Description: Knight Problem-Solving Strategy 4.1 Projectile Motion Problems is illustrated.

## Learning Goal:

To practice Problem-Solving Strategy 4.1 for projectile motion problems.
A rock thrown with speed $9.50 \mathrm{~m} / \mathrm{s}$ and launch angle $30.0^{\circ}$ (above the horizontal) travels a horizontal distance of $d=18.0 \mathrm{~m}$ before hitting the ground. From what height was the rock thrown? Use the value $g=9.800 \mathrm{~m} / \mathrm{s}^{2}$ for the free-fall acceleration.

PROBLEM-SOLVING STRATEGY 4.1 Projectile motion problems
MODEL: Is it reasonable to ignore air resistance? If so, use the projectile motion model.
VISUALIZE: Establish a coordinate system with the $x$-axis horizontal and the $y$-axis vertical. Define symbols and identify what the problem is trying to find. For a launch at angle $\theta$, the initial velocity components are $v_{\mathrm{i} x}=v_{0} \cos \theta$ and $v_{\mathrm{i} y}=v_{0} \sin \theta$.
solve: The acceleration is known: $a_{x}=0$ and $a_{y}=-g$. Thus, the problem becomes one of two-dimensional kinematics. The kinematic equations are

| Horizontal | Vertical |
| :--- | :--- |
| $x_{\mathrm{f}}=x_{\mathrm{i}}+v_{\mathrm{i} x} \Delta t$ | $y_{\mathrm{f}}=y_{\mathrm{i}}+v_{\mathrm{i} y} \Delta t-\frac{1}{2} g(\Delta t)^{2}$ |
| $v_{\mathrm{f} x}=v_{\mathrm{i} x}=\mathrm{constant}$ | $v_{\mathrm{f} y}=v_{\mathrm{i} y}-g \Delta t$ |,

$\Delta t$ is the same for the horizontal and vertical components of the motion. Find $\Delta t$ from one component, and then use that value for the other component.
ASSESS: Check that your result has the correct units and significant figures, is reasonable, and answers the question.

## Model

Start by making simplifying assumptions: Model the rock as a particle in free fall. You can ignore air resistance because the rock is a relatively heavy object moving relatively slowly.

## Visualize

## Part A

Which diagram represents an accurate sketch of the rock's trajectory?

- View Available Hint(s) (1)

ANSWER:


## Part B

As stated in the strategy, choose a coordinate system where the $x$ axis is horizontal and the $y$ axis is vertical. Note that in the strategy, the $y$ component of the projectile's acceleration, $a_{y}$, is taken to be negative. This implies that the positive $y$ axis is upward. Use the same convention for your $y$ axis, and take the positive $x$ axis to be to the right.

Where you choose your origin doesn't change the answer to the question, but choosing an origin can make a problem easier to solve (even if only a bit). Usually it is nice if the majority of the quantities you are given and the quantity you are trying to solve for take positive values relative to your chosen origin. Given this goal, what location for the origin of the coordinate system would make this problem easiest?

## ANSWER:

At the point where the rock strikes the groundAt the point where the rock is releasedAt ground level below the peak of the trajectoryAt the peak of the trajectory- At ground level below the point where the rock is launched

It's best to place the origin of the coordinate system at ground level below the launching point because in this way all the points of interest (the launching point and the landing point) will have positive coordinates. (Based on your experience, you know that it's generally easier to work with positive coordinates.) Keep in mind, however, that this is an arbitrary choice. The correct solution of the problem will not depend on the location of the origin of your coordinate system.

Now, define symbols representing initial and final position, velocity, and time. Your target variable is $y_{\mathrm{i}}$, the initial $y$ coordinate of the rock. Your pictorial representation should be complete now, and similar to the picture below:


Solve

## Part C

Find the height $y_{\mathrm{i}}$ from which the rock was launched.

## Express your answer in meters to three significant figures.

View Available Hint(s) (3)
ANSWER:

$$
y_{\mathrm{i}}=0.5 g\left(t_{\mathrm{air}}\right)^{2}-\left(v_{0}\right)\left(\sin \left(\theta_{l}\right)\right)\left(t_{\text {air }}\right)=13.1
$$

## Assess

## Part D

A second rock is thrown straight upward with a speed $4.750 \mathrm{~m} / \mathrm{s}$. If this rock takes 2.188 s to fall to the ground, from what height $H$ was it released?

## Express your answer in meters to three significant figures.

- View Available Hint(s) (2)

ANSWER:

$$
H=h_{f}=13.1 \quad \mathrm{~m}
$$

Projectile motion is made up of two independent motions: uniform motion at constant velocity in the horizontal direction and free-fall motion in the vertical direction. Because both rocks were thrown with the same initial vertical velocity, $v_{\mathrm{i} y}=4.750 \mathrm{~m} / \mathrm{s}$, and fell the same vertical distance of 13.1 m , they were in the air for the same amount of time. This result was expected and helps to confirm that you did the calculation in Part C correctly.

