Description: Students use the PhET simulation "Projectile Motion" to understand how the trajectory of an object depends on its initial velocity, and to understand how air resistance affects the trajectory.

Constants | Periodic Table

Learning Goal:

To understand how the trajectory of an object depends on its initial velocity, and to understand how air resistance affects the trajectory.

For this problem, use the PhET simulation *Projectile Motion*. This simulation allows you to fire an object from a cannon, see its trajectory, and measure its height, range, and hang time (the amount of time in the air).

Start the simulation by selecting the option labeled "Intro". Press the red **Fire** icon to launch an object. You can choose the object by clicking on one of the objects in the scroll-down menu at top right. To adjust the cannon barrel's angle, click and drag on it. You can also adjust the **Initial Speed** of the object using the toolbar at the bottom left. Clicking **Air Resistance** displays the drag coefficient for the given object and enables air resistance during motion. To measure the height, range, or hang time, drag the box that contains these quantities from the top of the screen to the various points of interest along the object's trajectory (use the crosshairs to locate the exact point of interest).

Play around with the simulation. When you are done, click the **Erase** icon and set the object to a pumpkin prior to beginning Part A. For now leave **Air Resistance** unchecked.

Part A

First, you will investigate purely vertical motion. The kinematics equation for vertical motion (ignoring air resistance) is given by

$$y(t) = y_0 + v_0 t - (1/2)gt^2,$$

where $y_0 = 0$ is the initial position, v_0 is the initial speed, and g is the acceleration due to gravity.

Drag the cannon downwards so it is at ground level, or 0 m (which represents the initial height of the object), then fire the pumpkin straight upward (at an angle of 90°) with an initial speed of 14 $\mathrm{m/s}$.

How long does it take for the pumpkin to hit the ground?

Express your answer with the appropriate units.

ANSWER:

2.85 s

Also accepted: 2.9 s

Notice that this value could be determined from the kinematics equation. Given that the initial and final height of the pumpkin is 0 m, the kinematics equation becomes $(v_0 - 0.5gt) = 0$, or $t = 2v_0/g = 2.9\ \mathrm{s}$. This calculation is interesting because it shows that, for vertical motion, the time the pumpkin is in the air is proportional to its initial speed.

Part B

When the pumpkin is shot straight upward with an initial speed of 14 $\mathrm{m/s}$, what is the maximum height above its initial location?

Express your answer with appropriate units.

View Available Hint(s) (1)

ANSWER:

9.99 m

Also accepted: 10 m

Notice that this value could be determined from the kinematics equation. Since you found it takes 2.85 s for the pumpkin to reach the ground, it must take 1.43 s to reach the maximum height, which gives $y(t = 1.43\ \mathrm{s}) = (14\ \mathrm{m/s})(1.43\ \mathrm{s}) - (1/2)(9.8\ \mathrm{m/s^2})(1.43\ \mathrm{s})^2 = 10\ \mathrm{m}$.

Part C

If the initial speed of the pumpkin is doubled, how does the maximum height change? (Note: for this part, as well as later parts, you will need to use the zoom in and out buttons to see the full trajectories)

▸ **View Available Hint(s)** (1)

ANSWER:

- ◯ The maximum height increases by a factor of 1.4 (square root of 2).
- ◯ The maximum height increases by a factor of two.
- ◉ The maximum height increases by a factor of four.

Since the amount of time it takes to reach the maximum height doubles, and since its average velocity in going upward also doubles (the average velocity is equal to half the initial velocity), the height it reaches before stopping increases by a factor of four (distance is equal to the average velocity multiplied by the time duration).

Part D

Erase all the trajectories, and fire the pumpkin vertically again with an initial speed of 14 $\mathrm{m/s}$. As you found earlier, the maximum height is 9.99 m. If the pumpkin isn't fired vertically, but at an angle less than 90°, it can reach the same maximum height if its initial speed is faster. Set the initial speed to 22 $\mathrm{m/s}$, and find the angle such that the maximum height is roughly the same. Experiment by firing the pumpkin with many different angles. What is this angle?

ANSWER:

- ◯ 35°
- ◉ 40°
- ◯ 45°
- ◯ 50°
- ◯ 55°

Notice that the initial speed in the vertical direction is given by $(22\ \mathrm{m/s})\sin(40^\circ) = 14\ \mathrm{m/s}$. The pumpkin launched at this angle reaches the same height as the vertically launched pumpkin because they have the same initial speeds in the vertical direction.

Part E

In the previous part, you found that a pumpkin fired with an initial speed of 22 $\mathrm{m/s}$ and an angle of 40° reaches the same height as a pumpkin fired vertically with an initial speed of 14 $\mathrm{m/s}$. Which pumpkin takes longer to land?

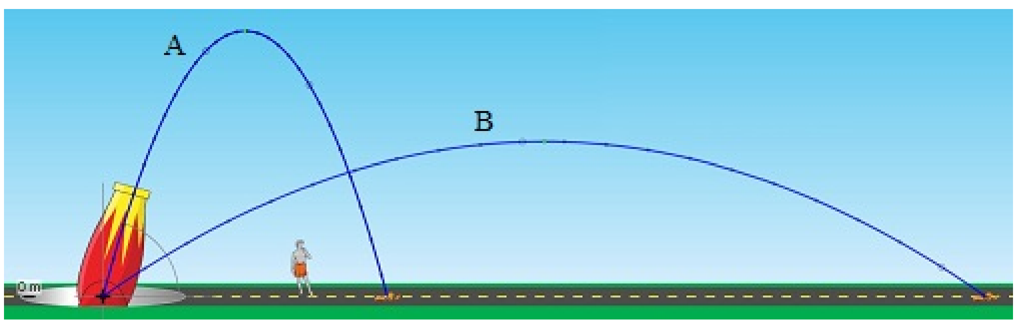
ANSWER:

- (•) Both pumpkins are in the air the same amount of time.
- ( ) The pumpkin fired vertically stays in the air longer.
- ( ) The pumpkin fired at an angle of 40° stays in the air longer.

The vertical component of the velocity determines how long the pumpkin will be in the air (and its maximum height). The horizontal component of the pumpkin's velocity does not affect this hang time.

Part F

The figure shows two trajectories, made by two pumpkins launched with different angles and possibly different initial speeds.



Based on the figure, for which trajectory was the pumpkin in the air for the greatest amount of time?

▸ **View Available Hint(s)** (1)

ANSWER:

- It's impossible to tell solely based on the figure.
- Trajectory A
- The pumpkins are in the air for the same amount of time.
- Trajectory B

All that matters is the vertical height of the trajectory, which is based on the component of the initial velocity in the vertical direction (v_0 $\sin\theta$). The higher the trajectory, the more time the pumpkin will be in the air, regardless of the pumpkin's range or horizontal velocity.

Part G

The range is the horizontal distance from the cannon when the pumpkin hits the ground. This distance is given by the product of the horizontal velocity (which is constant) and the amount of time the pumpkin is in the air (which is determined by the vertical component of the initial velocity, as you just discovered).

Set the initial speed to 14 $\mathrm{m/s}$, and fire the pumpkin several times while varying the angle between the cannon and the horizontal.

For which angle is the range a maximum (with the initial speed held constant)?

ANSWER:

- 0°
- 60°
- 30°
- 45°
- 90°

When the pumpkin is launched near a level ground, 45° is the optimum angle. If launched with a greater angle, it stays in the air longer, but its horizontal speed is slower, and it won't go as far. If launched with a smaller angle, its horizontal speed is faster, but it won't stay in the air as long and it won't go as far. The product between the horizontal speed and the amount of time in the air is largest when the angle is 45°.

Part H

How does the range of the pumpkin change if its initial velocity is tripled (keeping the angle fixed and less than 90°)?

ANSWER:

- ○ The pumpkin's range is eighteen times as far.
- ○ The pumpkin's range is three times as far.
- ◉ The pumpkin's range is nine times as far.

Since the vertical component of the velocity is three times as large, it takes three times as long to hit the ground. The horizontal component of the velocity is also tripled, and since the range is equal to the horizontal velocity times the amount of time the pumpkin is in the air, the range increases by a factor of nine. The results of this question and the previous question can be summarized by the range equation, which is

$$R = 2{v_0}^2 \sin(\theta)\cos(\theta)/g.$$

Part I

Now, let's see what happens when the cannon is high above the ground. Click on the cannon, and drag it upward as far as it goes (15 m above the ground). Set the initial velocity to 14 $\mathrm{m/s}$, and fire several pumpkins while varying the angle. For what angle is the range the greatest?

ANSWER:

- ○ 45°
- ○ 50°
- ◉ 30°
- ○ 40°
- ○ 20°

Since the cannon is very high off the ground, the pumpkin will be in the air for an appreciable amount of time even if the pumpkin is launched nearly horizontally. Thus, the amount of time the pumpkin is in the air isn't proportional to the vertical component of the initial velocity (as it was when the cannon was on the ground). This means that the initial horizontal velocity is more important, resulting in an optimal angle less than 45°. You should realize that the range equation given in Part H, $R = 2{v_0}^2\sin(\theta)\cos(\theta)/g$,is not valid when the initial height is not zero. You can also verify that, if you change the initial velocity, the optimal angle also changes!

Part J

So far in this tutorial, you have been launching a pumpkin. Let's see what happens to the trajectory if you launch something bigger and heavier, like a car. Compare the trajectory and range of the pumpkin to that of the car, using the same initial speed and angle (e.g., 45°). (Be sure that air resistance is still turned off.) Which statement is true?

ANSWER:

- ○ The trajectories differ; the range of the car is longer than that of the pumpkin.
- ◉ The trajectories and thus the range of the car and the pumpkin are identical.
- ○ The trajectories differ; the range of the car is shorter than that of the pumpkin.

Since we are ignoring air resistance, the trajectory of the object does not depend on its mass or size. In the next part, you will turn on air resistance and discover what changes.

Part K

In the previous part, you discovered that the trajectory of an object does not depend on the object's size or mass. But if you have ever seen a parachutist or a feather falling, you know this isn't really true. That is because we have been neglecting air resistance, and we will now study its effects here.

For the following parts, select the "Lab" mode of the simulation found at the bottom of the screen. Notice that you can adjust the mass and diameter of the object being launched. Turn on **Air Resistance** by checking the box. Fire a cannonball with an initial speed of 18 $\mathrm{m/s}$ and an angle of 45°. Compare the trajectory to the case without air resistance. How do the trajectories differ?

ANSWER:

- ○ The trajectory with air resistance has a longer range.
- ◉ The trajectory with air resistance has a shorter range.
- ○ The trajectories are identical.

Air resistance is a force due to the object ramming through the air molecules, and is always in the opposite direction to the object's velocity. This means the air resistance force will slow the object down, resulting in a shorter range (the simulation assumes the air is still; there is no strong tailwind).

Part L

What happens to the trajectory of the cannonball when you increase the diameter while keeping the mass constant?

ANSWER:

- ○ The size of the object doesn't affect the trajectory.
- ○ Increasing the size makes the range of the trajectory increase.
- ◉ Increasing the size makes the range of the trajectory decrease.

Since the surface area increases if the diameter increases, the object is sweeping through more air, causing more collisions, and a greater force of air drag (in fact, if the diameter is doubled, for a given speed, the force of air drag is increased by a factor of four). This greater force of air drag causes the object to slow down more quickly, resulting in a slower average speed and a shorter range.

Part M

You might think that it is never a good approximation to ignore air resistance. However, often it is. Fire the cannonball without air resistance, and then fire it with air resistance (same angle and initial speed). Then, adjust the mass of the cannonball (increase it and decrease it) and see what happens to the trajectory. Don’t change the diameter. When does the range with air resistance approach the range without air resistance?

ANSWER:

- ( ) The range with air resistance approaches the range without air resistance as the mass of the cannonball is decreased.
- ( ) It never does. Regardless of the mass, the range with air resistance is always shorter than the range without.
- (•) The range with air resistance approaches the range without air resistance as the mass of the cannonball is increased.

As the mass is increased, the force of gravity on the cannonball becomes larger. The force due to air drag just depends on the speed and the size of the object, so it doesn't change if the mass changes. As the mass gets large enough, the force of gravity becomes much larger than the air drag force in the vertical direction, and so the air drag force becomes negligible. This results in a trajectory nearly the same as when air resistance is turned off. Thus, for small, dense objects (like rocks and bowling balls), air resistance is typically unimportant, but for objects with a low density (like feathers) or a very large surface area (like parachutists), air resistance is very important.