Description: Discuss adding vectors and illustrate with components and with geometry.

Constants | Periodic Table

Learning Goal:

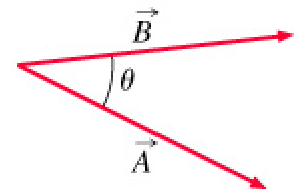
To understand how vectors may be added using geometry or by representing them with components.

Fundamentally, vectors are quantities that possess both magnitude and direction. In physics problems, it is best to think of vectors as arrows, and usually it is best to manipulate them using components. In this problem we consider the addition of two vectors using both of these methods. We will emphasize that one method is easier to conceptualize and the other is more suited to manipulations.

Consider adding the vectors $\vec{A}$ and $\vec{B}$, which have lengths A and B, respectively, and where $\vec{B}$ makes an angle θ from the direction of $\vec{A}$.

In vector notation the sum is represented by

$$\vec{C} = \vec{A} + \vec{B}.$$



Addition using geometry

Part A

Which of the following procedures will add the vectors $\vec{A}$ and $\vec{B}$?

ANSWER:

Typesetting math: 100%

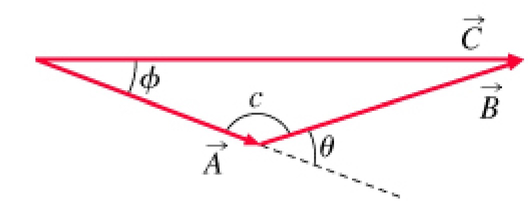
- (selected) Put the tail of $\vec{B}$ on the arrow of $\vec{A}$; $\vec{C}$ goes from the tail of $\vec{A}$ to the arrow of $\vec{B}$
- Put the tail of $\vec{A}$ on the tail of $\vec{B}$; $\vec{C}$ goes from the arrow of $\vec{B}$ to the arrow of $\vec{A}$
- Put the tail of $\vec{A}$ on the tail of $\vec{B}$; $\vec{C}$ goes from the arrow of $\vec{A}$ to the arrow of $\vec{B}$
- Calculate the magnitude as the sum of the lengths and the direction as midway between $\vec{A}$ and $\vec{B}$.

It is equally valid to put the tail of $\vec{A}$ on the arrow of $\vec{B}$; then $\vec{C}$ goes from the tail of $\vec{B}$ to the arrow of $\vec{A}$.

Part B

Find C, the length of $\vec{C}$, the sum of $\vec{A}$ and $\vec{B}$.

Express C in terms of A, B, and angle θ, using radian measure for known angles.



▸ **View Available Hint(s)** (2)

ANSWER:

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$C =$ $\sqrt{A^2 + B^2 + 2AB\cos(\theta)}$

Also accepted: $\sqrt{A^2 + B^2 - 2AB\cos(\pi - \theta)}$

Part C

Find the angle ϕ that the vector $\vec{C}$ makes with vector $\vec{A}$.

Express ϕ in terms of C and any of the quantities given in the problem introduction (A, B, and/or θ) as well as any necessary constants. Use radian measure for known angles. Use asin for arcsine

▸ **View Available Hint(s)** (1)

ANSWER:

$\phi =$ $\mathrm{asin}\left(\frac{B\sin(\pi - \theta)}{C}\right)$

Also accepted: $\mathrm{asin}\left(\frac{B\sin(\pi - \theta)}{\sqrt{A^2 + B^2 + 2AB\cos(\theta)}}\right)$, $\mathrm{acos}\left(\frac{A + B\cos(\theta)}{C}\right)$, $\mathrm{atan}\left(\frac{B\sin(\theta)}{A + B\cos(\theta)}\right)$, $\mathrm{acos}\left(\frac{A^2 + C^2 - B^2}{2AC}\right)$

Addition using vector components

Part D

To manipulate these vectors using vector components, we must first choose a coordinate system. In this case choosing means specifying the angle of the *x* axis. The *y* axis must be perpendicular to this and by convention is oriented $\pi/2$ radians counterclockwise from the *x* axis.

Indicate whether the following statement is true or false:
There is only one unique coordinate system in which vector components can be added.

ANSWER:

- ○ true
- ◉ false

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Part E

The key point is that you are *completely free* to choose any coordinate system you want in which to manipulate the vectors. It is a matter of convenience only, and so you must consider which orientation will simplify finding the components of the given vectors and interpreting the results in that coordinate system to get the required answer. Considering these factors, and knowing that you are going to be required to find the length of $\vec{C}$ and its angle with respect to $\vec{A}$, which of the following orientations simplifies the calculation the most?

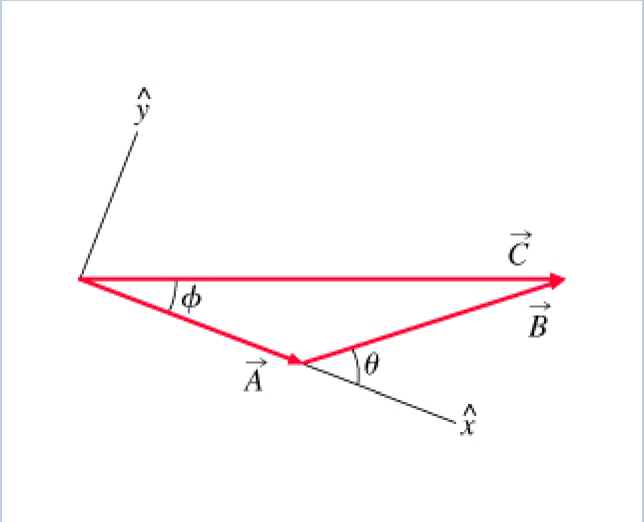
ANSWER:

- The *x* axis should be oriented along $\vec{A}$
- The *x* axis should be oriented along $\vec{B}$
- The *x* axis should be oriented along $\vec{C}$

Part F

Find the components of $\vec{B}$ in the coordinate system shown.

Express your answer as an ordered pair: `x component, y component` ; in terms of B and θ. Use radian measure for known angles.



ANSWER:

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B_x, B_y = $B\cos(\theta),\ B\sin(\theta)$

Part G

In the same coordinate system, what are the components of $\vec{C}$?

Express your answer as an ordered pair separated by a comma. Give your answer in terms of variables defined in the introduction (A, B, and θ). Use radian measure for known angles.

ANSWER:

C_x, C_y = $A + B\cos(\theta),\ B\sin(\theta)$

This should show you how easy it is to add vectors using components. Subtraction is similar except that the components must be subtracted rather than added, and this makes it important to know whether you are finding $\vec{D} = \vec{A} - \vec{B}$ or $\vec{E} = \vec{B} - \vec{A}$. (Note that $\vec{E} = -\vec{D}$.)

Although adding vectors using components is clearly the easier path, you probably have no immediate picture in your mind to go along with this procedure. Conversely, you probably think of adding vectors in the way we've drawn the figure for Part B.

This justifies the following maxim: *Think about vectors geometrically; add vectors using components.*

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