$$I_\text{p} = 4.07\times10^{-3}\text{ A} = 4.07\text{ mA}.$$

ASSESS: It takes ¼ of a period, or $\frac{1}{4}$ (6.75 ms) = 1.69 ms, to reach peak current after the fully charged capacitor begins to discharge.

81. **INTERPRET:** This problem deals with impedance in an *RLC* AC circuit.

DEVELOP: The lowest impedance occurs at resonance, in which case $\omega = \omega_0 = 1/\sqrt{LC}$. At this point, the reactance is zero and the impedance is equal to *R*. For the situation here, $\omega_0 = 2\pi f_0$ and $f_0 = 95$ Hz. We want to find *R* and *L*.

EVALUATE: **(a)** The circuit is at resonance, so $Z = R = 18$ kΩ.

(b) Combining $\omega_0 = 1/\sqrt{LC}$ and $\omega_0 = 2\pi f_0$ gives $\omega_0^2 = \frac{1}{LC}$. Solving for L gives

$$L = \frac{1}{\omega_0^2 C} = \frac{1}{\left[2\pi(95\text{ Hz})\right]^2\left(14\times10^{-6}\text{ F}\right)} = 0.20\text{ H}.$$

ASSESS: At resonance, $X_L = X_C$, so $X_L - X_C = 0$ which makes $Z = R$.

82. **INTERPRET:** This problem is about the power factor.

DEVELOP: The average power in an AC circuit is $\langle P\rangle = I_\text{rms}V_\text{rms}\cos\phi$, where $\cos\phi$ is the power factor. We want to find $\cos\phi$, and we know the other quantities.

EVALUATE: Using $\langle P\rangle = I_\text{rms}V_\text{rms}\cos\phi$, we have

$$490\text{ W} = (4.6\text{ A})(120\text{ V})\cos\phi$$

$$\cos\phi = 0.89.$$

ASSESS: The power factor is 1 for a purely resistive circuit, but less than that here due to the presence of capacitors and/or inductors.

83. **INTERPRET:** This problem deals with inductive and capacitive reactance.

DEVELOP: The inductive reactance is $X_L = \omega L$, and the capacitive inductance is $X_C = \frac{1}{\omega C}$. At ω_1, we know that $X_C = 16X_L$, so $\frac{1}{\omega_1 C} = 16\omega_1 L$. At ω_2, we want $X_C = X_L$, so $\frac{1}{\omega_2 C} = \omega_2 L$. We first relate ω_2 to ω_1, and then use that result to relate f_2 to f_2 using $\omega = 2\pi f$.

EVALUATE: We don't know *L* or *C* for this circuit. If we take the ratio of the reactances at the two frequencies, *L* and *C* will cancel. This gives

$$\frac{1/\omega_1 C}{1/\omega_2 C} = \frac{16\omega_1 L}{\omega_2 L}$$

$$\frac{\omega_2}{\omega_1} = \frac{16\omega_1}{\omega_2}$$

$$\omega_2^2 = 16\omega_1^2$$

$\omega_2 = 4\omega_1$.
Using $\omega = 2\pi f$, we get
$2\pi f_2 = 4(2\pi f_1)$
$f_2 = 4f_1$.
ASSESS: At f_2 the circuit is in resonance because $X_L = X_C$.

84. **INTERPRET:** This problem deals with an LC tuning circuit.
DEVELOP: To tune to a frequency, that frequency should be the resonant frequency $\omega_0 = 1/\sqrt{LC}$ of the circuit. Using $\omega = 2\pi f$, we have $2\pi f_0 = 1/\sqrt{LC}$. Solving for C we get $C = \dfrac{1}{L(2\pi f_0)^2}$. We want to find C and know the other quantities.
EVALUATE: At 87.5 MHz, f_0 = 87.5 MHz. Therefore C is

$$C = \frac{1}{L(2\pi f_0)^2} = \frac{1}{(0.220\times 10^{-6}\text{ H})\left[2\pi(87.5\times 10^{6}\text{ Hz})\right]^2} = 1.05\times 10^{-11}\text{ F} = 15.0\text{ pF}.$$

At 108 MHz we have

$$C = \frac{1}{L(2\pi f_0)^2} = \frac{1}{(0.220\times 10^{-6}\text{ H})\left[2\pi(108\times 10^{6}\text{ Hz})\right]^2} = 9.87\times 10^{-12}\text{ F} = 9.87\text{ pF}.$$

ASSESS: The capacitance in the tuner can easily be changed by varying the area in common between the two plates.

85. **INTERPRET:** In this problem, we are dealing with an RLC AC circuit.
DEVELOP: The maximum voltage across the capacitor can never exceed 400 V. As the frequency of the sine-wave generator is varied, the impedance of the circuit changes and reaches its minimum value at the resonant frequency $\omega_0 = 1/\sqrt{LC}$. At this point, the current is a maximum, I_p, and the impedance is $Z = R$. The voltage across the capacitor at this point is $V_C = I_p X_C = \dfrac{V}{R}\left(\dfrac{1}{\omega_0 C}\right)$. In this equation, V is the voltage of the generator. The maximum voltage across the capacitor, V_{Cp}, will occur when V has its peak value, V_p. Therefore $V_{Cp} = \dfrac{V_p}{R}\left(\dfrac{1}{\omega_0 C}\right)$. Using $\omega_0 = 1/\sqrt{LC}$, we can write the capacitive reactance as $\dfrac{1}{\omega_0 C} = \dfrac{1}{\dfrac{1}{\sqrt{LC}}C} = \dfrac{1}{\sqrt{\dfrac{C}{L}}} = \sqrt{\dfrac{L}{C}}$. Therefore the peak capacitor voltage is $V_{Cp} = \dfrac{V_p}{R}\sqrt{\dfrac{L}{C}}$. We know that V_{Cp} = 400 V, so we can solve for the peak generator voltage V_p.
EVALUATE: Using $V_{Cp} = \dfrac{V_p}{R}\sqrt{\dfrac{L}{C}}$ gives

$$400\text{ V} = \frac{V_p}{18\,\Omega}\sqrt{\frac{1.5\text{ H}}{250\times 10^{-6}\text{ F}}}$$

$V_p = 93$ V.
ASSESS: At frequencies other than the resonant frequency, the peak V_p could be higher than 93 V because the impedance would be greater than 18 Ω. But we want to protect the capacitor in all possible cases, so $V_p = 93$ V will always be safe.

86. **INTERPRET:** This problem is about an undriven LC circuit having some resistance.
DEVELOP: The charge on the capacitor is $q(t) = q_p e^{-Rt/2L}\cos\omega t$. The capacitor voltage is $V_C = q/C$, so we can express it as a function of time, which gives $V_C(t) = \dfrac{q_p}{C}e^{-Rt/2L}\cos\omega t = V_p e^{-Rt/2L}\cos\omega t$. For the capacitor voltage to peak at 28 V, $\cos\omega t$ must have its maximum value, which is 1. So for part **(a)**, we want to solve the equation

$V_C = V_\text{p} e^{-Rt/2L}$ for t. In part **(b)**, we want to know how many cycles occur during this time. To do this, we know that $\omega = 1/\sqrt{LC} = 2\pi/T$, so $T = 2\pi\sqrt{LC}$, which will give us the time for one cycle.

EVALUATE: (a) Solve $V_C = V_\text{p} e^{-Rt/2L}$ for t.

$28\text{ V} = 35\text{ V } e^{-Rt/2L}$

$0.80 = e^{-Rt/2L}$

$\ln(0.80) = -Rt/2L$

$$t = -\frac{2L\ln(0.80)}{R} = -\frac{2(0.027\text{ H})\ln(0.80)}{0.640\ \Omega} = 1.9\times10^{-2}\text{ s} = 19\text{ ms.}$$

(b) The period (time for one cycle) is

$$T = 2\pi\sqrt{LC} = 2\pi\sqrt{(0.027\text{ H})(3.3\times10^{-6}\text{ F})} = 1.9\times10^{-3}\text{ s} = 1.9\text{ ms.}$$

If N is the number of cycles, then $NT = 19$ ms, so

$N(1.9\text{ ms}) = 19\text{ ms}$

$N = 10$ cycles.

ASSESS: In 10 cycles, the voltage amplitude has decreased from 35 V to 28 V, which is a percent decrease of (35 V – 28 V)/(35 V) = 0.20 = 20%. The lost energy has gone into heat in the resistor.

Chapter 29: Alternate Problem Set in *Mastering Physics*

75. **INTERPRET:** This problem is about displacement current.

DEVELOP: The displacement current is $I_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$, and the electric flux is $\Phi_E = EA$ since the electric field is uniform over the area. Therefore $I_d = \varepsilon_0 \dfrac{d(EA)}{dt} = \varepsilon_0 A \dfrac{dE}{dt}$. We want $\dfrac{dE}{dt}$ and we know I_d and A.

EVALUATE: Using $I_d = \varepsilon_0 A \dfrac{dE}{dt}$ gives

$$1.33\times10^{-6}\text{ A} = (8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2)(1.10\times10^{-4}\text{ m}^2)\ \frac{dE}{dt}$$

$$\frac{dE}{dt} = 1.37\times10^{9}\text{ V/m}\cdot\text{s} = 1.37\text{ G V/m}\cdot\text{s}.$$

ASSESS: The field is changing rapidly, but our result does *not* mean that the field actually reaches 1.37 GV/m.

76. **INTERPRET:** In this problem, we use the speed of electromagnetic waves.

DEVELOP: The total path length is the distance the electromagnetic waves travel in 290 ms at the speed of light. So we use $x = ct$ to find the path length.

EVALUATE: $x = ct = (3.0\times10^{8}\text{ m/s})(0.290\text{ s}) = 8.7\times10^{7}\text{ m} = 87\text{ Mm}$.

ASSESS: We have assumed that there are no time delays during electronic processing of the signal at the receiver or transmitter, so 87 Mm is the *maximum* that the distance could be.

77. **INTERPRET:** This problem deals with the transmission of polarized light through a polarizing sheet.

DEVELOP: The law of Malus applies, so $S = S_0\cos^2\theta$. We know that $S = \frac{1}{3}S_0$ and we want to find the angle θ.

EVALUATE: $S = S_0\cos^2\theta$

$\frac{1}{3}S_0 = S_0\cos^2\theta$

$\cos^2\theta = \frac{1}{3}$

$\theta = 55°$.

ASSESS: As θ is increased, the transmitted intensity decreases.

78. **INTERPRET:** This problem deals with the electric and magnetic fields in a light beam, as well as its intensity and power.

DEVELOP: The fields in an electromagnetic wave are related by $E = cB$. The average intensity of the beam is $\bar{S} = \dfrac{E_p^2}{2\mu_0 c}$. The average intensity is also given by $\bar{S} = \bar{P}/A$, so the average power is $\bar{P} = \bar{S}A$.

EVALUATE: **(a)** For the peak fields, we have $E_p = cB_p$, so

$356\text{ V/m} = (3.00\times10^{8}\text{ m/s})B_p$

$B_p = 1.19\times10^{-6}\text{ T} = 1.19\ \mu\text{T}$.

(b) $$\bar{S} = \frac{E_p^2}{2\mu_0 c} = \frac{(356\text{ V/m})^2}{2(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(3.00\times10^{8}\text{ m/s})} = 168\text{ W/m}^2.$$

(c) $\bar{P} = \bar{S}A = \bar{S}\pi r^2 = (168\ \text{W/m}^2)\pi\left(\dfrac{0.950\times10^{-3}\ \text{m}}{2}\right)^2 = 1.19\times10^{-4}\ \text{W} = 119\ \mu\text{W}.$

ASSESS: The power (119 μW) of the beam is small, but the intensity (168 W/m^2) is large because the energy is concentrated over a small area.

79. **INTERPRET:** In this problem we must related the frequency and wavelength of light.
DEVELOP: For a light beam, $f\lambda = c$. In this case, f = 5.0 GHz. We first find the wavelength λ.
EVALUATE: Using $f\lambda = c$ gives
$(5.0\times10^9\ \text{Hz})\lambda = (3.0\times10^8\ \text{m/s})$
$\lambda = 0.060\ \text{m} = 6.0\ \text{cm}.$
The length of the antenna is half a wavelength, so the antenna length is 3.0 cm.
ASSESS: This antenna is just over an inch long.

80. **INTERPRET:** In this case, we need to relate the electric and magnetic fields in a laser beam.
DEVELOP: The fields in an electromagnetic wave are related by $E = cB$.
EVALUATE: For the peak fields
$E_\text{p} = cB_\text{p} = (3.0\times10^8\ \text{m/s})(2.8\times10^{-3}\ \text{T}) = 8.4\times10^5\ \text{V/m} = 0.84\ \text{MV/m}.$
Dielectric breakdown in air occurs at 3 MV/m, so this field is not strong enough to cause breakdown.
ASSESS: The largest that B_p could be and still just avoid breakdown is $B_\text{p} = \dfrac{E_\text{p}}{c} = \dfrac{3\times10^6\ \text{V/m}}{3.0\times10^8\ \text{m/s}} = 10^{-2}\ \text{T} = 10\ \text{mT},$ so the given wave is not very close to causing electrical breakdown of the air.

81. **INTERPRET:** This problem is about polarized light.
DEVELOP: The law of Malus applies, so $S = S_0\cos^2\theta$. Call S_0 the intensity of the light incident of the first polarizer, S_1 the intensity of the light emerging from the first polarizer (and incident on the second polarizer), and S_2 the intensity of light emerging from the second polarizer. Using the law of Malus and the given information, we know that
$S_1 = S_0\ \cos^2 60° = \tfrac{1}{4}S_0$
$S_2 = S_1\ \cos^2\theta = \left(\tfrac{1}{4}S_0\right)\cos^2\theta$
where θ is the angle between the polarizing axis of the *second* polarizer and the angle of polarization of the light incident on it (which is 60° from the vertical). We are given that $S_2 = 0.23S_0$, so
$\left(\tfrac{1}{4}S_0\right)\cos^2\theta = 0.23S_0$
EVALUATE: Solving $\left(\tfrac{1}{4}S_0\right)\cos^2\theta = 0.23S_0$ for θ gives
$\tfrac{1}{4}\cos^2\theta = 0.23$
$\theta = 16°.$
This is the angle of the second polarizer with respect to the light that passed through the first filter. That light was polarized at 60° from the vertical by the first polarizer, so there are two possibilities for the second polarizer. It could be at 60° + 16° = 76°, or it could be at 60° – 16° = 44°.
ASSESS: The intensity S_2 would be a maximum if $\theta = 0$, in which case the angles of the two polarizers would be aligned.

82. **INTERPRET:** This problem deals with intensity and power and the variation of intensity with distance.
DEVELOP: The intensity is $S = P/A$. The star and quasar have the same intensity S as viewed from Earth, but the power output of the quasar is 38 billion times that of the star, so $P_\text{quasar} = 38\times10^9 P_\text{star}$. At Earth, the intensity of the star is $S_\text{star} = P_\text{star}/4\pi r_\text{star}^2$ and the intensity of the quasar is $S_\text{quasar} = P_\text{quasar}/4\pi r_\text{quasar}^2$, where r_star and r_quasar are the distances of these bodies from Earth. We are given that the intensities are equal as viewed from Earth, so $P_\text{star}/4\pi r_\text{star}^2 = P_\text{quasar}/4\pi r_\text{quasar}^2$. Canceling and using $P_\text{quasar} = 38\times10^9 P_\text{star}$, we get $\dfrac{P_\text{star}}{r_\text{star}^2} = \dfrac{38\times10^9 P_\text{star}}{r_\text{quasar}^2}$. We can solve this equation for the distance of the quasar, r_quasar, which is what we want.

EVALUATE: Solving $\frac{P_{\text{star}}}{r_{\text{star}}^2} = \frac{38\times10^9 P_{\text{star}}}{r_{\text{quasar}}^2}$ for r_{quasar} gives

$r_{\text{quasar}} = r_{\text{star}}\sqrt{38\times10^9} = (4200\text{ ly})\sqrt{38\times10^9} = 8.2\times10^8\text{ ly} = 820\text{ Mly}.$

ASSESS: The distance of the quasar from Earth compared to the star's distance is $(8.2\times10^8\text{ ly})/(4200\text{ ly}) \approx 200{,}000$ times. So the quasar is 200,000 times farther from us than the star, yet it appears just as bright as the star.

83. **INTERPRET:** This problem deals with the intensity and power in an electromagnetic wave and how the intensity varies with distance.

DEVELOP: The average intensity is $\bar{S} = \frac{E_{\text{p}}^2}{2\mu_0 c}$, and we also know that the intensity is $\bar{S} = \bar{P}/A = \bar{P}/4\pi r^2$ for a source that radiates power uniformly in all directions. Therefore we can express the power as

$\bar{P} = \bar{S}A = \frac{E_{\text{p}}^2}{2\mu_0 c}\cdot 4\pi r^2 = \frac{2\pi E_{\text{p}}^2 r^2}{\mu_0 c}$. We want the average power $\bar{P}$, and we know that $E_{\text{p}} = 53.7$ mV/m = 0.0537 V/m at $r = 10.2$ km from the source.

EVALUATE: Using our result for $\bar{P}$ gives

$$\bar{P} = \frac{2\pi E_{\text{p}}^2 r^2}{\mu_0 c} = \frac{2\pi(0.0537\text{ V/m})^2(10{,}200\text{ m})^2}{(4\pi\times10^{-7}\text{T}\cdot\text{m/A})(3.00\times10^8\text{ m/s})} = 5{,}000\text{ W} = 5.00\text{ kW}.$$

ASSESS: According to the text, the peak electric field in this case (53.7 mV/m) is about 50 times as great as the field handled by a receiver in a cell phone tower.

84. **INTERPRET:** This problem involves radiation pressure and static equilibrium.

DEVELOP: The pressure exerted by a beam of electromagnetic waves is $p_{\text{rad}} = \frac{\bar{S}}{c}$ for a perfectly absorbing surface. We also know that $\bar{S} = \bar{P}/A$, where A is the cross-sectional area of the beam. (Careful here! Don't confuse p_{rad}, which is *pressure*, with $\bar{P}$, which is *power*!) To levitate the ping-pong ball, the force F_{rad} of the laser beam on it must equal its weight, so $F_{\text{rad}} = mg$. Since $p_{\text{rad}} = F_{\text{rad}}/A$, the force is $F_{\text{rad}} = p_{\text{rad}}A$. Therefore $p_{\text{rad}}A = mg$, which tells us that $\frac{\bar{S}}{c}A = mg$. But $\bar{S} = \bar{P}/A$, so $\left(\frac{\bar{P}/A}{c}\right)A = mg$, which gives $\bar{P} = mgc$.

EVALUATE: Using our previous result, we have

$\bar{P} = mgc = (0.0027\text{ kg})(9.81\text{ m/s}^2)(3.0\times10^8\text{ m/s}) = 7.9\times10^6\text{ W} = 7.9\text{ MW}.$

ASSESS: This is a lot of power just to balance a ping-pong ball! If the ball were painted white, making it a perfect reflector, the power could be cut in half, but it would still be very large.

Chapter 30: Alternate Problem Set in *Mastering Physics*

67. **INTERPRET:** In this problem, we are dealing with reflection at two intersecting plane mirrors.
DEVELOP: At reflection, the angle of incidence is equal to the angle of reflection. Light strikes the first mirror, bounces off of it, and then strikes the second mirror. We can think of the second mirror as having been rotated by ±0.5° from being perpendicular to the first mirror and see what effect this rotation has on the light reflected from the second mirror. Call θ the original angle of incidence with the second mirror if the mirrors were exactly perpendicular. If we now rotate the second mirror by 0.5° in a sense to increase the angle of incidence, the normal to the second mirror also rotates by 0.5°, so the angle of incidence is now $\theta + 0.5°$, but that is with respect to the rotated normal, which is 0.5° from the original normal. So the angle of reflection with respect to the *original* normal is $(\theta + 0.5°) + 0.5° = \theta + 1°$. If we rotated the second mirror the other way, the reflected ray would make an angle of $\theta - 1°$ with respect to the original normal.
EVALUATE: The angle of the second mirror is within ±0.5° of 90°, so the angle of the reflected ray is within ±2(0.5°) = ±1° of what it would be if the mirrors were perfectly perpendicular.
ASSESS: This relationship is true for other angles other than 0.5°: the angular error in the return beam is twice the angular error in the mirrors.

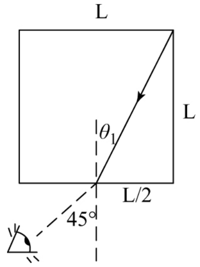
68. **INTERPRET:** This problem involves the speed of light in a transparent material.
DEVELOP: The speed of light in a material is related to the index of refraction of that material by $n = c/v$. From Table 30.1 we find that $n = 1.361$ for ethyl alcohol, so we can solve for v.
EVALUATE: $v = c/n = (2.9979\times10^8\ \text{m/s})/1.361 = 2.303\times10^8\ \text{m/s}$.
ASSESS: Our result gives $v < c$, as it should be. Note that we used a very precise value for c since we know n to 4 significant figures.

69. **INTERPRET:** This problem involves refraction of light.
DEVELOP: The light goes from glass into water. We use $n_1 \sin\theta_1 = n_2 \sin\theta_2$ (Snell's law). In this case, $n_1 = 1.52$ (glass), $n_2 = 1.333$ (water), $\theta_2 = 15.3°$, and we want θ_1.
EVALUATE: Applying $n_1 \sin\theta_1 = n_2 \sin\theta_2$ gives
$(1.52)\sin\theta_1 = (1.333)\sin(15.3°)$
$\theta_1 = 13.4°$.
ASSESS: We found that $\theta_2 > \theta_1$, so the light was bent away from the normal in the water. This is reasonable because $n_{\text{water}} < n_{\text{glass}}$.

70. **INTERPRET:** This problem deals with the critical angle for total internal reflection.
DEVELOP: At the critical angle θ_c, the angle of refraction is 90°, so Snell's law gives $n_1 \sin\theta_c = n_2 \sin(90°) = n_2$. Therefore $\sin\theta_c = n_2/n_1$. In this case, $\theta_c = 41.56°$, so $n_2/n_1 = \sin 41.56° = 0.6634$. Now consult Table 30.1 to find two materials whose refractive indices have this ratio.
EVALUATE: Avoid materials having similar indices of refraction since $n_2 \approx \frac{2}{3} n_1$. For diiodomethane and rutile, $n_2/n_1 = 1.738/2.62 = 0.6634$, so they have the proper ratio.
ASSESS: Any other materials having the ratio of $n_2/n_1 = 0.6634$ would also be possibilities.

71. **INTERPRET:** This problem involves Snell's law.
DEVELOP: In order to see through the opposite face of the cube, light from that face would have to exit the center of the cube at 45° with the normal on your side, since that is the direction you are looking. The largest angle of incidence in the cube would be for light rays from the corners that hit the center of the opposite face (by you), as

shown in the figure below. From that figure, we see that $\tan\theta_1 = \dfrac{L/2}{L} = 0.500$, which gives $\theta_1 = 26.565°$, which is the angle of incidence at the face of the cube by you. That light leaves the cube at an angle θ_2 on your side. Using Snell's law, $n_1 \sin\theta_1 = n_2 \sin\theta_2$, we can find θ_2. You are presumably in air, so $n_2 = 1.00$.



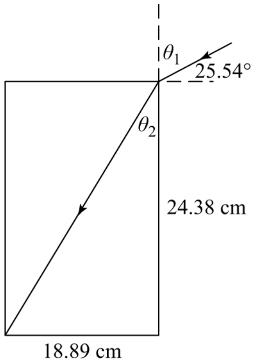
EVALUATE: Applying $n_1 \sin\theta_1 = n_2 \sin\theta_2$ gives
(1.52) sin 26.565° = (1.00) sin θ_2
$\theta_2 = 42.8°$.
The emerging light is not headed toward you, since your line of sight is 45° from the normal. Light from other parts of the opposite face would strike the normal inside the cube at angles even smaller than 26.565° and therefore would have angles of refraction in the air of even less than 42.8° and would thus not reach you. Therefore you could see no light from the opposite face of the cube.
ASSESS: If you moved closer to the normal, so your line of sight was 26.565° or less, you could see some light from the opposite face.

72. **INTERPRET:** In this problem, we must use the definition of the index of refraction as well as the critical angle.
DEVELOP: First use $n = c/v$ to find the index of refraction of the material. Then use $\sin\theta_c = n_2/n_1$ to find the critical angle θ_c. Use $n = 1.000$ for air.
EVALUATE: $n = c/v = c/(0.856c) = 1.168$. This is n_1, and $n_2 = 1.00$ for air. Now find θ_c.
$\sin\theta_c = n_2/n_1 = 1.00/1.168 = 0.8562$
$\theta_c = 58.9°$.
ASSESS: The critical angle would be different if the material (presumed to be a solid) were immersed in water.

73. **INTERPRET:** In this problem we are dealing with the critical angle and the polarizing angle.
DEVELOP: Call n_H the high index of refraction and n_L the low index of refraction, with similar notation for θ_H and θ_L. Total internal reflection occurs only when light goes from a high-n material to a low-n material, and in that case $\sin\theta_c = n_L/n_H$. At the polarizing angle of incidence, light goes from the low-n material to the high-n material, and the polarizing angle is given by $\tan\theta_p = n_H/n_L$.
EVALUATE: We know $\theta_c = 66.00°$, so
$\sin\theta_c = n_L/n_H$
$\sin(66.0°) = n_L/n_H = 0.9135$
At the polarizing angle, $\tan\theta_p = n_H/n_L$. We just saw that $n_L/n_H = 0.9135$, so $n_H/n_L = 1/0.9135 = 1.0946$.
Therefore
$\tan\theta_p = n_H/n_L = 1.0946$
$\theta_p = 47.6°$.
ASSESS: Note that $\theta_p \neq \theta_c$.

74. **INTERPRET:** This problem deals with the critical angle.
DEVELOP: At the critical angle, we have $\sin\theta_c = n_2/n_1$. In this case, $n_2 = 1.00$ (for the atmosphere) and $n_1 = 1.286$ (for the liquid methane). Solve for the critical angle θ_c.
EVALUATE: $\sin\theta_c = n_2/n_1 = 1.000/1.286 = 0.7776$
$\theta_c = 51.04°$.
ASSESS: Any light striking the surface of the lake from below at an angle of incidence greater than 51.04° would not escape the lake.

75. **INTERPRET:** This problem uses Snell's law since it deals with refraction.
DEVELOP: Refer to the figure below. The first light to reach the bottom of the tank would strike the upper right corner in the figure below. The angle of incidence for that light is $\theta_1 = 90° - 25.54° = 64.46°$. For the refracted light just to reach the bottom of the tank, the angle of refraction would have to be given by $\tan\theta_2 = \dfrac{18.89\text{ cm}}{24.38\text{ cm}} =$ 0.7748, so $\theta_2 = 37.77°$. Apply Snell's law, $n_1\sin\theta_1 = n_2\sin\theta_2$, at the surface of the liquid, with $n_1 = 1.000$ for air, and solve for n_2. Then compare this value to those in Table 30.1.



EVALUATE: Using $n_1\sin\theta_1 = n_2\sin\theta_2$ gives
$(1.000)\sin(64.46°) = n_2\sin(37.77°)$
$n_2 = 1.473$
From Table 30.1, we see that the liquid is glycerine.
ASSESS: As the sun continues to rise, θ_1 will decrease so more rays will reach the bottom of the tank.

76. **INTERPRET:** In this problem, we use the definition of index of refraction, but one which depends on depth in the material.
DEVELOP: Since $n = c/v$, the speed of light v will vary with depth x since n depends on x. Therefore $v(x) = c/n(x)$, and we are given $n(x)$. By the definition of v, we have $v = dx/dt$, so $dx = vdt$, which gives $\dfrac{dx}{v} = dt$. Using $v = c/n$, we can write this as $\dfrac{dx}{c/n(x)} = \dfrac{n(x)}{c}dx = dt$. Since we know $n(x)$, we can integrate to find the time T for the light to travel a distance d in the slab, so $\int_0^d \dfrac{n(x)}{c}dx = \int_0^T dt$.

EVALUATE: We carry out the integration in $\int_0^d \dfrac{n(x)}{c}dx = \int_0^T dt$. The time integral is just T. The integration over x is as follows.

$$T = \int_0^d \frac{n(x)}{c}dx = \int_0^d \frac{\left(n_1 + (n_2 - n_1)\sqrt{x/d}\right)}{c}dx = \frac{1}{c}\int_0^d n_1 dx + \frac{1}{c}\int_0^d (n_2 - n_1)\sqrt{x/d}\,dx$$

$$= \frac{n_1 d}{c} + \frac{n_2 - n_1}{c\sqrt{d}}\cdot\frac{d^{3/2}}{3/2} = \frac{(n_1 + 2n_2)d}{3c}.$$

ASSESS: First check units. n has no units, so the SI units of the final result are m/(m/s) = s, which are correct time units. Now check in the case where $n_1 = n_2 = n$. This gives $T = (n + 2n)d/3c = 3nd/3c = nd/c = (c/v)d/c = d/v$, which is correct for a constant speed v.

Chapter 31: Alternate Problem Set in *Mastering Physics*

85. **INTERPRET:** This problem deals with image formation by reflection at a plane mirror.

DEVELOP: The mirror is very small and is at floor level, so the only rays from your feet that reach it are at floor level and parallel to the floor. Since the mirror is tilted away from you at 35° with the vertical, these rays strike the mirror at 35° with respect to the normal, so the reflected rays leave the mirror at 35° with the normal. But since the normal is tilted at 35° above the floor, the reflected rays are traveling at 70° above the floor. For you to see your feet, the reflected rays must reach your eyes at 140 cm above the floor. Call x the distance of your feet from the mirror. Then $\tan 70° = \frac{140 \text{ cm}}{x}$. We want to solve for x, your distance from the mirror.

EVALUATE: Solve $\tan 70° = \frac{140 \text{ cm}}{x}$ for x, giving

x = (140 cm)/tan(70°) = 51 cm.

ASSESS: If the mirror were tilted at 45° with the vertical, the reflected rays would travel vertically, so no one could see their shoes. This might discourage sales!

86. **INTERPRET:** This problem is about image formation by a concave mirror.

DEVELOP: The mirror equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ applies. In this case, s = 12.5 cm and s' = –41.2 cm; it is negative because the image is behind the mirror. We want to find f.

EVALUATE: $\frac{1}{f} = \frac{1}{s} + \frac{1}{s'} = \frac{1}{12.5 \text{ cm}} + \frac{1}{-41.2 \text{ cm}}$

f = 17.9 cm.

ASSESS: In this situation, $s < f$, so the image should be virtual and behind the mirror, which it is.

87. **INTERPRET:** This problem is about image formation by a concave mirror.

DEVELOP: The mirror equation $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ applies, and the magnification is $M = -\frac{s'}{s}$. For a concave mirror, f is positive. In this case, $s = 2f$. We want to find the location, size, and characteristics of the image.

EVALUATE: **(a)** Solve $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ for s'.

$$\frac{1}{s'} = \frac{1}{f} - \frac{1}{s} = \frac{1}{f} - \frac{1}{2f} = \frac{1}{2f}$$

$s' = 2f$, so the image a distance $2f$ in front of the mirror.

(b) $M = -\frac{s'}{s} = -\frac{2f}{2f} = -1$, so the image is the same size as the object.

(c) s' is positive, so the image is real.

(d) M is negative, so the image is inverted.

ASSESS: The image is on the same side of the mirror as the object.

88. **INTERPRET:** This problem is about the image formed by a convex lens.

DEVELOP: A convex lens is a converging lens, so it has a positive focal length. The lens equation $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ applies, and the magnification is $M=-\frac{s'}{s}$. The image is real and inverted, so s' is positive and the magnification is negative. Since the image is twice as tall as the object, $M = -2$. We want to find the location of the object, s.

EVALUATE: $M=-\frac{s'}{s}=-2$, so $s' = 2s$. Now use $\frac{1}{f}=\frac{1}{s}+\frac{1}{s'}$ to find s.

$$\frac{1}{f}=\frac{1}{s}+\frac{1}{2s}$$

$$\frac{1}{f}=\frac{3}{2s}$$

$$s=\frac{3f}{2}$$

ASSESS: Check by substituting the result into the lens equation using $s = 3f/2$ and $s' = 2s = 2(3f/2) = 3f$.

$\frac{1}{3f/2}+\frac{1}{3f}=\frac{2}{3f}+\frac{1}{3f}=\frac{1}{3f}(2+1)=\frac{1}{f}$, so our result checks.

89. **INTERPRET:** This problem involves the image formed by a lens.

DEVELOP: We use the lens formula $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to find the location s' of the image. Then use $M=-\frac{s'}{s}$ to find the magnification. We know that $f = 28.5$ cm and $s = 9.50$ cm, and we want M.

EVALUATE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives

$$\frac{1}{9.50\text{ cm}}+\frac{1}{s'}=\frac{1}{28.5\text{ cm}}$$

$s' = -14.25$ cm.

Now find the magnification M.

$$M=-\frac{s'}{s}=-\frac{-14.25\text{ cm}}{9.50\text{ cm}}=+1.50.$$

The image is 1.50 times the height of the object.

ASSESS: A magnifying glass is often used for reading small print, so the image must be upright, which it is since M is positive.

90. **INTERPRET:** This problem is about the apparent depth of an object that is submerged in water.

DEVELOP: We can think of the water as a surface having an infinite radius of curvature. In that case, the equation $\frac{n_1}{s}+\frac{n_2}{s'}=\frac{n_2-n_1}{R}$ becomes $\frac{n_1}{s}+\frac{n_2}{s'}=0$ since $R\to\infty$. For this situation, we have $n_1 = 1.333$ (for water), $n_2 = 1.000$ (for air), and $s = 22.0$ cm. We want to find the location of the image, s'.

EVALUATE: Solving $\frac{n_1}{s}+\frac{n_2}{s'}=0$ for s' gives

$$\frac{1.333}{22.0\text{ cm}}=\frac{1.000}{s'}$$

$s' = -16.5$ cm.

Your feet appear to be 16.5 cm below the surface of the water.

ASSESS: The fact that s' is negative tells us that the image is virtual and the magnification is positive, so the image is upright.

91. **INTERPRET:** This problem involves the lens equation, lens power, and knowledge of image formation by the eye.
DEVELOP: Since you view the smartphone comfortably with the reading glasses, they must be forming an image of the phone at the near point of your eye. We use $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to find the location of your near point. That is where you would have to put the phone without the glasses to view it comfortably. We know that the power of a lens is $P = 1/f$, and that $P = 2.25$ diopters for the reading glasses. We also know that $s = 25$ cm with the glasses on. We want to find the distance to your near point, which is s' when you are wearing the glasses.
EVALUATE: First get the focal length of the lenses in the reading glasses using $P = 1/f$. Solving $1/f = 2.25$ diopters for f gives

$$f=\frac{1}{2.25}\text{ m }=0.444\text{ m}=44.4\text{ cm}.$$

Now apply $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ to find s'.

$$\frac{1}{25\text{ cm}}+\frac{1}{s'}=\frac{1}{44.4\text{ cm}}$$

$s' = -57$ cm, so the near point of your unaided eye is 57 cm in front of the eye. You should hold the smartphone 57 cm from your eye without the glasses.
ASSESS: A near point of 57 cm is not unusual. The image distance is negative because the glasses must form a virtual image in front of your eye at the near point.

92. **INTERPRET:** This problem is about a compound microscope.
DEVELOP: The magnification is $M=-\frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right)$. We know $f_o = 6.12$ mm and $f_e = 1.75$ cm, and we want to find the lens separation L so that $|M| = 275$.
EVALUATE: We disregard the minus sign since we are interested only in the magnitude of the magnification.
Using $M=\frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right)$ gives

$$275=\frac{L}{0.612\text{ cm}}\left(\frac{25\text{ cm}}{1.75\text{ cm}}\right)$$

$L = 11.8$ cm.
ASSESS: A length of 11.8 cm is about 4.6 inches, which is a bit small for a compound microscope. We could increase the length by increasing f_o, f_e, or both.

93. **INTERPRET:** This problem is about the image formed by a concave mirror.
DEVELOP: The mirror equation $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ applies, and the magnification is $M=-\frac{s'}{s}$. In this situation, $s = 15.0$ cm and $f = +25.0$ cm for a concave mirror. We want to know the type of image formed, its location, and the magnification.
EVALUATE: Using $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$ gives

$$\frac{1}{15.0\text{ cm}}+\frac{1}{s'}=\frac{1}{25.0\text{ cm}}$$

$s' = -37.5$ cm.

$$M=-\frac{s'}{s}=-\frac{-37.5\text{ cm}}{15.0\text{ cm}}=+2.50$$

Since s' is negative, the image is virtual, and since $s' = -37.5$ cm, the image is 37.5 cm behind the mirror. $M = +2.50$, so the image is upright and 2.50 times as large as the object.
ASSESS: The characteristics we have just found agree with those summarized in Table 31.1 when $s < f$, which is the case here.

94. **INTERPRET:** This problem deals with a compound microscope.

DEVELOP: The magnification is $M = -\frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right)$. We know that $M = 375$, $L = 10.0$ cm, and $f_e = 1.90$ cm, and we want to find the focal length f_o of the objective lens.

EVALUATE: We are interested only in the magnitude of the magnification, so we use $M = \frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right)$. This gives

$$375 = \frac{10.0\text{ cm}}{f_o}\left(\frac{25\text{ cm}}{1.90\text{ cm}}\right)$$

$f_o = 0.351$ cm $= 3.51$ mm.

ASSESS: A microscope normally views small objects that are placed very close to the objective lens, so the focal length of that lens can be as small as several millimeters, as we found here.

Chapter 32: Alternate Problem Set in *Mastering Physics*

75. **INTERPRET:** This problem is about the interference of light from two slits.

DEVELOP: For distances far from the slits compared to the separation of the slits, the formula $y = m\dfrac{\lambda L}{d}$ applies. In this problem, L = 75.0 cm and d = 25.0 μm, which means that $L >> d$ so we can apply the formula. For adjacent fringes, m varies by ±1, so the fringe separation Δy is $\Delta y = \Delta m\dfrac{\lambda L}{d} = (1)\dfrac{\lambda L}{d} = \dfrac{\lambda L}{d}$. We are given that Δy = 1.72 cm, and we want to find the wavelength λ of the light.

EVALUATE: Using $\Delta y = \dfrac{\lambda L}{d}$ gives

1.72 cm = $\dfrac{\lambda(75.0\text{ cm})}{25.0\times10^{-6}\text{ m}}$

λ = 5.73 $\times 10^{-7}$ m = 573 nm.

ASSESS: This wavelength is in the visible range of 400 nm – 700 nm.

76. **INTERPRET:** This problem involves the interference of light passing through 3 slits.

DEVELOP: The maxima are given by $d\sin\theta = m\lambda$, and the minima are given by $d\sin\theta = \left(m+\frac{1}{3}\right)\lambda$ and $d\sin\theta = \left(m+\frac{2}{3}\right)\lambda$. We know that the first maximum is at 15°, so the minima within 15° are found when m = 0, and there will be 2 of them. Since the first maximum is at 15°, we know that $d\sin15° = \lambda$, so λ/d = sin15° = 0.2588.

EVALUATE: The first minimum is at $d\sin\theta = \left(m+\frac{1}{3}\right)\lambda$ where m = 0, so

$d\sin\theta = \frac{1}{3}\lambda$

$\sin\theta = \frac{1}{3}\lambda/d = \frac{1}{3}(0.2588)$

θ = 4.95° ≈ 5°.

The second minimum is at $d\sin\theta = \left(m+\frac{2}{3}\right)\lambda$ where m = 0, so

$d\sin\theta = \frac{2}{3}\lambda$

$\sin\theta = \frac{2}{3}\lambda/d = \frac{2}{3}(0.2588)$

θ = 9.9° ≈ 10°.

ASSESS: For small angles, the minima are approximately equally spaced, but this is not true for larger angles.

77. **INTERPRET:** This problem deals with thin-film interference.

DEVELOP: The reflected light is strongly reflected, so constructive interference has occurred. Therefore the formula $2nd = \left(m+\frac{1}{2}\right)\lambda$ applies. We want to find n and use it to identify the substance making up the film.

We are given that d = 95.2 nm and λ = 661.8 nm. For the thinnest film, m = 0. We can find n and then compare it to the substances in Table 30.1.

EVALUATE: Using $2nd = \left(m+\frac{1}{2}\right)\lambda$ gives

$2n$(95.2 nm) = $\frac{1}{2}$(661.8 nm)

n = 1.738.

From Table 30.1, we see that diiodomethane has precisely this index of refraction, so it is the material of the film.
ASSESS: It is conceivable that other materials, not shown in the table, have the same index of refraction, but no others in Table 30.1 have that value.

78. **INTERPRET:** This problem is about the angular resolution of a telescope mirror determined by the diffraction limit and the Rayleigh criterion for a circular aperture.

DEVELOP: The Rayleigh criterion for circular apertures is $\theta_{\text{min}} = \dfrac{1.22\lambda}{D}$. We are given that λ = 1.8 μm and D = 6.0 m, and we want to find θ_{min}.

EVALUATE: Using $\theta_{\text{min}} = \dfrac{1.22\lambda}{D}$ gives

$$\theta_{\text{min}} = \frac{1.22(1.8\times10^{-6}\text{ m})}{6.0\text{ m}} = 3.7\times10^{-7}\text{ rad} = 0.37\text{ μrad.}$$

ASSESS: The resolution could be better using visible light because its wavelengths are all less than those of infrared light, and violet light would be better than red light.

79. **INTERPRET:** This problem involves double-slit interference.
DEVELOP: The bright fringes occur when $d\sin\theta = m\lambda$. For the third-order fringe, m = 3. We are given that λ = 435 nm and θ = 35.8°, and we want to find the slit spacing d.
EVALUATE: Using $d\sin\theta = m\lambda$ gives
$d\sin(35.8°) = 3(435\times10^{-9}\text{ m})$
$d = 2.23\times10^{-6}\text{ m} = 2.23\text{ μm}$.
ASSESS: Although d is small on a macroscopic scale, it is much larger than the wavelength of the light.

80. **INTERPRET:** This problem is about double-slit interference.
DEVELOP: For bright fringes, $d\sin\theta = m\lambda$ and for dark fringes $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda$. We know that the wavelength of one spectral line is 568 nm. Call λ the wavelength of the other spectral line.
EVALUATE: For the fourth-order bright fringe of the known wavelength line, we know that $d\sin\theta = 4\lambda$ = 4(568 nm) = 2272 nm. For the fifth-order dark fringe of the unknown wavelength we know that $d\sin\theta = \left(5+\frac{1}{2}\right)\lambda$ = 5.50 λ. These two fringes occur at the same place, so θ is the same for both of them. Equating the two expressions for $d\sin\theta$ gives
5.50 λ = 2272 nm
λ = 413 nm.
ASSESS: Both spectral lines are in the visible region of 400 nm – 700 nm.

81. **INTERPRET:** This problem is about a diffraction grating.
DEVELOP: The interference maxima occur when $d\sin\theta = m\lambda$. We know that the third-order (m = 3) maximum for 656-nm light occurs at 37.1°. We can use this information to find d. Then we use that result to find the width of 7780 lines.
EVALUATE: Using $d\sin\theta = m\lambda$ gives
$d\sin(37.1°) = 3(656\text{ nm})$
d = 3263 nm.
The width of 7780 lines is (7780)(3262 nm) = 2.54×10^{7} nm = 0.0254 m = 2.54 cm.
ASSESS: This grating is an inch wide, which is typical for gratings in physics optics labs.

82. **INTERPRET:** This problem is about the resolution of a telescope using the Rayleigh criterion.

DEVELOP: For a spherical aperture, $\theta_{\text{min}} = \dfrac{1.22\lambda}{D}$. For the Thirty-Meter Telescope, D = 30 m. We are using light of wavelength 1.0 μm, so we can find θ_{min}. Next we can calculate the angle that an Earth-like planet would subtend if it were 4 light-years away, and finally we compare this angle to the angular resolution of the Thirty-Meter Telescope. The radius of Earth is 6.37×10^{6} m and 1 ly = 9.46×10^{15} m.

EVALUATE: The angular resolution of the telescope is

$$\theta_{\min} = \frac{1.22\lambda}{D} = \frac{1.22(1.0\times10^{-6}\ \text{m})}{30\,\text{m}} = 4.07\times10^{-8}\ \text{rad}.$$

The angle Earth would subtend if it were 4 ly away is

$$\theta_{\text{Earth}} = \frac{D_{\text{Earth}}}{r} = \frac{2(6.37\times10^{6}\ \text{m})}{4(9.46\times10^{15}\ \text{m})} = 3.37\times10^{-10}\ \text{rad}.$$

Since $\theta_{\text{Earth}} < \theta_{\min}$, this telescope could *not* resolve Earth-like planets that are 4 light-years away.

ASSESS: How close would Earth-like planets have to be to be resolved by this telescope? Solving $\theta_{\text{Earth}} = \frac{D_{\text{Earth}}}{r}$ for *r*, gives $r = \frac{D_{\text{Earth}}}{\theta_{\min}}$. The distance would be

$$r = \frac{D_{\text{Earth}}}{\theta_{\min}} = \frac{2(6.37\times10^{6}\ \text{m})}{4.07\times10^{-8}\,\text{rad}} = 3.13\times10^{14}\ \text{m} = 0.033\ \text{ly}.$$ Unfortunately there are no stars this close to us.

83. **INTERPRET:** This problem is about thin-film interference.
DEVELOP: The reflected violet light is visible so constructive interference has occurred. For that condition, $2nd = \left(m+\frac{1}{2}\right)\lambda$. We know that λ = 395 nm and d = 55.2 nm, and we want to find n. In this case, m = 0 because this is the last visible color to be seen.
EVALUATE: Using $2nd = \left(m+\frac{1}{2}\right)\lambda$ gives

$2n(55.2\ \text{nm}) = \frac{1}{2}(395\ \text{nm})$

$n = 1.79.$

ASSESS: The index of refraction is greater than 1, as it must be, and a value of 1.79 is very reasonable for many materials as Table 30.1 shows.

84. **INTERPRET:** This problem is about the angular resolution of a telescope mirror using the Rayleigh criterion.
DEVELOP: We know that at 1200 km, the mirrors could resolve an object that is 40 cm in diameter using 660-nm light. We can use this information to find $\theta_{\min}$ for the mirrors. Then we can use the Rayleigh criterion $\theta_{\min} = \frac{1.22\lambda}{D}$ to find the diameter D of each of the mirrors.
EVALUATE: The angle the telescopes could resolve from an altitude of 1200 km is

$$\theta_{\min} = \frac{40\ \text{cm}}{1.2\times10^{6}\,\text{m}} = \frac{0.40\ \text{m}}{1.2\times10^{6}\,\text{m}} = 3.33\times10^{-7}\ \text{rad}.$$ This is their angular resolution. Applying this result to the Rayleigh criterion to find D gives

$$\theta_{\min} = \frac{1.22\lambda}{D}$$

$$D = \frac{1.22\lambda}{\theta_{\min}} = \frac{1.22(6.60\times10^{-9}\ \text{m})}{3.33\times10^{-7}\ \text{rad}} = 2.4\ \text{m}.$$

ASSESS: These mirrors are the same size as the mirror in the Hubble Space Telescope.

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80. **INTERPRET:** This problem involves interferometry. The scientists think that the light travels at different speeds perpendicular to the direction of motion of Earth and parallel to the direction of the motion.

DEVELOP: In the original interference pattern, the bright fringes are a distance λ apart. Due to the (believed) differences in the speed of light, the light traveling perpendicular to the direction of motion of Earth travels at a different speed from the light traveling perpendicular to at motion. Therefore there should be a time delay in the arrival of the two beams, resulting in an interference pattern. This time delay is equivalent to a path difference between the two beams. We first find the time delay between the beams, and then express it as a fraction of the period of the light. The fractional change in the period will be the same as the fractional change in the expected path difference, and this will be the distance that the bright fringes are expected to be shifted. The light traveling perpendicular to the direction of Earth's motion takes time $t_{\perp} = \dfrac{2L}{\sqrt{c^2 - v^2}}$ to travel a distance of $2L$, where v is the orbital speed of the Earth. For the light traveling parallel to Earth's motion, the time is $t_{\parallel} = \dfrac{L}{c-v} + \dfrac{L}{c+v}$ = $\dfrac{2cL}{c^2 - v^2}$. The time difference between the arrival of the two beams, Δt, is $\Delta t = t_{\parallel} - t_{\perp}$, which gives Δt = $\dfrac{2cL}{c^2 - v^2}$ $- \dfrac{2L}{\sqrt{c^2 - v^2}} = 2L\left(\dfrac{c}{c^2 - v^2} - \dfrac{1}{\sqrt{c^2 - v^2}}\right)$. The fraction that Δt is of the period T of the light is $\Delta t / T$. We know that $f\lambda = c$ and $f = 1/T$, so $T = \lambda / c$. Therefore we get $\dfrac{\Delta t}{T} = \dfrac{2L}{\lambda / c}\left(\dfrac{c}{c^2 - v^2} - \dfrac{1}{\sqrt{c^2 - v^2}}\right) = \dfrac{2Lc}{\lambda}\left(\dfrac{c}{c^2 - v^2} - \dfrac{1}{\sqrt{c^2 - v^2}}\right)$.

EVALUATE: First find $\Delta t / T$. From Appendix E, we know that the orbital speed of Earth is v = 29,800 m/s. Using our results above, we have

$$\frac{\Delta t}{T} = \frac{2Lc}{\lambda}\left(\frac{c}{c^2 - v^2} - \frac{1}{\sqrt{c^2 - v^2}}\right)$$

$$\frac{\Delta t}{T} = \frac{2(11\text{ m})(3.00\times 10^8\text{ m/s})}{550\times 10^{-9}\text{ m}}\left(\frac{3.00\times 10^8\text{ m/s}}{(3.00\times 10^8\text{ m/s})^2 - (29{,}800\text{ m/s})^2} - \frac{1}{\sqrt{(3.00\times 10^8\text{ m/s})^2 - (29{,}800\text{ m/s})^2}}\right)$$

$$\frac{\Delta t}{T} = 0.20.$$

This result means that the scientists expected the bright fringes to move $0.20\,\lambda$ from their original positions.

ASSESS: Such small path differences are well within the capability of modern Michelson interferometers. But of course no change in the pattern would be observed since the speed of light is c in all directions.

81. **INTERPRET:** This problem involves length contraction.

DEVELOP: The proper length is 55 ly as measured in the frame of the two stars at rest. The formula $\Delta x' = \Delta x\sqrt{1 - v^2/c^2}$ applies here, with Δx = 55 ly and $\Delta x'$ = 27 ly, and we want to find v in terms of c.

EVALUATE: Using $\Delta x' = \Delta x\sqrt{1-v^2/c^2}$ gives

27 ly = (55 ly)$\sqrt{1-v^2/c^2}$

$(27/55)^2 = 1 - (v/c)^2$

$v/c = 0.87$

$v = 0.87c.$

ASSESS: The distance is greater in the rest frame of the stars. From the rocket's point of view, the stars are moving at $0.87c$, so the distance between them has contracted.

82. **INTERPRET:** This problem involves length contraction.

DEVELOP: The formula $\Delta x' = \Delta x\sqrt{1-v^2/c^2}$ applies here, with Δx = 75.0 m and $v = 0.900c$, and we want to find $\Delta x'$.

EVALUATE: $\Delta x' = (75.0\text{ ly})\sqrt{1-(0.900c/c)^2}$ = 32.7 m.

ASSESS: The length you measure is less than the proper length, as it should be.

83. **INTERPRET:** This problem deals with relativistic momentum.

DEVELOP: The object is moving along a straight line, so we write momentum along that line. The relativistic momentum is $p = \dfrac{mu}{\sqrt{1-u^2/c^2}}$. When the speed has increased by 5.5%, the momentum has increased by 12%. Thus the new momentum p_2 and the new speed u_2 are $u_2 = u + 0.055u = 1.055u$ and $p_2 = p + 0.12p = 1.12p$. The momentum is then $1.12p = \dfrac{m(1.055u)}{\sqrt{1-(1.055u)^2/c^2}}$. We want to find the original speed u.

EVALUATE: If we take the ratio of the new momentum to the original momentum, m and p will cancel out, leaving u as the only unknown. This gives

$$\frac{1.12p}{p} = \frac{\dfrac{m(1.055u)}{\sqrt{1-(1.055u)^2/c^2}}}{\dfrac{mu}{\sqrt{1-u^2/c^2}}}$$

$$1.12 = 1.055\frac{\sqrt{1-u^2/c^2}}{\sqrt{1-(1.055u)^2/c^2}}$$

$$\left(\frac{1.12}{1.055}\right)^2 = \frac{1-u^2/c^2}{1-(1.055u)^2/c^2} = \frac{1-u^2/c^2}{1-1.113u^2/c^2}.$$

Collecting terms and solving for u gives

$u = 0.71c.$

ASSESS: The nonrelativistic momentum $p = mu$ would increase by 5.5% if the speed increased by 5.5%, but the percent increase in the relativistic momentum is over twice as great for the same percent increase in the speed.

84. **INTERPRET:** This problem involves time dilation.

DEVELOP: The formula $\Delta t' = \Delta t\sqrt{1-v^2/c^2}$ applies. To narrow the age gap from 45 y to 10 y, you must age 35 y less than your beloved on Earth, so $\Delta t' = \Delta t - 35\text{ y}$, where Δt is the time elapsed in the Earth frame and $\Delta t'$ is the time elapsed in the rocket frame. Therefore $\Delta t - 35\text{ y} = \Delta t\sqrt{1-v^2/c^2}$. We are given that $v = 0.90c$. We want to find Δt, $\Delta t'$, and the distance from Earth you should travel.

EVALUATE: First solve $\Delta t - 35\text{ y} = \Delta t\sqrt{1-v^2/c^2}$ for Δt.

$$\Delta t\left(1-\sqrt{1-v^2/c^2}\right) = 35\text{ y}$$

$$\Delta t = \frac{35\text{ y}}{1-\sqrt{1-(0.90c)^2/c^2}} = 62\text{ y}.$$

This means that on Earth, 62 y have elapsed during your trip, so your beloved has aged 62 y. The time in the rocket is $\Delta t' = \Delta t - 35\text{ y} = 62\text{ y} - 35\text{ y} = 27\text{ y}$, which is the amount you have aged during the trip. To find the distance you travel, use the Earth reference frame since we know the rocket speed in that frame. The time to reach your maximum distance from Earth is $\frac{1}{2}(62\text{ y}) = 31\text{ y}$. The distance your rocket travels is $v\,\Delta t = (0.90c)(31\text{ y}) = 28\text{ ly}$.

The final answers are

(a) 28 ly **(b)** 62 y **(c)** 27 y.

ASSESS: To you the distance traveled will be less than 28 ly because it has been length-contracted.

85. **INTERPRET:** This problem involves time dilation.

DEVELOP: The formula $\Delta t' = \Delta t\sqrt{1 - v^2/c^2}$ applies. First find the time interval $\Delta t'$ in the sample's reference frame, which is the proper frame. In your frame it is $\Delta t = 15.0$ min, and $v = 0.924c$. During each half-life, the number of atoms decreases by a factor of one-half.

EVALUATE: Find the time interval $\Delta t'$ in the sample's rest frame.

$\Delta t' = \Delta t\sqrt{1 - v^2/c^2} = (15.0\text{ min})\sqrt{1 - (0.924c)^2/c^2} = 5.736$ min.

In 5.736 min, the number of atoms decreased from 1000 to 125. To see how many half-lives this took, look at what happens during each half-life. After the first half-life there would be 500 atoms left, after the second half-life there would be 250 atoms left, and after the third half-life there would be 125 atoms left. So it took 3 half-lives for the number of atoms to decrease from 1000 to 125, and this time was 5.736 min. Therefore the half-life is one-third of this time, or (5.736 min)/3 = 1.91 min.

ASSESS: In your reference frame, it took 15.0 min for three half-lives, so the half-life would be 5.00 min.

86. **INTERPRET:** This problem requires use of the Lorentz transformation equation for time.

DEVELOP: Call the Earth reference frame the unprimed frame with the origin at Earth and the x-axis pointing from Earth to the sun, and let the rocket frame be the primed frame. The Lorentz transformation equation for time is $t' = \gamma\left(t - vx/c^2\right)$. In the Earth frame, $t_B - t_A = +2.45$ min because event A occurs 2.45 min before event B. The events occur $8.33\ \text{l}\cdot\text{min}$ apart, so $x_B - x_A = 8.33\ \text{l}\cdot\text{min} = 8.33c$ min. In the rocket (primed) frame, $t'_B - t'_A =$ –5.74 min because event B occurs 5.74 min before event A. Using the Lorentz transformation equation to get $t'_B - t'_A$ gives

$$t'_B - t'_A = \gamma\left[\left(t_B - vx_B/c^2\right) - \left(t_A - vx_A/c^2\right)\right] = \gamma\left[\left(t_B - t_A\right) - \left(\frac{v}{c^2}\left(x_B - x_A\right)\right)\right].$$

EVALUATE: Putting in the numbers for the time and position intervals gives

$-5.74\text{ min} = \gamma(+2.45\text{ min} - 8.33\text{ min } v/c)$.

Calling $v/c \equiv u$, we have

$$-5.74 = \frac{1}{\sqrt{1-u^2}}(2.45 - 8.33u)$$

$$-5.74\sqrt{1-u^2} = 2.45 - 8.33u$$

Squaring and solving for u gives the equation

$3.105u^2 - 1.239u - 0.8178 = 0$.

Using the quadratic formula and taking the positive root gives $u = 0.75$, so

$v = 0.75c$.

ASSESS: Careful! We cannot use the formula $\Delta t' = \Delta t\sqrt{1 - v^2/c^2}$ because events A and B did not occur at the same point in either reference frame.

87. **INTERPRET:** In this problem, we want to calculate the space-time interval in two different reference frames. We will also need to use the Lorentz transformation equation for x.

DEVELOP: Call the Earth reference frame the unprimed frame with the origin at Earth and the x-axis pointing from Earth to the sun, and let the rocket frame be the primed frame. In the Earth frame, $t_B - t_A$ = +2.45 min because event A occurs 2.45 min before event B. The events occur 8.33 $1\cdot\text{min}$ apart, so $x_B - x_A$ = 8.33 $1\cdot\text{min}$ = 8.33c min. In the rocket (primed) frame, $t'_B - t'_A$ = –5.74 min because event B occurs 5.74 min before event A. For motion along only the x-axis, the space-time interval is $\Delta s^2 = c^2(\Delta t)^2 - (\Delta x)^2$. In the rocket frame, we will need to use the Lorentz transformation equation to find x'_B, which is $x'_B = (x_B - vt_B)\gamma$. For this, we need to find the speed of the rocket. Using the Lorentz transformation equation to get $t'_B - t'_A$ gives

$t'_B - t'_A = \gamma\left[\left(t_B - vx_B/c^2\right) - \left(t_A - vx_A/c^2\right)\right] = \gamma\left[\left(t_B - t_A\right) - \left(\frac{v}{c^2}\left(x_B - x_A\right)\right)\right]$. Since we know the time intervals as well as $x_B - x_A$, we can solve for the rocket speed v.

EVALUATE: First find the rocket speed. We just found that

$t'_B - t'_A = \gamma\left[\left(t_B - t_A\right) - \left(\frac{v}{c^2}\left(x_B - x_A\right)\right)\right]$. Putting in the numbers for the time and position intervals gives

–5.74 min = γ (+2.45 min – 8.33 min v/c).

Calling $v/c \equiv u$, we have

$$-5.74 = \frac{1}{\sqrt{1-u^2}}(2.45 - 8.33u)$$

$$-5.74\sqrt{1-u^2} = 2.45 - 8.33u$$

Squaring and solving for u gives the equation

$3.105u^2 - 1.239u - 0.8178 = 0$.

Using the quadratic formula and taking the positive root gives $u = 0.75$, so

$v = 0.75c$. Now we can find the space-time intervals in the two frames.

(a) In the Earth frame: Δt = +2.45 min and Δx = 8.33 $1\cdot\text{min}$ = 8.33c min, so we have

$\Delta s^2 = c^2(\Delta t)^2 - (\Delta x)^2 = c^2(2.45\text{ min})^2 - (8.33c\text{ min})^2 = -63.4\ (1\cdot\text{min})^2$.

(b) In the rocket frame: $\Delta t'$ = –5.74 min. We use $x'_B = (x_B - vt_B)\gamma$ to find x'_B.

$$x'_B = (x_B - vt_B)\gamma = \frac{8.33\ 1\cdot\text{min} - (0.75\ c)(2.45\text{ min})}{1 - \sqrt{1-(0.75c)^2/c^2}} = 9.816\ 1\cdot\text{min}$$

$\Delta s'^2 = c^2(\Delta t')^2 - (\Delta x')^2 = c^2(-5.74\text{ min})^2 - (9.816\ 1\cdot\text{min})^2 = -63.4\ (1\cdot\text{min})^2$.

We see what the space-time interval is –63.4 $(1\cdot\text{min})^2$ in both reference frames.

ASSESS: We call the space-time interval an *invariant* because it is the same in all reference frames.

88. **INTERPRET:** In this problem, we use relativistic momentum and relativistic kinetic energy.

DEVELOP: First use $p = \frac{mu}{\sqrt{1-u^2/c^2}}$ to find v since we know the momentum and mass of the proton. Next use this u in $K = (\gamma - 1)mc^2$ to find the kinetic energy. The rest energy of a proton is 938 MeV.

EVALUATE: **(a)** Solving $p = \frac{mu}{\sqrt{1-u^2/c^2}}$ for u gives

$$u^2/c^2 = \frac{p^2}{(mc)^2 + p^2} = \frac{1}{(mc/p)^2 + 1} = \frac{1}{\left[\frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{5.84\times10^{-19}\text{ kg}\cdot\text{m/s}}\right]^2 + 1}$$

$u = 0.759c$.

(b) $K = (\gamma - 1)mc^2 = \left(\dfrac{1}{\sqrt{1-u^2/c^2}} - 1\right)mc^2 = \left(\dfrac{1}{\sqrt{1-(0.759)^2}} - 1\right)(938 \text{ MeV})$

$K = 502$ MeV.

ASSESS: The proton's kinetic energy is a little over half of its rest energy.

89. **INTERPRET:** This problem involves time dilation.

DEVELOP: Of your allotted 6 years to complete your PhD, 2 years are spent at rest on Earth, and 3 months (0.25 year) are spent on the planet, which is at rest relative to Earth. This leaves you only 6.00 y – 2.25 y = 3.75 y to make the round trip to the planet and back. It takes light 32.0 y to make the round trip, so you will need to travel fast enough so that the time elapsed for you is only 3.75 y. We assume that you travel the same speed v both going and returning. As observed from Earth, the distance you travel is 32.0 ly, so $v\,\Delta t$ = 32.0 ly, which gives Δt = (32 ly)/v. Using $\Delta t' = \Delta t\sqrt{1-v^2/c^2}$, $\Delta t'$ is the time interval as observed in the rocket, so $\Delta t'$ = 3.75 y. Combining the two equations involving the time intervals gives $\Delta t' = \dfrac{\Delta t}{v}\sqrt{1-v^2/c^2}$. We know Δt and $\Delta t'$, so we can solve for v, the rocket speed, which is what we want to find.

EVALUATE: Using $\Delta t' = \dfrac{\Delta t}{v}\sqrt{1-v^2/c^2}$ gives 3.75 y = $\dfrac{32.0 \text{ ly}}{v}\sqrt{1-v^2/c^2}$. We can express 32.0 ly as 32.0c y, so we get

3.75 y = $\dfrac{32.0c \text{ y}}{v}\sqrt{1-v^2/c^2} = \dfrac{32.0 \text{y}}{v/c}\sqrt{1-v^2/c^2}$

Calling $z \equiv v/c$, this equation becomes

3.75 = $\dfrac{32.0}{z}\sqrt{1-z^2}$

Squaring and solving for z gives

$z^2[1 + (3.75/32)^2] = 1$

$z = 0.993$, so $v = 0.993c$.

ASSESS: As viewed from Earth, the total round trip will take time $\Delta t = x/v$, so Δt = (32.0 ly)/(0.993c) = 32.2 y. But for you in the rocket, it will last only 3.75 y. It took you 6 y to get your PhD, but the time on Earth is 32.2 y + 2.25 y = 34.45 y, which must set some sort of record for length!

Chapter 34: Alternate Problem Set in *Mastering Physics*

88. **INTERPRET:** This problem is about blackbody radiation, so the Stefan-Boltzmann law applies.

DEVELOP: The radiated power of an ideal blackbody is $P = \sigma A T^4$. Call T_1 the initial temperature of the blackbody and T_2 its final temperature. At T_2 it radiates twice as much power as at T_1, so $P_2 = 2P_1$. We want to find the factor by which the temperature increased, so we want T_2/T_1.

$\lambda_{\text{median}} T = 4.11\ \text{mm}\cdot\text{K}$

EVALUATE: Take the ratio of the power at the two temperatures, knowing that $P_2 = 2P_1$.

$$\frac{P_2}{P_1} = \frac{2P_1}{P_1} = \frac{\sigma A T_2^4}{\sigma A T_1^4} = \frac{T_2^4}{T_1^4}$$

$$2 = \frac{T_2^4}{T_1^4}$$

$T_2 = 2^{1/4} T_1 \approx 1.19 T_1$.

ASSESS: Because the power is proportional to T^4, a small increase in temperature produces a larger increase in the radiated power.

89. **INTERPRET:** This problem is about the median wavelength radiated by blackbody radiation.

DEVELOP: The median wavelength is given by $\lambda_{\text{median}} T = 4.11\ \text{mm}\cdot\text{K}$. We know that λ_{median} = 1.0 μm, and we want to find the temperature T.

EVALUATE: λ_{median} = 1.0 μm = 1.0×10^{-3} mm, so using $\lambda_{\text{median}} T = 4.11\ \text{mm}\cdot\text{K}$ gives

$(1.0\times 10^{-3}\ \text{mm})T = 4.11\ \text{mm}\cdot\text{K}$

T = 4100 K.

ASSESS: This star is somewhat cooler than our Sun, which has a surface temperature around 5500 K.

90. **INTERPRET:** This problem deals with the energy of a photon.

DEVELOP: The energy of a single photon of light is $E = hf$. We know that E = 3.3 yJ = 3.3×10^{-24} J, and we want to find the frequency f of the light.

EVALUATE: Using $E = hf$ gives

$3.3\times 10^{-24}\ \text{J} = (6.63\times 10^{-34}\ \text{J}\cdot\text{s})f$

$f = 5.0\times 10^{9}$ Hz = 5.0 GHz.

Therefore this is a 5-Gz network.

ASSESS: The wavelength of these photons is $\lambda = c/f = \dfrac{3.00\times 10^8\ \text{m/s}}{5.0\times 10^9\ \text{Hz}}$ = 0.060 m = 6.0 cm, so they are in the microwave region of the electromagnetic spectrum, according to Figure 29.10.

91. **INTERPRET:** This problem deals with the energy of a photon.

DEVELOP: The energy of a single photon of light is $E = hf$, and $f\lambda = c$, so $E = hc/\lambda$. We are given that λ = 241.3 nm, and we want to find the energy E.

EVALUATE: $E = \dfrac{hc}{\lambda} = \dfrac{(6.62607\times10^{-34}\ \text{J}\cdot\text{s})(2.99792\times10^{8}\ \text{m/s})}{241.3\times10^{-34}\ \text{m}}$

$E = 8.232265\times10^{-19}$ J. Converting this to eV gives

$E = (8.232265\times10^{-19}\ \text{J})[(1\ \text{eV})/(1.60218\times10^{-19}\ \text{J})] = 5.138$ eV.

ASSESS: This energy is close to many photoelectric work functions, as Table 34.1 shows. We used very precise values for *h* and *c* to retain accuracy because we know λ to 4 significant figures.

92. **INTERPRET:** This problem involves matter waves and the de Broglie wavelength.

DEVELOP: The de Broglie wavelength is h/p and $p = mv$ (if $v << c$). We know *m* and that λ = 1.25 μm, and we want to find the speed *v*.

EVALUATE: Solving $\lambda = h/mv$ for *v* gives

$$v = \frac{h}{m\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.67\times10^{-27}\ \text{kg})(1.25\times10^{-6}\ \text{m})} = 0.318\ \text{m/s} = 31.8\ \text{cm/s}.$$

ASSESS: Since 31.8 cm/s << *c*, we were correct in using the nonrelativistic momentum $p = mv$.

93. **INTERPRET:** This problem is about the photoelectric effect.

DEVELOP: The maximum kinetic energy is $K_{\text{max}} = hf - \phi$ and $f\lambda = c$, so $K_{\text{max}} = hc/\lambda - \phi$. For aluminum, Table 34.1 tells us that ϕ = 4.28 eV. We know that λ = 221 nm and want to find K_{max}.

EVALUATE: First find hc/λ and express it in eV.

$$\frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{221\times10^{-9}\ \text{m}} = 9.00\times10^{-19}\ \text{J} = 5.62\ \text{eV}.$$

The maximum kinetic energy is

$K_{\text{max}} = hc/\lambda - \phi = 5.62\ \text{eV} - 4.28\ \text{eV} = 1.34$ eV.

ASSESS: The maximum kinetic energy is considerably less than the work function, so most of the photon energy went into just dislodging the electron from the aluminum.

94. **INTERPRET:** This problem is about the energy of a photon.

DEVELOP: The energy of a single photon of light is $E = hf$, and $f\lambda = c$, so $E = hc/\lambda$. We know the energies of two photons and want to find their wavelengths. We'll need to convert the energies from eV to J.

EVALUATE: Converting the photon energies, we get 1.87 eV = 2.992×10^{-19} J and 2.88 eV = 4.608×10^{-19}. Now solve $E = hc/\lambda$ for λ, giving $\lambda = hc/E$. The energies are

$$\lambda_1 = \frac{hc}{E_1} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{2.992\times10^{-19}\ \text{J}} = 6.65\times10^{-7}\ \text{m} = 665\ \text{nm}.$$

$$\lambda_2 = \frac{hc}{E_2} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{4.608\times10^{-19}\ \text{J}} = 4.32\times10^{-7}\ \text{m} = 432\ \text{nm}.$$

ASSESS: The wavelengths are close to opposite ends of the visible region of the electromagnetic spectrum.

95. **INTERPRET:** This problem involves Compton scattering of a photon off of an electron.

DEVELOP: The change in wavelength of the photon is $\Delta\lambda = \dfrac{h}{mc}(1-\cos\theta)$. For an electron, h/mc = 0.00243 nm = 2.43 pm, The change in wavelength is $\Delta\lambda = \lambda_2 - \lambda_1$. We know that θ = 128° and λ_2 = 155.8 pm, and we want to find λ_1. So we solve for λ_1 giving $\lambda_1 = \lambda_2 - \Delta\lambda$.

EVALUATE: First use $\Delta\lambda = \dfrac{h}{mc}(1-\cos\theta)$ to find $\Delta\lambda$.

$\Delta\lambda$ = (2.43 pm)(1 – cos128°) = 3.926 pm.

Now use $\lambda_1 = \lambda_2 - \Delta\lambda$ to find λ_1.

$\lambda_1 = \lambda_2 - \Delta\lambda$ = 155.8 pm – 3.926 pm = 151.9 pm.

ASSESS: The wavelength changed by only 3.926/151.9 = 0.026 = 2.6%.

96. **INTERPRET:** This problem involves the electron energy levels in the Bohr hydrogen atom.

DEVELOP: The total energy of an electron in the n^{th} level of hydrogen is $E = -\dfrac{13.6\text{ eV}}{n^2}$. This is the minimum energy needed to remove that electron from the atom; that is, to ionize the atom. The atom in this problem absorbs a photon of wavelength λ = 1.46 μm, and the energy of that photon is $E = hc/\lambda$. We can find the energy of the photon, which is the same as the energy needed to ionize the atom, and use that to find the state n of the atom.

EVALUATE: First find the photon energy.

$$E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.46\times10^{-6}\text{ m}} = 1.362\times10^{-19}\text{ J} = 0.8515\text{ eV}.$$

Now use $E = -\dfrac{13.6\text{ eV}}{n^2}$ to find n. The amount of energy we must give the electron to remove it is $\dfrac{13.6\text{ eV}}{n^2}$, so

$$0.8515\text{ eV} = \frac{13.6\text{ eV}}{n^2}$$

$n^2 = 16$

$n = 4$, so the electron was originally in the $n = 4$ excited state.

ASSESS: As n gets larger and larger, we need less and less energy to ionize the atom.

97. **INTERPRET:** This problem involves the uncertainty principle.

DEVELOP: The uncertainty principle can be stated as $\Delta x\Delta p \ge \hbar$. The minimum energy of the electron is its kinetic energy K_{min} when the electron has its minimum speed v_{min}. This speed cannot be zero because they we would know the momentum exactly, in violation of the uncertainty principle. Using $K = p^2/2m$, we get $p_{\text{min}} = \sqrt{2mK_{\text{min}}}$. We cannot know the momentum any more accurately than p_{min}. We don't know which way the electron is moving; it could be to the right or to the left, so the *minimum* uncertainty in the momentum is $\Delta p_{\text{min}} = 2p_{\text{min}} = 2\sqrt{2mK_{\text{min}}}$. We want to find Δx which is the width of the quantum well. So the most precise measurement we can have is when we use the equal sign in the uncertainty principle, in which case $\Delta x\Delta p = \hbar$. Using what we just found for Δp_{min}, we get

$$\Delta x\Delta p_{\text{min}} = \hbar$$

$$\Delta x\left(2\sqrt{2mK_{\text{min}}}\right) = \hbar$$

$$\Delta x = \frac{\hbar}{2\sqrt{2mK_{\text{min}}}}.$$

EVALUATE: $K_{\text{min}} = 0.24\text{ meV} = 0.00024\text{ eV} = 3.84\times10^{-23}$ J. Now find Δx using $\Delta x = \dfrac{\hbar}{2\sqrt{2mK_{\text{min}}}}$.

$$\Delta x = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})/2\pi}{2\sqrt{2\left(9.11\times10^{-31}\text{ kg}\right)\left(3.84\times10^{-23}\text{ J}\right)}} = 6.3\times10^{-9}\text{ m} = 6.3\text{ nm}.$$

ASSESS: A ground-state Bohr hydrogen atom is about 0.1 nm in diameter, so this well is about 60 times as wide as a Bohr hydrogen atom.

Chapter 35: Alternate Problem Set in *Mastering Physics*

65. **INTERPRET:** This problem is about the probability density and the wave function.

DEVELOP: The probability density (in this case probability per unit length) at a point x is $\psi^2(x)$. The particle is most likely to be found where $\psi^2(x)$ is a maximum. In this case, that is at $x = 0$ since e^{-x^2/a^2} is a maximum when $x = 0$. We want to compare the probability density at $x = 2a$ to the density at $x = 0$.

EVALUATE: Take the ratio $\dfrac{\psi^2(2a)}{\psi^2(0)}$, which gives

$$\frac{\psi^2(2a)}{\psi^2(0)} = \frac{Ae^{-(2a)^2/a^2}}{Ae^{-0}} = e^{-4} = 0.018 = 1.8\%.$$

The particle is about 1.8% as likely to be found at $x = 2a$ as it is at $x = 0$.

ASSESS: As x gets larger than $2a$, the probability of finding it gets even smaller than 1.8% because e^{-x^2/a^2} keeps decreasing.

66. **INTERPRET:** This particle deals with the energy levels of a particle in an infinite square well.

DEVELOP: The energy levels are given by $E = \dfrac{n^2h^2}{8mL^2}$, with $n = 1$ for the ground state. Let n be the original state. The final state is the $n = 3$ state. The energy difference between these two states is $\Delta E = E_n - E_3 = \dfrac{n^2h^2}{8mL^2} - \dfrac{3^2h^2}{8mL^2}$, so $\Delta E = \dfrac{h^2}{8mL^2}\left(n^2 - 9\right)$. The ground state is $E_1 = \dfrac{h^2}{8mL^2}$, and we are given that $\Delta E = 72E_1$. Therefore $72E_1 = E_1(n^2 - 9)$.

EVALUATE: Solve $72E_1 = E_1(n^2 - 9)$ for n.

$72 = n^2 - 9$

$n = 9$.

ASSESS: The particle goes from a high-energy state to a low-energy state, so it would give up energy in the form of a photon during this transition.

67. **INTERPRET:** In this problem, we model an alpha particle in a uranium nucleus as a particle in an infinite square well.

DEVELOP: The energy levels for an infinite square well are $E = \dfrac{n^2h^2}{8mL^2}$. The lowest energy is 180 keV = 2.88×10^{-14} J for the $n = 1$ state, and the alpha particle mass is 4 u, where u = 1.661×10^{-27} kg. We want to find the width of the well L, which will be an estimate of the diameter of the uranium nucleus.

EVALUATE: Solving $E = \dfrac{n^2h^2}{8mL^2}$ for L gives $L = \sqrt{\dfrac{h^2}{8mE}}$. Putting in numbers gives

$$L = \sqrt{\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{8\left[4\left(1.661\times10^{-27}\ \text{kg}\right)\right]\left(2.88\times10^{-14}\ \text{J}\right)}} = 1.7\times10^{-14}\ \text{m} = 17\ \text{fm}.$$

ASSESS: This diameter is typical of large nuclei, such as uranium, so our infinite well model is reasonable.

68. **INTERPRET:** This problem deals with the energy levels of a quantum harmonic oscillator.

DEVELOP: The energy levels are given by $E_n = \left(n + \tfrac{1}{2}\right)\hbar\omega$. The energy difference between adjacent states is

$\Delta E = E_{n+1} - E_n = \left(n+1+\tfrac{1}{2}\right)\hbar\omega - \left(n+\tfrac{1}{2}\right)\hbar\omega = \hbar\omega$.

We know that $\Delta E = 8.4$ eV, and we want to find the ground state energy, which is $E_1 = \tfrac{1}{2}\hbar\omega$.

EVALUATE: Using $\Delta E = \hbar\omega$, we have

$8.4 \text{ eV} = \hbar\omega$

$E_1 = \tfrac{1}{2}\hbar\omega = \tfrac{1}{2}(8.4 \text{ eV}) = 4.2 \text{ eV}$.

ASSESS: We did not need to know which adjacent states had an energy difference of 8.4 eV because the energy levels are equally spaced. This is *not* true of all systems, such as electron energy levels or the energy levels of infinite wells.

69. **INTERPRET:** This problem is about the energy levels of an electron in a cubical box.

DEVELOP: The energy levels are given by $E = E = \dfrac{h^2}{8mL^2}\left(n_x^2 + n_y^2 + n_z^2\right)$. The ground state is the lowest energy state, and that is when all the values of n are equal to 1. Therefore $E_{\text{grd}} = \dfrac{3h^2}{8mL^2}$. The first excited state is the state in which two of the values of n are equal to 1 and the third one is equal to 2, so the energy of that state is $E_{\text{first}} = \dfrac{h^2}{8mL^2}\left(1^2 + 1^2 + 2^2\right) = \dfrac{6h^2}{8mL^2}$. The energy difference between these two states is $\Delta E = E_{\text{first}} - E_{\text{grd}} = \dfrac{6h^2}{8mL^2} - \dfrac{3h^2}{8mL^2} = \dfrac{3h^2}{8mL^2}$. This is the energy emitted by the photon, so $E_{\text{photon}} = \dfrac{3h^2}{8mL^2}$. We want the photon's wavelength, so we use $E = hc/\lambda$. Therefore $\dfrac{hc}{\lambda} = \dfrac{3h^2}{8mL^2}$, which gives $\lambda = \dfrac{8mcL^2}{3h}$.

EVALUATE: Using $\lambda = \dfrac{8mcL^2}{3h}$, we get

$$\lambda = \frac{8(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})(935\times10^{-12}\text{ m})^2}{3\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)} = 9.61\times10^{-7}\text{ m} = 961\text{ nm}.$$

ASSESS: The emitted light has a wavelength longer than that of visible light. A smaller box would result in a shorter wavelength.

70. **INTERPRET:** This problem involves electron transitions in an infinite square well.

DEVELOP: The energy levels are $E = \dfrac{n^2h^2}{8mL^2}$. The energy difference between two adjacent states (n + 1 and n) is

$\Delta E = E_{n+1} - E_n = \dfrac{(n+1)^2h^2}{8mL^2} - \dfrac{n^2h^2}{8mL^2}$, so

$\Delta E = \dfrac{h^2}{8mL^2}\left(n^2 + 2n + 1 - n^2\right) = \dfrac{h^2}{8mL^2}(2n+1)$. The energy of a photon is $E = hc/\lambda$, and this is equal to the energy difference between the two states. Thus $\dfrac{hc}{\lambda} = \dfrac{h^2}{8mL^2}(2n+1)$, which simplifies to $\dfrac{c}{\lambda} = \dfrac{h}{8mL^2}(2n+1)$. We want to know the value of n, so we solve for n.

EVALUATE: Solving $\dfrac{c}{\lambda} = \dfrac{h}{8mL^2}(2n+1)$ for n gives $n = \dfrac{1}{2}\left(\dfrac{8mcL^2}{\lambda h} - 1\right)$. Therefore

$$n = \frac{1}{2}\left(\frac{8\left(9.11\times10^{-31}\text{ kg}\right)(3.00\times10^{8}\text{ m/s})(1.53\times10^{-9}\text{ m})^2}{(454\times10^{-9}\text{ m})(6.63\times10^{-34}\text{ J}\cdot\text{s})} - 1\right)$$ = 8, so the states are 8 and 9. The transition is from the $n = 9$ state to the $n = 8$ state.

ASSESS: In an infinite well, the energy levels are *not* equally spaced, so the energy of the emitted photon depends on the particular adjacent states.

71. **INTERPRET:** This problem involves the probability in a square well, so we'll have to use $\psi^2(x)$.

DEVELOP: The probability density is $\psi^2(x)$, so the probability of finding the particle in the region between $x = \pm a$ is $P = \int_{-a}^{a}\psi^2(x)dx$. For a square well, the wave functions are given by $\psi(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$, with $n = 1$ for the ground state, giving $\psi(x) = \sqrt{\frac{2}{L}}\sin(\pi x/L)$ for the ground state wave function. This equation is for axes chosen with the origin at one end of the well. In this problem, we are interested in an interval centered on the midpoint of the well, so it is more convenient to put the origin at the midpoint of the well. In that case, the wave function becomes a cosine instead of a sine, so $\psi(x) = \sqrt{\frac{2}{L}}\cos(\pi x/L)$. Now the probability of finding the particle within $\pm a$ of the center of the well is $P = \int_{-a}^{a}\psi^2(x)dx = \int_{-a}^{a}\left(\sqrt{\frac{2}{L}}\cos(\pi x/\mathrm{L})\right)^2 dx = \int_{-a}^{a}\frac{2}{L}\cos^2(\pi x/\mathrm{L})dx$. The wave function is symmetric about the center of the well, so we can change the limits to give $P = 2\int_0^a \frac{2}{L}\cos^2(\pi x/L)dx = \frac{4}{L}\int_0^a \cos^2(\pi x/L)dx$. We want the probability of finding the particle within the interval from $-a$ to $+a$ to be 70%, so $P = 0.70$. The length of this interval is $2a$. We want to know the percent of the well's width that we'd have to search so $P = 0.70$, which means that we want to know what percent $2a$ is of L. Thus we want to find $2a/L$. To do this, we must solve the equation $0.70 = \frac{4}{L}\int_0^a \cos^2(\pi x/L)dx$ for $2a/L$.

EVALUATE: Carry out the integration to solve $0.70 = \frac{4}{L}\int_0^a \cos^2(\pi x/L)dx$. Using integral tables, we find that $\int \cos^2(bx)dx = \frac{x}{2} + \frac{\sin(2bx)}{4b}$, and in this case $b = \pi/L$. Therefore our integration gives

$$0.70 = \frac{4}{L}\int_0^a \cos^2(\pi x/L)dx = \frac{4}{L}\left[\frac{x}{2} + \frac{\sin(2\pi x/L)}{4\pi/L}\right]_0^a = \frac{2a}{L} + \frac{1}{\pi}\sin(2\pi a/L).$$

Letting $z \equiv 2a/L$, this equation can be written as

$$0.70 = z + \frac{\sin \pi z}{\pi}.$$

We cannot solve this equation algebraically but must use numerical methods. The result is $z = 2a/L = 0.40$, so $2a = 0.40L$. Therefore there is a 70% chance of finding the particle within $\pm a = \pm 0.20L$ of the center of the well. We would have to search 40% of the well's width with that interval centered on the midpoint of the well.

ASSESS: If we did not limit the region to be centered on the middle of the well, the region would be different from $0.40L$ because the wave function (and hence the probability density) depends on the location in the well.

72. **INTERPRET:** This problem involves the probability in a square well, so we'll have to use $\psi^2(x)$.

DEVELOP: Calling the origin at one end of the well, the central one-third of the well extends from $x = L/3$ to $x = 2L/3$. The normalized wave function is $\psi(x) = \sqrt{\frac{2}{L}}\sin(n\pi x/L)$. The probability of finding a particle in the central one-third of the well is $P = \int_{L/3}^{2L/3}\psi^2(x)dx = \int_{L/3}^{2L/3}\left(\sqrt{\frac{2}{L}}\sin(n\pi x/L)\right)^2 dx = \frac{2}{L}\int_{L/3}^{2L/3}\sin^2(n\pi x/L)dx$. Carrying out the integration using integral tables, we use $\int \sin^2(bx)dx = \frac{x}{2} - \frac{\sin(2bx)}{4b}$, and in this case $b = n\pi/L$. This gives $P = \frac{2}{L}\left[\frac{x}{2} - \frac{\sin(2n\pi x/L)}{4n\pi/L}\right]_{L/3}^{2L/3} = \frac{1}{3} - \frac{\sin(4n\pi/3) - \sin(2n\pi/3)}{2n\pi}$. We know that in 400 out of 1000 of the systems, the particle was in the central one-third of the well. This means that the probability of finding it in that region is

400/1000 = 0.40, so we have 0.40 = $\frac{1}{3}-\frac{\sin(4n\pi/3)-\sin(2n\pi/3)}{2n\pi}$. We want to find n, which must be a positive integer.

EVALUATE: Using 0.40 = $\frac{1}{3}-\frac{\sin(4n\pi/3)-\sin(2n\pi/3)}{2n\pi}$, try integer values of n until the right-hand side of this equation is equal to 0.40.

n = 1: $\frac{1}{3}-\frac{\sin(4\pi/3)-\sin(2\pi/3)}{2\pi} = 0.61 \neq 0.40$, so n = 1 is not a solution.

n = 2: $\frac{1}{3}-\frac{\sin(8\pi/3)-\sin(4\pi/3)}{4\pi} = 0.195 \neq 0.40$, so n = 2 is not a solution.

n = 3: $\frac{1}{3}-\frac{\sin(12\pi/3)-\sin(6\pi/3)}{6\pi} = 0.33 \neq 0.40$, so n = 3 is not a solution.

n = 4: $\frac{1}{3}-\frac{\sin(16\pi/3)-\sin(8\pi/3)}{8\pi} = 0.40$, so n = 4 *is* the solution.

The system is in the n = 4 quantum state.

ASSESS: The wave function for this state is $\psi(x)=\sqrt{\frac{2}{L}}\sin\left(4\pi x/L\right)$.

73. **INTERPRET:** This problem is about normalizing a wave function so it will involve $\psi^2(x)$.

DEVELOP: The probability of finding a particle in an interval from a to b is $P=\int_a^b \psi^2(x)dx$. The particle must be somewhere in the universe, so the probability of finding it between $\pm\infty$ must be 1. Therefore $\int_{-\infty}^{+\infty}\psi^2(x)dx=1$. The given wave function is zero for $x \le -L$ and for $x \ge L$, we need only integrate from $-L$ to $+L$. The wave function has a different form between $-L$ and 0 than between 0 and $+L$, so we break the integral into two parts, giving $1=\int_{-L}^{L}\psi^2(x)dx=\int_{-L}^{0}\left[A(L+x)\right]^2dx+\int_0^L\left[A(L-x)\right]^2dx$. We want to find A so the probability is 1.

EVALUATE: Carry out the integration.

$$1=\int_{-L}^{0}\left[A(L+x)\right]^2dx+\int_0^L\left[A(L-x)\right]^2dx$$

$$1 = A^2\int_{-L}^{0}(L^2+2Lx+x^2)dx+A^2\int_0^L(L^2-2Lx+x^2)dx$$

$$1 = A^2\left(L^2x+Lx^2+\frac{x^3}{3}\right)\Bigg|_{-L}^{0}+A^2\left(L^2x-Lx^2+\frac{x^3}{3}\right)\Bigg|_0^L$$

$$1 = A^2\left[-\left(-L^3+L^3-\frac{L^3}{3}\right)\right]+A^2\left(L^3-L^3+\frac{L^3}{3}\right) = \frac{2L^3}{3}A^2$$

$$A=\sqrt{\frac{3}{2}}\frac{1}{L^{3/2}}.$$

ASSESS: Check the units of A. Since $\psi^2(x)$ is a probability density, it has SI units of m^{-1}. For this wave function, $\psi^2(x)$ is of the form $A^2(L\pm x)^2$. Using our result for A, $\psi^2(x)$ has units of $(\mathrm{m}^{-3/2})^2(\mathrm{m})^2 = \mathrm{m}^{-3}\mathrm{m}^2 = \mathrm{m}^{-1}$, which are correct. Therefore our units for A are correct.

74. **INTERPRET:** This problem is about the energy levels of an infinite square well.

DEVELOP: The energy levels are given by $E=\frac{n^2h^2}{8mL^2}$, with n = 1 for the ground state. So the ground state energy is $E=\frac{h^2}{8mL^2}$. We know that $E_A = 5E_B$ and want to find how the widths of these two wells are related, so we can find L_B/L_A.

EVALUATE: Use $E = \dfrac{h^2}{8mL^2}$ to take the ratio of the two ground state energies. We know that $E_A = 5E_B$, so $E_A/E_B = 5$. This gives

$$\frac{E_A}{E_B} = 5 = \frac{h^2/8mL_A^2}{h^2/8mL_B^2} = \left(\frac{L_B}{L_A}\right)^2.$$

$$\frac{L_B}{L_A} = \sqrt{5}$$

$$L_B = \sqrt{5}L_A \approx 3.34L_A.$$

ASSESS: $E \propto \dfrac{1}{L^2}$, so a smaller L should result in a larger E. Since $E_A = 5E_B$, we should find that $L_A < L_B$, which we do.

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80. **INTERPRET:** This problem involves the energy levels and ionization energy of hydrogen.

DEVELOP: The ionization energy is the minimum amount of energy needed to remove an electron from the atom. The total energy of an electron in state n is $E = -\dfrac{13.6\text{ eV}}{n^2}$, so the ionization energy is $E = \dfrac{13.6\text{ eV}}{n^2}$. We use this formula to calculate the ionization energy for each of the specified states.

EVALUATE: **(a)** $E_2 = (13.6\text{ eV})/2^2 = 3.40\text{ eV}$.

(b) $E_5 = (13.6\text{ eV})/5^2 = 0.544\text{ eV}$.

(c) $E_{95} = (13.6\text{ eV})/95^2 = 1.51\times 10^{-3}\text{ eV} = 1.51\text{ meV}$.

ASSESS: As n gets larger, the ionization energy decreases because the strength of the electrical attraction of the nucleus decreases.

81. **INTERPRET:** This problem is about the orbital angular momentum of an electron in hydrogen.

DEVELOP: The magnitude of the orbital angular momentum L is $L = \sqrt{l(l+1)}\hbar$, where $l = 0, 1, \ldots, n-1$. In this case, we are given that $L = \sqrt{42}\hbar$, and we want to find the principal quantum number n.

EVALUATE: $\sqrt{l(l+1)} = \sqrt{42} = \sqrt{6(7)}$, so $l = 6$. Since this is the maximum possible angular momentum for this state, l must have its maximum value, which is $n - 1$, so $6 = n - 1$, which gives $n = 7$.

ASSESS: For the $n = 7$ state, l could also be equal to 0, 1, 2, 3, 4, or 5, but these would not give the *maximum* angular momentum.

82. **INTERPRET:** This problem is about the total angular momentum of an electron in hydrogen.

DEVELOP: The magnitude of the total angular momentum J is $J = \sqrt{j(j+1)}\hbar$, where $j = l \pm \frac{1}{2}$ for $l \neq 0$. In this case, $J = \sqrt{\frac{15}{4}}\hbar$ and $J = \sqrt{\frac{35}{4}}\hbar$, so we know that $j(j+1) = \frac{15}{4}$ and $j(j+1) = \frac{35}{4}$. We can write these fractions as $\frac{15}{4} = \frac{3}{2}\cdot\frac{5}{2}$ and $\frac{35}{4} = \frac{5}{2}\cdot\frac{7}{2}$.

EVALUATE: <u>In the first case</u>: $j(j+1) = \frac{3}{2}\cdot\frac{5}{2}$, so $j = \frac{3}{2}$. Since $j = l - \frac{1}{2}$, we have $j = l - \frac{1}{2} = \frac{3}{2}$, which gives $l = 2$.

<u>In the second case</u>: $j(j+1) = \frac{5}{2}\cdot\frac{7}{2}$, so $j = \frac{5}{2}$. Therefore $j = l + \frac{1}{2} = \frac{5}{2}$, so $l = 2$.

Thus this is an $l = 2$ state, which is a D state. These are the *maximum* possible values of J, so l must be at its maximum possible value, with is $n - 1$. Therefore $n - 1 = 2$, so $n = 3$. The atom is in a $3D$ state.

ASSESS: If we did not know that J was a maximum, n could be ≥ 3.

83. **INTERPRET:** This problem involves the harmonic oscillator energy levels and the exclusion principle.

DEVELOP: The energy levels for a quantum harmonic oscillator are given by $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$. We know that the frequency is $f = 180\text{ THz} = 180\times 10^{12}\text{ Hz}$ and the energy of the most energetic electron is $10.8\text{ eV} = 1.728\times 10^{-18}\text{ J}$, so we can find n. Since we know f, we can write the energy levels in terms of the frequency as $E_n = \left(n+\frac{1}{2}\right)\hbar\omega = \left(n+\frac{1}{2}\right)\left(\dfrac{h}{2\pi}\right)(2\pi f) = \left(n+\frac{1}{2}\right)hf$. Once we have n, we apply the exclusion principle, which says that no two electrons can be in exactly the same state at the same time.

EVALUATE: Find n using the most energetic electron using $E_n = \left(n+\frac{1}{2}\right)hf$, giving

$1.728\times 10^{-18}\text{ J} = \left(n+\frac{1}{2}\right)(6.63\times 10^{-34}\text{ J}\cdot\text{s})(180\times 10^{12}\text{ Hz})$

$n+\frac{1}{2} = 14.5$

$n = 14.$

The states start with $n = 0$, so there are $n + 1$ states for every value of n, which means that in this case there are 15 possible states. By the exclusion principle, each state can have only two electrons in it (each having opposite spin), so this state could contain one or two electrons. Thus this well could contain 29 or 30 electrons.

ASSESS: The lower states are all full, so we could put 1 or 2 additional electrons in the $n = 14$ state.

84. **INTERPRET:** This problem is about the splitting of energy levels in sodium.

DEVELOP: The energy difference ΔE between the two levels is $\Delta E = 0.6934\text{ meV} = 1.11095\times 10^{-22}$ J.

$\Delta E = E_2 - E_1 = \dfrac{hc}{\lambda_2} - \dfrac{hc}{\lambda_1} = hc\left(\dfrac{1}{\lambda_2} - \dfrac{1}{\lambda_1}\right)$. We know that the longer wavelength is $\lambda_1 = 330.298$ nm and we want the shorter wavelength λ_2.

EVALUATE: Using $\Delta E = hc\left(\dfrac{1}{\lambda_1} - \dfrac{1}{\lambda_2}\right)$ gives

$$1.11095\times 10^{-22}\text{ J} = (6.62607\times 10^{-34}\text{ J}\cdot\text{s})(2.99792\times 10^{8}\text{ m/s})\left(\frac{1}{\lambda_2} - \frac{1}{330.298\times 10^{-9}\text{ m}}\right)$$

$\lambda_2 = 3.3024\times 10^{-7}\text{ m} = 330.24\text{ nm}.$

ASSESS: We used more than the usual number of significant figures for the constants in our calculation to retain accuracy since the two wavelengths are very close together due to the very small energy difference of 0.6934 meV. The difference in the wavelengths is only about 0.06 nm.

85. **INTERPRET:** This problem is about quantized orbital angular momentum of Earth.

DEVELOP: The orbital angular momentum L is $L = \sqrt{l(l+1)}\hbar$. We want to find l for Earth. The planet's angular momentum is $L = I\omega = (mr^2)(2\pi/T)$, so we have $\dfrac{2\pi mr^2}{T} = \sqrt{l(l+1)}\hbar$. The period is $T = 1\text{ y} = 3.156\times 10^7$ s, and from Appendix E we find that $m = 5.97\times 10^{24}$ kg and $r = 149.6\times 10^6$ km.

EVALUATE: We need to solve $\dfrac{2\pi mr^2}{T} = \sqrt{l(l+1)}\hbar$ for l. We know that l will be extremely large, so we can replace $l(l + 1)$ by l^2, which gives $\dfrac{2\pi mr^2}{T} = l\hbar$. Putting in the numbers gives

$$\frac{2\pi(5.97\times 10^{24}\text{ kg})(149.6\times 10^{9}\text{ m})^2}{3.156\times 10^{7}\text{ s}} = l\left(\frac{6.63\times 10^{-34}\text{ J}\cdot\text{s}}{2\pi}\right)$$

$l = 2.5\times 10^{74}.$

ASSESS: There is precious little chance that quantum effects will play a role in Earth's orbital motion!

86. **INTERPRET:** This problem deals with the orbital angular momentum vector, its z-component, as well as the orbital and orbital magnetic quantum numbers.

DEVELOP: The z-component of the angular momentum vector is $L_z = m_l\hbar$, where m_l is the orbital magnetic quantum number and can take the values 0, ±1, … , ±l. We can also express the z-component of the angular momentum as $L_z = L\cos\theta$, where θ is the angle that $\vec{L}$ makes with the z-axis. We also know that $L = \sqrt{l(l+1)}\hbar$. Combining these quantities gives $L_z = m_l\hbar = L\cos\theta = \sqrt{l(l+1)}\hbar\cos\theta$, so $\sqrt{l(l+1)}\hbar\cos\theta = m_l\hbar$. We are given that

$\cos\theta = 1/\sqrt{6}$, and we want to find l and m_l. Using $\sqrt{l(l+1)}\hbar\cos\theta = m_l\hbar$ with $\cos\theta = 1/\sqrt{6}$ and squaring gives

$l(l+1)\left(\frac{1}{6}\right) = m_l^2$, so $\dfrac{1}{6} = \dfrac{m_l^2}{l(l+1)}$.

EVALUATE: We now find out what values of l and m_l satisfy the equation $\frac{1}{6}=\frac{m_l^2}{l(l+1)}$, starting with the lowest value of l.

$l = 0$: This will not work because it leads to 0/0 , which is indeterminate.

$l = 1$ and $m_l = \pm 1$: $(\pm 1)^2/2 = ½$, which is not 1/6, so these values do not work.

$l = 2$ and $m_l = \pm 2$: $(\pm 2)^2/6 = 2/3$, which is not 1/6, so these values do not work.

$l = 2$ and $m_l = \pm 1$: $(\pm 1)^2/6 = 1/6$, which does work, so $l = 2$. But is m_l equal to +1 or –1? We know that the angle that $\vec{L}$ makes with the z-axis is 65.9°, which has a positive z-component, so we must use $m_l = +1$. Therefore we know that

(a) $l = 2$ and **(b)** $m_l = +1$.

ASSESS: Other angles are possible for different values of m_l, but not for the conditions given.

87. **INTERPRET:** This problem involves the energy levels of an infinite square well and the exclusion principle.

DEVELOP: The energy levels are $E_n=\frac{n^2h^2}{8mL^2}$. The exclusion principle says that no two electrons can be in exactly the same at the same time. Therefore there can be no more than two electrons per energy level (having opposite spins). Since there are 16 electrons, they must occupy the 8 lowest energy levels, so n varies from 1 to 8. We know that the total energy for all 16 electrons is 25.0 eV, which means that

$2(E_1 + E_2 + \ldots E_8) = 25.0$ eV. The factor of 2 is present because there are 2 electrons for each energy level. We want to find the minimum width L of the well.

EVALUATE: Adding the energies gives $E_n=2\left(\frac{h^2}{8mL^2}+\frac{2^2h^2}{8mL^2}+\ldots+\frac{8^2h^2}{8mL^2}\right)=25.0\text{ eV}$

$$2(1+4+9+16+25+36+49+64)\left(\frac{h^2}{8mL^2}\right)=25.0\,\text{eV}.$$

Using $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$, $m = 9.11\times10^{-31}$kg, and 25.0 eV = 4.00×10^{-18} J, we solve for L, which gives $L = 2.49\times10^{-9}$ m = 248 nm.

ASSESS: This width is typical of the wavelength of ultraviolet light, which is shorter than that of visible light. It is *much* larger than the diameter of a typical atom, which is around 0.1 nm.

88. **INTERPRET:** This problem is about the probability density of hydrogen in an S state.

DEVELOP: The radial probability density for hydrogen is $P(r) = 4\pi r^2\psi^2(r)$. The most probably locations to find an electron is where this density is greatest. We are given that this is between 12 and 15 Bohr radii from the nucleus for these atoms. We want to know the quantum state of these atoms.

EVALUATE: Figure 36.4 in the text shows a graph of $P(r)$ as a function of r/a_0 for three S states of hydrogen. From this graph, we see that $P(r)$ for the 3s state has a relatively flat maximum where r/a_0 is between 12 and 15. Therefore these atoms are in the 3S state.

ASSESS: The other two densities in the figure peak at much smaller values of r/a_0, so electrons for atoms in those states would be more likely to be found closer to the nucleus than for the 3S state.

89. **INTERPRET:** This problem is about the electron capacity of atom shells.

DEVELOP: The electron capacity of atoms for the first few principal quantum number states is 2 (for $n = 1$), 8 (for $n = 3$), 18 (for $n = 3$).

EVALUATE: **(a)** The given atom has 18 electrons in its outermost shell, so that must be the $n = 3$ shell. The principal quantum number is $n = 3$.

(b) The total number of electrons is 18, which is the same as the number of protons. From the periodic chart in the text (or Table 36.2), we see that this atom is argon.

ASSESS: The electron capacity of an atom is $2n^2$, which in this case would be $2(3^2) = 18$, which agrees with our result.

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72. **INTERPRET:** This problem is about the rotational energy levels in oxygen.

DEVELOP: The rotational energy levels are given by $E_{\text{rot}} = \dfrac{\hbar^2}{2I}l(l+1)$. To the excite the lowest (l = 0) state, the energy of the absorbed photon must be equal to the energy difference ΔE_{rot} between the lowest and next highest state (l = 1). The photon energy is therefore $E_{\text{photon}} = hc/\lambda = \Delta E_{\text{rot}}$. In terms of l, the energy difference is $\Delta E_{\text{rot}} = E_1 - E_0 = \dfrac{\hbar^2}{2I}1(1+1) - 0 = \dfrac{\hbar^2}{I}$, so $\dfrac{hc}{\lambda} = \dfrac{\hbar^2}{I}$. We know that λ = 3.48 mm, and we want the rotational inertia I.

EVALUATE: Solve $\dfrac{hc}{\lambda} = \dfrac{\hbar^2}{I}$ for I, giving $I = \dfrac{(h/2\pi)^2\lambda}{hc} = \dfrac{h\lambda}{4\pi^2 c}$. Putting in the numbers gives

$$I = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)(0.00348\ \text{m})}{4\pi^2(3.00\times10^{8}\ \text{m/s})} = 1.95\times10^{-46}\ \text{kg}\cdot\text{m}^2.$$

ASSESS: The SI units of our answer are $\dfrac{(\text{J}\cdot\text{s})(\text{m})}{\text{m/s}} = (\text{kg}\cdot\text{m}^2/\text{s}^2)(\text{s})(\text{m})(\text{s/m}) = \text{kg}\cdot\text{m}^2$, which are the correct SI units for rotational inertia.

73. **INTERPRET:** This problem deals with the vibrational levels of a hydrogen molecule.

DEVELOP: The vibrational energy levels are given by $E_{\text{vib}} = \left(n+\tfrac{1}{2}\right)\hbar\omega$. The energy spacing between two adjacent levels is $\Delta E = \left(n+1+\tfrac{1}{2}\right)\hbar\omega - \left(n+\tfrac{1}{2}\right)\hbar\omega = \hbar\omega = (h/2\pi)(2\pi f) = hf$. We know that ΔE = 546 meV = 8.736×10^{-20} J, and we want to find the frequency f.

EVALUATE: Using $\Delta E = hf$ gives

8.736×10^{-20} J = (6.63×10^{-34} J·s)f

$f = 1.32\times10^{14}$ Hz.

ASSESS: This frequency is close to the frequency of visible light, but a bit lower.

74. **INTERPRET:** This problem requires the conversion of units.

DEVELOP: A chemist expresses the ionic cohesive energy of NaCl as 181 kcal/mol, but a physicist would express it in units of eV/molecule. So we must convert from 181 kcal/mol to units of eV/molecule. We use the following equalities: 1 cal = 4.184 J and 1 eV = 1.602×10^{-19} J, and the fact that 1 mol contains 6.022×10^{23} molecules.

EVALUATE: Using the equalities indicated above, the conversion gives

$$\left(\frac{181{,}000\ \text{cal}}{1\ \text{mol}}\right)\left(\frac{1\ \text{mol}}{6.022\times10^{23}\ \text{molecules}}\right)\left(\frac{4.184\ \text{J}}{1\ \text{cal}}\right)\left(\frac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = 7.85\ \text{eV}.$$

ASSESS: The two values are so different because the chemist uses a value for a macroscopic quantity (a mole) but the physicist uses a value for a microscopic quantity (a molecule).

75. **INTERPRET:** This problem is about the wavelength of light emitted by a 3.86-eV band gap semiconductor.

DEVELOP: The band gap energy is 3.86 eV = 6.176×10^{-19} J. We need to find the wavelength of a photon with this energy to see if it is in the ultraviolet region of the electromagnetic spectrum. The energy of a photon is $E = hc/\lambda$.

EVALUATE: Solving $E = hc/\lambda$ for λ gives $\lambda = hc/E$, so

$$\lambda = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{6.176\times10^{-19}\ \text{J}} = 3.22\times10^{-7}\ \text{m} = 322\ \text{nm}.$$

This wavelength is in the ultraviolet part of the electromagnetic spectrum.

ASSESS: For a visible LED, a larger band gap would be needed.

76. **INTERPRET:** In this problem, we want to relate the band gap energy of LEDs to the wavelength of the emitted light.

DEVELOP: The energy of the emitted photon is equal to the band gap energy ΔE_b, so $\Delta E_b = E_{\text{photon}} = hc/\lambda$. We know the band gap energies and want the corresponding wavelengths.

EVALUATE: Use $\Delta E_b = hc/\lambda$ for each wavelength, and express the final answer in eV. For the 455-nm gap, we have $\Delta E_b = (6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(455\times10^{-9}\ \text{m}) = 2.73$ nm.

Similar calculations for the other energies give the other wavelengths. The results are 2.73 nm, 2.28 nm, 2.02 nm, 1.93 nm, 1.86 nm.

ASSESS: All of these energies are less than the ionization energy of hydrogen, which is 13.6 eV.

77. **INTERPRET:** This problem deals with the rotational energy of a molecule.

DEVELOP: The rotational energy levels are given by $E_{\text{rot}} = \frac{\hbar^2}{2I}l(l+1)$, where $l = 0, \pm1, \pm2, \ldots$. During a transition between two states, the energy difference between the states is equal to the energy of the emitted photon. We know that during the $l = 1 \to l = 0$ transition, a photon of energy 1.25 meV is emitted. We want to find the energy of the photon emitted during the $l = 2 \to l = 1$ transition.

EVALUATE: For the $l = 1 \to l = 0$ transition:

$$\Delta E_{1\to0} = E_1 - E_0 = \frac{\hbar^2}{2I}1(1+1) - 0 = \frac{\hbar^2}{I} = 1.25\ \text{meV}.$$

For the $l = 2 \to l = 1$ transition:

$$\Delta E_{2\to1} = E_2 - E_1 = \frac{\hbar^2}{2I}2(2+1) - \frac{\hbar^2}{2I}1(1+1) = \frac{\hbar^2}{2I}(6-2) = \frac{2\hbar^2}{I}.$$

But from the $l = 1 \to l = 0$ transition, we saw that $\frac{\hbar^2}{I} = 1.25\ \text{meV}$. Therefore

$$\Delta E_{2\to1} = 2(1.25\ \text{meV}) = 2.50\ \text{meV}.$$

ASSESS: The energy difference between adjacent rotational states gets progressively greater as l increases, as our results show.

78. **INTERPRET:** This problem involves the vibrational and rotational energy states of a deuterium molecule.

DEVELOP: The vibrational energies of the molecule are $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$, and the rotational energies are $E_{\text{rot}} = \frac{\hbar^2}{2I}l(l+1)$. The energy of a given state $E_{n,l}$ is the sum of the vibrational and rotational energies, which means that $E_{n,l} = E_{\text{vib}} + E_{\text{rot}} = \left(n+\frac{1}{2}\right)\hbar\omega + \frac{\hbar^2}{2I}l(l+1)$. The energy $E = hc/\lambda$ of the emitted photon is equal to the *difference* in energy ΔE between the two states. We know that $I = 9.17\times10^{-48}\ \text{kg}\cdot\text{m}^2$ and $\lambda = 3.41\ \mu\text{m}$, and we want to find the classical frequency f of vibration.

EVALUATE: In the $n = 1$, $l = 0$ state, $E_{\text{rot}} = 0$, so $E_{1,0} = \left(1+\frac{1}{2}\right)\hbar\omega = \frac{3}{2}\hbar\omega$. In the $n = 0$, $l = 2$ state, $E_{0,2} = \frac{1}{2}\hbar\omega + \frac{\hbar^2}{2I}2(2+1) = \frac{1}{2}\hbar\omega + \frac{3\hbar^2}{I}$. The energy difference between these two states is equal to hc/λ, so

$$\frac{hc}{\lambda} = \frac{3}{2}\hbar\omega - \left(\frac{1}{2}\hbar\omega + \frac{3\hbar^2}{I}\right) = \hbar\omega - \frac{3\hbar^2}{I} = \left(\frac{h}{2\pi}\right)(2\pi f) - \frac{3(h/2\pi)^2}{I}$$

$$\frac{c}{\lambda} = f - \frac{3h}{4\pi^2 I}$$

$$f = \frac{3.00\times10^{8}\ \text{m/s}}{3.41\times10^{-6}\ \text{m}} + \frac{3(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{4\pi^2(9.17\times10^{-48}\text{kg}\cdot\text{m}^2)} = 9.35\times10^{13}\ \text{Hz} = 93.5\ \text{THz}.$$

ASSESS: This frequency is than the frequency of visible light.

79. **INTERPRET:** This problem involves rotational energy levels and rotational inertia.

DEVELOP: The rotational energy levels are $E_{\text{rot}} = \dfrac{\hbar^2}{2I}l(l+1)$, where $l = 0, \pm1, \ldots$

The rotational inertia about an axis through the center of the molecule is $I = m(r/2)^2 + m(r/2)^2 = \frac{1}{2}mr^2$, where r is the spacing between nuclei. The energy difference between the first excited state and the ground state is $\Delta E = E_1 - E_0 = \dfrac{\hbar^2}{2I}1(1+1) - 0 = \dfrac{\hbar^2}{I}$. This is the energy of the emitted photon, which is hc/λ, so $\dfrac{hc}{\lambda} = \dfrac{\hbar^2}{I}$.

Using $I = \frac{1}{2}mr^2$, this becomes $\dfrac{hc}{\lambda} = \dfrac{\hbar^2}{\frac{1}{2}mr^2} = \dfrac{2(h/2\pi)^2}{mr^2}$. Solving for r, we get $r = \sqrt{\dfrac{h\lambda}{2\pi^2 mc}}$. For hydrogen, m is the mass of the proton.

EVALUATE: Putting the numbers into $r = \sqrt{\dfrac{h\lambda}{2\pi^2 mc}}$ gives

$$r = \sqrt{\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(81\times10^{-6}\ \text{m})}{2\pi^2(1.67\times10^{-27}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}} = 7.4\times10^{-11}\ \text{m} = 74\ \text{pm}.$$

ASSESS: Atomic nuclei are around 10^{15} m or more in diameter, so the spacing here is much greater than that.

80. **INTERPRET:** This problem involves the potential energy of a LiCl crystal.

DEVELOP: The potential energy U of an ionic crystal is given by $U = -\alpha\dfrac{ke^2}{r_0}\left[\dfrac{r_0}{r} - \dfrac{1}{n}\left(\dfrac{r_0}{r}\right)^n\right]$, where r_0 is the equilibrium separation. When $r = r_0$, the energy U is the ionic cohesive energy, which is –8.40 eV = -1.344×10^{-18} J for LiCl. We are given that $\alpha \approx 1.748$ (the same as for NaCl) and $n = 7$. We want to find the equilibrium separation, r_0. In the equation for U, we can let $r = r_0$ and then solve for r_0 since we know U.

EVALUATE: Set $r = r_0$ in $U = -\alpha\dfrac{ke^2}{r_0}\left[\dfrac{r_0}{r} - \dfrac{1}{n}\left(\dfrac{r_0}{r}\right)^n\right]$, giving $U = -\alpha\dfrac{ke^2}{r_0}\left(1 - \dfrac{1}{n}\right)$. Solving for r_0 gives

$r_0 = -\alpha\dfrac{ke^2}{U}\left(1 - \dfrac{1}{n}\right)$. Putting in the numbers gives

$$r_0 = -(1.748)\frac{(9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{-1.344\times10^{-18}\ \text{J}}\left(1 - \frac{1}{7}\right)$$

$r_0 = 2.57\times10^{-10}$ m = 0.257 nm.

ASSESS: This separation is slightly smaller than the spacing of 0.282 nm in NaCl.

81. **INTERPRET:** This problem is about the Fermi energy in a solid.

DEVELOP: The Fermi temperature T_F is the temperature at which kT is equal to the Fermi energy E_F. In this problem, the Fermi temperature is given as being 100 times room temperature, so T_F = 100(293 K) = 29,300 K. We want to find the Fermi energy in eV, so find $E_F = kT_F$ expressed in eV.

EVALUATE: Using $E_F = kT_F$ and converting to eV gives

$E_F = (1.38\times10^{-23}\ \text{J/K})(29{,}300\ \text{K})[(1\ \text{eV})/(1.60\times10^{-19}\ \text{J})] = 2.53\ \text{eV}.$

ASSESS: This energy is somewhat less than the ionization energy of hydrogen, which is 13.6 eV.

Chapter 38: Alternate Problem Set in *Mastering Physics*

90. **INTERPRET:** This problem is about isotopes and nuclear symbols.

DEVELOP: The two chlorine isotopes must both have 17 protons (or else they wouldn't be chlorine), so the mass difference between them must be due to additional neutrons in the heavier isotope. The mass of the most common isotope is 35 u and that of the heavier one is 35 u + (0.0571)(35 u), so the extra mass is (0.0571)(35 u) = 2 u. Therefore the heavier isotope contains 2 more neutrons than the common isotope, so its mass number is A = 35 + 2 = 37. Using the standard notation ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$, A is the sum of the protons and neutrons (nucleons) and Z is the number of protons, which is 17 for chlorine.

EVALUATE: **(a)** Using ${}^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X}$, we see that the heavier isotope is ${}^{37}_{17}\mathrm{Cl}$.

(b) A is the sum of the protons and neutrons. Since A = 37 and Z = 17, the number of neutrons in the nucleus is $A - Z$ = 37 – 17 = 20 neutrons.

ASSESS: At the beginning of this solution, we found that the heavier isotope contains two additional nucleons than the common isotope. These cannot be protons because if the nucleus had two additional protons, it would be potassium, not chlorine.

91. **INTERPRET:** This problem is about conversion of units.

DEVELOP: The activity level is 485 Bq/L, and you need to convert it to nCi/L, using the fact that 1 Ci = 3.7×10^{10} Bq. After doing the conversion, compare the result to the allowable standard of 10 nCi/L.

EVALUATE: $$(485\ \text{Bq/L})\left(\frac{1\ \text{Ci}}{4.7\times10^{10}\ \text{Bq}}\right)\left(\frac{1\ \text{nCi}}{10^{-9}\ \text{Ci}}\right) = 13\ \text{nCi/L}.$$

This sample contains over 13 nCi/L, so it does *not* meet the safety standard.

ASSESS: To meet the standard of 10 nCi/L, it would have to have an activity in Bq of (485 Bq/L)(10/13) = 370 Bq.

92. **INTERPRET:** In this problem, we are dealing with the rate of radioactive decay.

DEVELOP: The decay rate is given by $R = R_0 2^{-t/t_{1/2}}$. If R = 1.0% of its original level in 2.0 h, we want to find the half-life $t_{1/2}$.

EVALUATE: Take natural (or base 10) logs of both sides of $R = R_0 2^{-t/t_{1/2}}$ and solve for $t_{1/2}$.

$$\ln(R/R_0) = \ln\left(2^{-t/t_{1/2}}\right) = -\frac{t}{t_{1/2}}\ln 2$$

$$t_{1/2} = -\frac{(2.0\ \text{h})\ln 2}{\ln\left(\frac{0.010R_0}{R_0}\right)} = 0.30\ \text{h} = 18\ \text{min}.$$

ASSESS: With an 18-min half-life, the sample would have to be produced (possibly in a reactor) just before it is ready to be used.

93. **INTERPRET:** This problem is about nuclear binding energy.

DEVELOP: Helium-3 has 3 nucleons in its nucleus, so its total binding energy E_b is 3(2.23 MeV) = 6.69 MeV. If we call m the mass of the helium-3 nucleus, we know that $mc^2 + E_\text{b} = Zm_\text{p}c^2 + (A - Z)m_\text{n}c^2$. From Table 38.2 we get m_p and m_n, and we know that Z = 2 and A = 3. We want to find the mass of the helium-3, m.

EVALUATE: Using $mc^2 + E_\text{b} = Zm_\text{p}c^2 + (A - Z)m_\text{n}c^2$, we have

mc^2 + 6.69 MeV = 2(938.272 MeV/c^2)c^2 + (3 – 2)(939.566 MeV/c^2)c^2

mc^2 = 2809.42 MeV.

Converting to u gives
(2809.42 MeV)(1 u/931.494 MeV) = 3.016 u.
ASSESS: The mass should be about 3 u since helium-3 contains 3 nucleons, which agrees with our result.

94. **INTERPRET:** This problem deals with the energy produced by nuclear fission and the thermal efficiency of a power plant.
DEVELOP: The purpose of this power plant is to convert heat to electrical energy. We know that the efficiency of this plant is 34%, so the electrical power output is 34% of the thermal power that comes from nuclear fission, so $P_{\rm electr} = 0.34P_{\rm fission}$. We want to know the rate at which power is extracted from fission and the electrical power produced.
EVALUATE: **(a)** There are 1.2×10^{20} fission events per second, and each one generates 200 MeV of energy, so the power from fission is
$P_{\rm fission} = (1.2\times10^{20}$ events/s)(200 MeV/event) $= 2.4\times10^{22}$ MeV/s.
Now convert this power to GW.

$$P_{\rm fission} = \left(2.4\times10^{22}\ \text{MeV/s}\right)\left(\frac{1.6\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right)\left(\frac{1\ \text{GW}}{10^9\ \text{W}}\right) = 3.8\ \text{GW}.$$

(b) $P_{\rm electr} = 0.34P_{\rm fission} = (0.34)(3.8\ \text{GW}) = 1.3\ \text{GW}$.
ASSESS: The waste power in this plant would be 66% of the power input from fission, which is (0.66)(3.8 GW) = 2.5 GW.

95. **INTERPRET:** This problem deals with the binding energy per nucleon of a nucleus.
DEVELOP: Call M the mass of the iridium-193 nucleus. Then we know that $Mc^2 + E_{\rm b} = Zm_{\rm p}c^2 + (A - Z)m_{\rm n}c^2$. For iridium-193, $Z = 77$ and $A = 193$, so the number of neutrons is $A - Z =$ 193 – 77 = 116. Table 38.2 gives the needed masses. We can solve for $E_{\rm b}$ and then divide by 193 to get the binding energy per nucleon. We are given the *atomic* mass of iridium-193 is 192.96 u, but this is the mass of the nucleus plus 77 electrons. So we must subtract the mass of those electrons to get the nuclear mass. Therefore M = 192.96 u – 77(0.000548579 u) = 192.918 u.
EVALUATE: First convert M to MeV/c^2, using the fact that 1 u = 931.494 MeV/c^2, which gives $M = 1.797017\times10^5$ MeV/c^2. Now use $Mc^2 + E_{\rm b} = Zm_{\rm p}c^2 + (A - Z)m_{\rm n}c^2$ and cancel the c^2, giving
1.797017×10^5 MeV + $E_{\rm b}$ = (77)(938.272 MeV) + (116)(939.566 MeV)
$E_{\rm b}$ = 1534.865 MeV.
The binding energy per nucleon is
(1534.865 MeV)/193 = 7.95 MeV/nucleon.
ASSESS: Comparing our result with the graph in Figure 38.9, we see that our result looks just about right for an isotope having 193 nucleons.

96. **INTERPRET:** This problem involves the radioactive decay of carbon-14.
DEVELOP: The decay rate of a radioactive isotope is given by $R = R_0 2^{-t/t_{1/2}}$. For carbon-14, the half-life is $t_{1/2}$ = 5730 y, and our sample is 12,400 y old. We want to find R/R_0.
EVALUATE: Solve $R = R_0 2^{-t/t_{1/2}}$ for R/R_0.
$R/R_0 = 2^{-t/t_{1/2}} = R/R_0 = 2^{-t/t_{1/2}} = 2^{(12{,}400\ \text{y})/(5730\ \text{y})} = 0.223.$
Therefore the rate R is 22.3% of the activity rate of a modern sample.
ASSESS: In round numbers, 12,400 y ≈ 2 half-lives, so the rate should be down by a factor of roughly $\frac{1}{2}\left(\frac{1}{2}\right) = \frac{1}{4}$ = 25%, which is quite close to our result.

97. **INTERPRET:** This problem is about radioactive decay.
DEVELOP: Both uranium isotopes decay, but with different half-lives, so their ratio will change over time. Call $N_{238,0}$ the number of uranium-238 atoms on Earth 4.54 billion years (by) ago, and $N_{235,0}$ the number of uranium-235 atoms present at that time. The number of both isotopes decreases as time goes by. They each obey the equation $N = N_0 2^{-t/t_{1/2}}$, but with different half-lives and different values of N_0. We know that 4.54 by ago, 23.8% of the uranium atoms were U-235 atoms, so 76.2% were U-238 atoms. Therefore the ratio of U-235 to U-238 atoms was

originally $N_{235,0}/N_{238,0} = 0.238/0.762 = 0.3123$. We know their half-lives are 4.46 by (U-238) and 0.704 by (U-235). We want to find the ration today and 5.0 billion years from now.

EVALUATE: The ration $R(t)$ of U-235 to U-238 atoms at any time t is

$$R(t) = \frac{N_{235,t}}{N_{238,t}} = \frac{N_{235,0}2^{-t/(0.704\text{ by})}}{N_{238,0}2^{-t/(4.46\text{ by})}} = (0.3123)2^{-t(1/0.704\text{ by} - 1/4.46\text{ by})}.$$

(a) For today we use $t = 4.54$ by.

$R(4.54\text{ by}) = (0.3123)2^{-(4.54\text{ by})(1/0.704\text{ by} - 1/4.46\text{ by})} = 0.0072394.$

Therefore $N_{235}/N_{238} = 0.0072394$ today.

The total number of uranium atoms is $N_{235} + N_{238}$, so the percent that is U-235 is

$$\frac{N_{235}}{N_{235}+N_{238}} = \frac{N_{235}/N_{238}}{N_{235}/N_{238}+1} = \frac{0.0072394}{0.0072394+1} = 0.00719 = 0.719\%.$$

(b) We follow the same approach as in part **(a)**, except we use $t = 9.54$ by.

$R(9.54\text{ by}) = (0.3123)2^{-(9.54\text{ by})(1/0.704\text{ by} - 1/4.46\text{ by})} = 0.0001146$

So $N_{235}/N_{238} = 0.0001146$ at 5.0 by from now.

The total number of uranium atoms is $N_{235} + N_{238}$, so the percent that is U-235 is

$$\frac{N_{235}}{N_{235}+N_{238}} = \frac{N_{235}/N_{238}}{N_{235}/N_{238}+1} = \frac{0.0001146}{0.0001146+1} = 0.000115 = 0.0115\%.$$

ASSESS: The ratio of U-235 atoms to U-238 atoms keeps decreasing because U-235 has a much shorter half-life than U-238.

98. **INTERPRET:** This problem is about nuclear fusion.

DEVELOP: The thermal power input in the plant comes from the D-D fusion. The two D-D fusion reactions of principal interest are ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_1\text{He} + {}^1_0n \quad (3.27\text{ MeV})$ and ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^3_2\text{He} + {}^1_1\text{H} \quad (4.03\text{ MeV})$.

The quantities in parentheses show the energy release per fusion event. We want to find the rate at which the fusion reactions produce energy.

EVALUATE: The reactions shown have nearly equal probability, so the average energy per fusion event is just the average of the two values given, which is (3.27 MeV + 4.03 MeV)/2 = 3.65 MeV. The fuel is heavy water (D_2O), which contains two deuterium nuclei per molecule, so each D_2O molecule is fuel for one fusion reaction, each of which releases 3.65 MeV on the average. The power plant is designed to use 1750 kg of heavy water per year. First we find the number of D_2O molecules in this amount of water. Each molecule has a mass of about 2(2 u) + 16 u = 20 u, and 1 u = 1.66×10^{-27} kg. So the number of D_2O molecules in 1750 kg is $(1750\text{ kg})\left(\dfrac{1\text{ molecule}}{20(1.66\times10^{-27}\text{ kg})}\right) =$ 5.27×10^{28} molecules.

The total energy released when the deuterium in these molecules fuses is

(3.65 MeV/molecule)(5.27×10^{28} molecules) = 1.924×10^{29} MeV = 3.078×10^{16} J.

This energy is released in one year, so the thermal power produced by fusion is

$$P_{\text{fusion}} = \frac{3.078\times10^{16}\text{ J}}{3.156\times10^{7}\text{ s)}} = 9.75\times10^{8}\text{ W} = 975\text{ MW}.$$

ASSESS: The thermal efficiency of such a plant could be extremely high because the plasma temperature could be around 600 MK. For an ideal (Carnot) engine with ambient air or water as the cold reservoir, the efficiency would be $e_{\text{Carnot}} = 1 - T_c/T_h = 1 - (300\text{ K})/(600\text{ MK}) = 0.99999 = 99.999\%$! There would hardly be any waste heat. Even if the efficiency were only 1% of the ideal Carnot, the efficiency would still be 99.9%.

99. **INTERPRET:** This problem involves the generation time and multiplication factor of a nuclear fission reactor.

DEVELOP: The multiplication factor k is the number of neutrons from one fission event that cause another fission event to occur. The generation time τ is the average time between fission events. The fission starts with one fission event, and after each generation time the total number N of fission events that have occurred is $N = 1 + k + k(k) + k[k(k)] + \ldots = 1 + k + k^2 + k^3 + \ldots$

After n generation times, the total number of fission events is

$N = 1 + k + k^2 + k^3 + \ldots + k^n$.

This is a geometric series, and its sum is given by $\dfrac{k^{n+1}-1}{k-1}$, so we have

$N = \dfrac{k^{n+1}-1}{k-1}$. In this case, N is the number of U-235 nuclei in 12.4 tonnes (12,400 kg) of uranium. Knowing that the mass of U-235 is nearly equal to 235 u, we can find the number of U-235 atoms. Then we can solve for n. Finally the total time for all the U-235 nuclei to undergo fission is $t = n\tau$, which is the time we want to find. We are given that τ = 685 ms and k = 1.00112 for this reactor.

EVALUATE: First find the number of U-235 nuclei in 12.4 tonnes of uranium. We know that 3.75% of the total mass of uranium is U-235, so the mass of U-235 is (0.0375)(12,400 kg) = 465 kg. The mass of each U-235 atom is essentially 235 u = $235(1.66\times10^{-27}$ kg), so the number of U-235 atoms in 465 kg is

$N = \dfrac{465 \text{ kg}}{(235)(1.66\times10^{-27}\text{ kg})} = 1.19\times10^{27}$ molecules of U-235.

Now use this value of N and solve the equation $N = \dfrac{k^{n+1}-1}{k-1}$ for n.

$1 + N(k-1) = k^{n+1}$

$\ln\left[1+n(k-1)\right] = (n+1)\ln k \quad n+1 = \dfrac{\ln\left[1+N(k-1)\right]}{\ln k} = \dfrac{\ln\left[1+(1.19\times10^{27})(1.00112-1)\right]}{\ln(1.00112)} = 49{,}630 \approx n.$

It takes 49,630 generation times to use up all the U-235. The total time is $t = n\tau$, so

t = (0.685 s)(49,630) = 33,990 s = 9.44 h.

ASSESS: With a time of 9.44 h, this event will not be an explosion. However the nuclear engineers will still need to act fast to correct the situation.

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65. **INTERPRET:** This problem involves the application of the uncertainty principle to virtual photons.
DEVELOP: Apply the uncertainty principle in terms of energy and time to the virtual photon. This is $\Delta E\Delta t \geq \hbar$, which gives $\Delta E \geq \dfrac{\hbar}{\Delta t}$. The energy cannot be know to less than ΔE, so the minimum energy the photon can have is $\Delta E = E_{\text{min}} = hc/\lambda$. Therefore $\dfrac{hc}{\lambda} = \dfrac{\hbar}{\Delta t}$. Solving for λ gives $\lambda = \dfrac{hc\Delta t}{\hbar} = 2\pi c\Delta t$. We know that $\Delta t = 0.336$ fs $= 0.336\times 10^{-15}$ s, and we want to find λ.
EVALUATE: Using $\lambda = 2\pi c\Delta t$ we have
$\lambda = 2\pi(3.00\times 10^{8}\text{ m/s})(0.336\times 10^{-15}\text{ s}) = 6.33\times 10^{-7}\text{ m} = 633\text{ nm}$.
ASSESS: If we had "virtual eyes," we could see this virtual photon.

66. **INTERPRET:** This problem involves the application of the uncertainty principle to the range of a force.
DEVELOP: In terms of energy and time, the uncertainty principle is $\Delta E\Delta t \geq \hbar$. The particle travels almost at the speed of light, so we can approximate $R = c\,\Delta t$, so $\Delta t = R/c$. Therefore $\Delta E\Delta t \geq \hbar$ can be approximated as $\Delta E\left(\dfrac{R}{c}\right) = \hbar$, so $R = \dfrac{\hbar c}{\Delta E}$. The energy cannot be know to be less than ΔE, so $\Delta E = E_{\text{min}} = mc^2$. Therefore R becomes $R = \dfrac{\hbar c}{mc^2} = \dfrac{\hbar}{mc}$.
EVALUATE: Using $R = \dfrac{\hbar}{mc}$ gives

$$R = \frac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})/2\pi}{(4.5\times 10^{-36}\text{ kg})(3.00\times 10^{8}\text{ m/s})} = 7.8\times 10^{-8}\text{ m} = 78\text{ nm}.$$

ASSESS: This range is much greater than that of the strong force, but not infinite like the electromagnetic force.

67. **INTERPRET:** This problem is about using the conservation laws during particle interactions.
DEVELOP: The proposed reaction is $\text{p} + \overline{\text{p}} \rightarrow 2\text{p} + 2\overline{\text{p}}$. It involves baryons, so we much see if it violates any of the conservation laws for baryons.
EVALUATE: We check the laws in the initial and final states.
Baryon number: $B_{\text{i}} = 1 + (-1) = 0$; $B_{\text{f}} = 2 + (-2) = 0$
Strangeness: $s_{\text{i}} = s_{\text{f}} = 0$
Lepton number: $L_{\text{i}} = L_{\text{f}} = 0$
Charge: $q_{\text{i}} = e + (-e) = 0$; $q_{\text{f}} = 2e + (-2e) = 0$
All of the above conservation laws are satisfied, so the reaction is possible, provided that the colliding protons have enough kinetic energy to create an additional proton-antiproton pair.
ASSESS: The minimum kinetic energy K_{min} the colliding protons would need is the rest energy of two protons, so $K_{\text{min}} = 2m_{\text{p}}c^2$, since the proton and antiproton have equal masses.

68. **INTERPRET:** This is a problem on conversion of units.
DEVELOP: We want to convert 20.8 km/s/Mly to units of km/s/Mpc. We have the following equalities from Appendix C: 1 pc $= 3.09\times 10^{16}$ m $= 3.09\times 10^{13}$ km and
1 ly $= 9.46\times 10^{15}$ m $= 9.46\times 10^{12}$ km.

EVALUATE: $\left(20.8\dfrac{\text{km/s}}{\text{Mly}}\right)\left(\dfrac{1\text{ ly}}{9.46\times10^{12}\text{ km}}\right)\left(\dfrac{3.09\times10^{13}\text{ km}}{1\text{ pc}}\right)=67.9\dfrac{\text{km/s}}{\text{Mpc}}.$

ASSESS: The value to use depends on whether you are measuring distances in parsecs or light-years.

69. **INTERPRET:** In this problem, we use Hubble's law.

DEVELOP: Hubble's law states that for galaxies $v = H_0 d$. At the present time, we know that $H_0 = 20.8$ km/s/Mly, and we are given that $v = 27{,}000$ km/s. We want to find the distance d of the galaxy.

EVALUATE: Using $v = H_0 d$, we have

27,000 km/s = (20.8 km/s/Mly)d

d = 1300 Mly = 1.3 Gly.

ASSESS: This distance is about 0.43 Gpc.

70. **INTERPRET:** This problem involves applying the uncertainty principle to the range of a force.

DEVELOP: In terms of energy and time, the uncertainty principle is $\Delta E \Delta t \geq \hbar$. The photon travels at the speed of light, so $R = c\,\Delta t$, which gives $\Delta t = R/c$. We can approximate using $\Delta E \Delta t = \hbar$, with $\Delta E = mc$ and $\Delta t = R/c$, which gives $m = \dfrac{\hbar}{Rc}$. We know $R = 7\times10^{19}$ m and want to find m.

EVALUATE: Using our approximation of $m = \dfrac{\hbar}{Rc}$ gives

$$m = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})/2\pi}{(7\times10^{19}\text{ m})(3.00\times10^{8}\text{ m/s})} = 5\times10^{-63}\text{ kg.}$$

The value $R = 7\times10^{19}$ m is a lower limit; R could be larger. In that case, m would be smaller, so our result for m is an *upper limit* on m.

ASSESS: The de Broglie wavelength of a photon of this mass would be about $\lambda = h/mv$ = $h/[(5\times10^{-63}\text{ kg})\,(3.0\times10^{8}\text{ m/s})] = 4\times10^{20}\text{ m} = 4\times10^{17}$ km. It would be extremely hard to localize such a photon for study!

71. **INTERPRET:** This problem is about the quark composition of a kaon K^0 particle.

DEVELOP: From Table 39.1, we see that the kaon (K^0) is a meson having strangeness $s = 1$, baryon number 0, and spin 0. We are given that one of its components is a down quark, and we want to find the other quark component(s).

EVALUATE: From Table 39.2, we see that the charge of the down quark is $-\frac{1}{3}e$. Since the K^0 is a meson, it consists of 2 quarks. Its charge is 0, so the other quark must have charge $+\frac{1}{3}e$. The K^0 has strangeness +1, and the down quark has strangeness 0, so the other quark must have strangeness +1. Therefore the properties of the other quark are: $s = +1$ and $q = +\frac{1}{3}e$. Only the antistrange quark $\overline{s}$ has these properties, so it must be the other quark component of the kaon.

ASSESS: Mesons consist of a quark and antiquark, which agrees with our result.

72. **INTERPRET:** This problem involves the head-on collision of a proton with an antiproton. The protons have speed $0.5414c$, so we must use relativistic formulas.

DEVELOP: The reaction is $\text{p} + \overline{\text{p}} \rightarrow \Lambda^0 + \overline{\Lambda}^0$. The protons each have speeds of $0.5414c$. The maximum mass that the lambda particles can have is if they are created at rest, in which case all the proton energy would go to the rest energy of the lambdas. In this case, $E_\text{p} + E_{\overline{\text{p}}} = E_\Lambda + E_{\overline{\Lambda}}$. The two protons have equal mass, as do the lambdas, so we can write the energies as $2m_\text{p}\gamma c^2 = 2m_\Lambda c^2$, which simplifies to $m_\Lambda = m_\text{p}\gamma$. We know that $m_\text{p} = 938.272\text{ MeV}/c^2$ (from Table 38.2), and we want to know what the maximum mass of the lambdas could be.

EVALUATE: Using $m_\Lambda = m_\text{p}\gamma = m_\Lambda = \dfrac{m_\text{p}}{\sqrt{1-v^2/c^2}}$ gives

$$m_\Lambda = \frac{938.272\text{ MeV}/c^2}{\sqrt{1-(0.5414c)^2/c^2}} = 1116\text{ MeV}/c^2.$$

ASSESS: The lambda mass is about 20% greater than the proton mass.

73. **INTERPRET:** In this problem, we use Hubble's law and the Doppler effect.

DEVELOP: The galaxy is receding from Earth. Hubble's law, $v = H_0 d$, gives the speed since we know that the distance is $d = 345$ Mly. After we find v, we use the Doppler effect (from Section 14.8) to find the wavelength, which gives $\lambda' = \lambda\left(1+\frac{v}{c}\right)$.

EVALUATE: First find the speed v.

$v = H_0 d = (20.8\text{ km/s/Mly})(345\text{ Mly}) = 7176\text{ km/s} = 7.176\times 10^6\text{ m/s}$.

Now use $\lambda' = \lambda\left(1+\frac{v}{c}\right)$ to find the new wavelength.

$$\lambda' = \lambda\left(1+\frac{v}{c}\right) = \lambda' = (486.1\text{ nm})\left(1+\frac{7.176\times 10^6\text{ m/s}}{3.00\times 10^8\text{ m/s}}\right) = 497.7\text{ nm.}$$

ASSESS: This galaxy is moving away from us due to the expansion of the universe, so the light is red-shifted to a longer wavelength, as we just found.

74. **INTERPRET:** This problem is about blackbody radiation.

DEVELOP: For a blackbody, the median wavelength is given by $\lambda_{\text{median}}T = 4.11\text{ mm}\cdot\text{K}$. We know the median wavelength is 1.37 μm and want to find the temperature T.

EVALUATE: Using $\lambda_{\text{median}}T = 4.11\text{ mm}\cdot\text{K}$ gives

$(1.37\times 10^{-6}\text{ m})T = 0.00411\ \text{m}\cdot\text{K}$

$T = 3000\text{ K} = 3.00\text{ kK}$.

ASSESS: This temperature is more than 1000 times the present-day temperature of 2.7 K.