Description: This problem explains how to find components of a vector using trigonometric functions

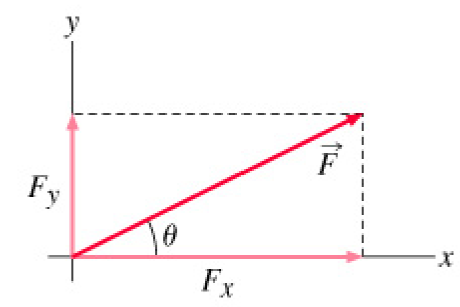
Constants | Periodic Table

Learning Goal:

To introduce you to vectors and the use of sine and cosine for a triangle when resolving components.

Vectors are an important part of the language of science, mathematics, and engineering. They are used to discuss multivariable calculus, electrical circuits with oscillating currents, stress and strain in structures and materials, and flows of atmospheres and fluids, and they have many other applications. Resolving a vector into components is a precursor to computing things with or about a vector quantity. Because position, velocity, acceleration, force, momentum, and angular momentum are all vector quantities, resolving vectors into components is *the most important skill* required in a mechanics course.

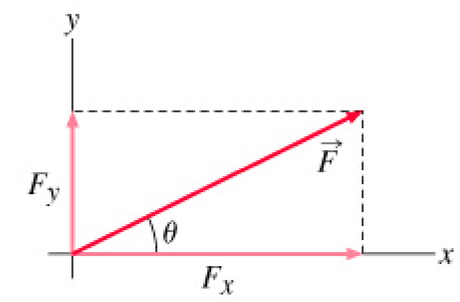
shows the components of $\vec{F}$, F_x and F_y, along the *x* and *y* axes of the coordinate system, respectively. The components of a vector depend on the coordinate system's orientation, the key being the angle between the vector and the coordinate axes, often designated θ.



Part A

shows the standard way of measuring the angle. θ is measured *to* the vector *from* the *x* axis, and counterclockwise is positive.

Express F_x and F_y in terms of the length of the vector F and the angle θ, with the components separated by a comma.



ANSWER:

F_x, F_y = $F\cos(\theta)$, $F\sin(\theta)$

Also accepted: $\left|\vec{F}\right|\cos(\theta)$, $\left|\vec{F}\right|\sin(\theta)$, $\left|\vec{F}\right|\cos(\theta)$, $F\sin(\theta)$, $F\cos(\theta)$, $\left|\vec{F}\right|\sin(\theta)$

In principle, you can determine the components of *any* vector with these expressions. If $\vec{F}$ lies in one of the other quadrants of the plane, θ will be an angle larger than 90 degrees (or $\pi/2$ in radians) and $\cos(\theta)$ and $\sin(\theta)$ will have the appropriate signs and values.

Unfortunately this way of representing $\vec{F}$, though mathematically correct, leads to equations that must be simplified using trig identities such as

$$\sin(180^\circ + \phi) = -\sin(\phi)$$

and

$$\cos(90^\circ + \phi) = -\sin(\phi).$$

These must be used to reduce all trig functions present in your equations to either $\sin(\phi)$ or $\cos(\phi)$. Unless you perform this followup step flawlessly, you will fail to recoginze that

$$\sin(180^\circ + \phi) + \cos(270^\circ - \phi) = 0,$$

and your equations will not simplify so that you can progress further toward a solution. Therefore, it is best to express all components in terms of either $\sin(\phi)$ or $\cos(\phi)$, with ϕ between 0 and 90 degrees (or 0 and $\pi/2$ in radians), and determine the signs of the trig functions by knowing in which quadrant the vector lies.

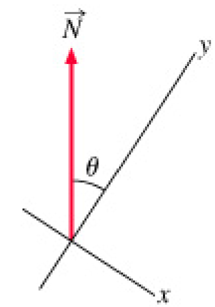
Part B

When you resolve a vector $\vec{F}$ into components, the components *must have the form* $|\vec{F}|\cos(\theta)$ or $|\vec{F}|\sin(\theta)$. The signs depend on which quadrant the vector lies in, and there will be one component with $\sin(\theta)$ and the other with $\cos(\theta)$.

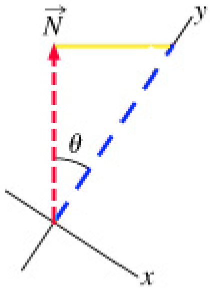
In real problems the optimal coordinate system is often rotated so that the *x* axis is not horizontal. Furthermore, most vectors will not lie in the first quadrant. To assign the sine and cosine correctly for vectors at arbitrary angles, you must figure out which angle is θ and then properly reorient the definitional triangle.

As an example, consider the vector $\vec{N}$ shown in labeled "tilted axes," where you know the angle θ between $\vec{N}$ and the *y* axis.

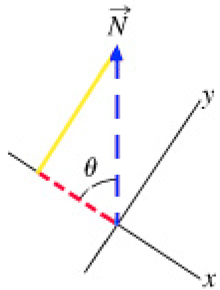
Which of the various ways of orienting the definitional triangle must be used to resolve $\vec{N}$ into components in the tilted coordinate system shown? (In the figures, the hypotenuse is blue (long dashes), the side adjacent to θ is red (short dashes), and the side opposite is yellow (solid).)



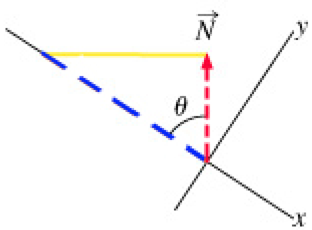
Tilted Axes



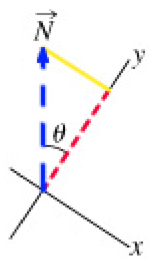
(1)



(2)



(3)



(4)

Indicate the number of the figure with the correct orientation.

▸ **View Available Hint(s)** (2)

ANSWER:

- ○ 1
- ○ 2
- ○ 3
- ◉ 4

Part C

Choose the correct procedure for determining the components of a vector in a given coordinate system from this list:

ANSWER:

- ○ Align the adjacent side of a right triangle with the vector and the hypotenuse along a coordinate direction with θ as the included angle.
- ◉ Align the hypotenuse of a right triangle with the vector and an adjacent side along a coordinate direction with θ as the included angle.
- ○ Align the opposite side of a right triangle with the vector and the hypotenuse along a coordinate direction with θ as the included angle.
- ○ Align the hypotenuse of a right triangle with the vector and the opposite side along a coordinate direction with θ as the included angle.

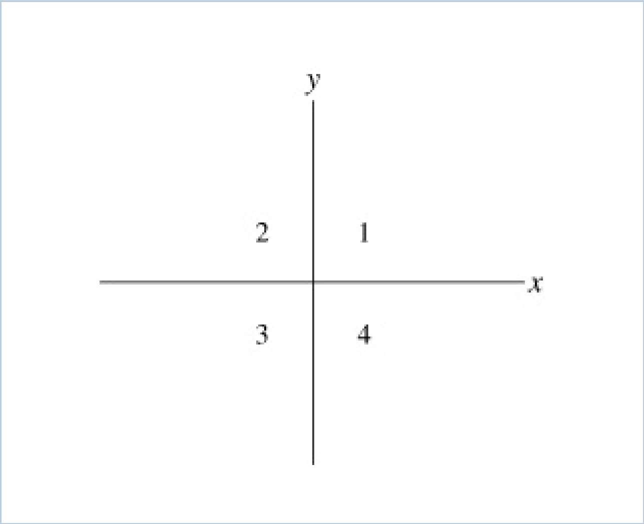
Part D

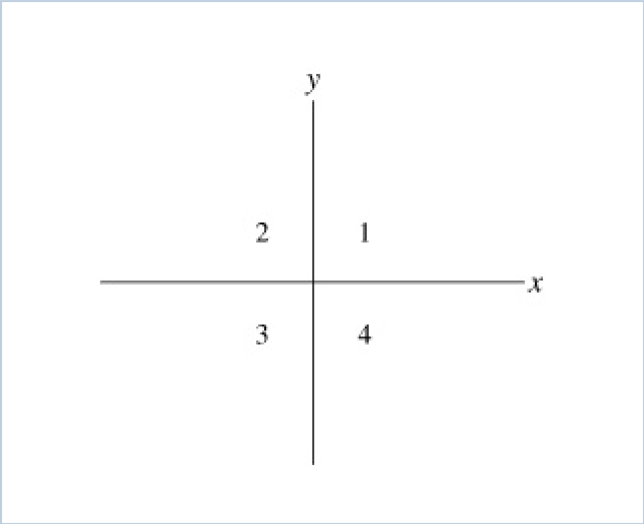
The space around a coordinate system is conventionally divided into four numbered *quadrants* depending on the signs of the *x* and *y* coordinates . Consider the following conditions:

A. $x > 0, y > 0$
B. $x > 0, y < 0$
C. $x < 0, y > 0$
D. $x < 0, y < 0$

Which of these lettered conditions are true in which the numbered quadrants shown in ?

Write the answer in the following way: If A were true in the third quadrant, B in the second, C in the first, and D in the fourth, enter "`3, 2, 1, 4`" as your response.





ANSWER:

1,4,2,3

Part E

Now find the components N_x and N_y of $\vec{N}$ in the tilted coordinate system of **Part B**.

Express your answer in terms of the length of the vector N and the angle θ, with the components separated by a comma.

ANSWER:

N_x, N_y = $-N\sin(\theta),\ N\cos(\theta)$

Also accepted: $-\left|\vec{N}\right|\sin(\theta),\ \left|\vec{N}\right|\cos(\theta),\ -\left|\vec{N}\right|\sin(\theta),\ N\cos(\theta),\ -N\sin(\theta),\ \left|\vec{N}\right|\cos(\theta)$